Confidence management: on interpersonal comparisons in teams *

Benoît S. Y. Crutzen, Otto H. Swank and Bauke Visser†

March 2010

Abstract

Organizations differ in the degree to which they differentiate employees by ability. We analyze how the effect of differentiation on employee morale may explain this variation. We characterize sufficient conditions for the manager to refrain from differentiation. She refrains from differentiation when employees are of similar ability, especially if absolute levels are high. Avoiding differentiation boosts the self-image of employees. To limit the negative effects of differentiation, the manager's strategy often relies on the coarsest message set possible. The likelihood that the manager differentiates depends on the presence of synergies between employees and on the convexity of the cost of effort function. Finally, we show that in the absence of commitment no differentiation is chosen too often.

Keywords: feedback, differentiation, cheap talk, comparison, morale, information disclosure

*We gratefully acknowledge feedback from the audience at presentations at ESEM 2008 meeting in Milan, the Erasmus School of Economics, European University Institute, University of Essex, the University of Groningen, and HEC Montreal. Special thanks are due to Leon Bettendorf, Pascal Courty, Robert Dur, Mike Gibbs, Maarten Goos, Marco Haan, Giuseppe Moscarini, Karl Schlag and, in particular, Vladimir Karamychev. Part of this paper was written while Bauke Visser was a Fernand Braudel Fellow at the EUI.

†Department of Economics, Erasmus School of Economics, Erasmus University Rotterdam. P.O. Box 1738, 3000 DR Rotterdam. The Netherlands. Emails: crutzen@ese.eur.nl; swank@ese.eur.nl; bvisser@ese.eur.nl.

Abstract

Organizations differ in the degree to which they differentiate employees by ability. We analyze how the effect of differentiation on employee morale may explain this variation. We characterize sufficient conditions for the manager to refrain from differentiation. She refrains from differentiation when employees are of similar ability, especially if absolute levels are high. Avoiding differentiation boosts the self-image of employees. To limit the negative effects of differentiation, the manager's strategy often relies on the coarsest message set possible. The likelihood that the manager differentiates depends on the presence of synergies between employees and on the convexity of the cost of effort function. Finally, we show that in the absence of commitment no differentiation is chosen too often.

"There's differentiation for all of us in our first 20 years. Why should it stop in the workplace?" (Jack Welch, former CEO of General Electric).

"Comparisons are odious" (Saying recorded from the mid Fifteenth century)

1 Introduction

Providing incentives through rewards or organizational choices often implies differentiation among employees on the basis of merit. In thousands of firms, employees who perform relatively well receive bonuses, are promoted, or are given challenging tasks. At the same time, numerous other firms are highly reluctant to differentiate employees on the basis of merit. Bewley (1999) interviewed more than 300 businesspeople to answer questions concerning internal pay structures. He was surprised by the extent to which "employers chose to impose bureaucratic constraints on their decision making" (Bewley, 1999, p. 75). Employers mentioned the repercussions of differentiation on internal harmony and morale as the main reason why they were reluctant to distinguish between employees on the basis of their performances.[1]

[1] Studies of internal labour markets also show that managers sometimes, but not always, eschew from differentiation on the basis of merit. In particular, performance differences sometimes exert an influence on pay differences, but the employees' position held is also very important (Doeringer and Piore, 1971, Baker, Jensen, and Murphy, 1988, Baker, Gibbs and Holmstrom, 1994a and 1994b, Gibbs, 1994). Does this mean that good performance is rewarded through promotion? Not always. Seniority and formal rule also play an important role in promotion decisions. In this context, Prendergast (1999) observes that bureaucracy is a central feature of organizations.

The observation that some firms differentiate on the basis of merit while others do not calls for an explanation. Offering an explanation for this limited use of differentiation is the main objective of the paper. Moreover, we want our explanation to be consistent with Bewley's conclusion that some firms abstain from differentiation because it may undermine morale.

We develop a cheap talk model in which a manager who runs a unit in an organization considers whether and how to use differentiation to maximize the value of production of her unit. Our model has four key features. First, the manager has got a more accurate estimate of the ability of the employees working in her unit than they have themselves. Second, ability and effort are complements in the objective function of both the manager and the employees. Third, we allow for the possibility of production synergies between the employees; we then talk about teams of employees. Finally, the manager decides what message to send *conditional* on the observed ability levels. The message should be incentive compatible (no commitment).

Our paper builds on recent work that emphasizes that employees have imperfect knowledge about their abilities and try to infer information from how they are treated by their supervisors (see Benabou and Tirole, 2003; Ishida, 2006, and Swank and Visser, 2007). The idea that people increase their self-knowledge by viewing themselves through the eyes of others is well-known to social psychologists. It is known as the "looking-glass self". Benabou and Tirole (2003), BT, formalizes this concept to show that rewards may have a hidden cost, because a reward may signal that the manager does not trust the employee. Giving a challenging task to a subordinate, by contrast, signals confidence and consequently motivates. In BT an employee wants to have an accurate perception of his ability to make a proper decision on how much effort to exert.[2] With effort and ability being complements, the more the employee is confident about his ability, the more effort he exerts.[3] In such an environment, managers want employees to believe they are as able as possible.

[2]In BT, a person wants to learn about himself to maker better decisions. Psychologists have paid much attention to a person's desire to obtain accurate self-knowledge from evaluations. One source of information are other persons' appraisals (Felson, 1993; Baumeister, 1998). A problem is that accurate feedback on abilities is rare (Jones and Wortman, 1973). Feedback tends to be too positive (Brown and Dutton, 1995).

[3]BT also pays some attention to the case in which effort and ability are substitutes.

The present paper studies the looking-glass self in a context in which agents gain self-knowledge by considering how they are being treated in comparison with other agents. That is, we study multi-agent settings. The informed manager may communicate unverifiable information about the abilities to the uninformed employees through the judicious use of some organizational practice. We focus on the manager's incentives to differentiate between her employees by means of ordinary speech - e.g., providing confidence and encouragement. An alternative interpretation of our model is that it deals with any organizational practice that can be used by an informed manager to communicate unverifiable information about the employees. One could think, for example, of a manager who may either delegate a task to the more able agent or may perform the task himself.

There are potentially two ways in which the manager's decision to delegate can affect her unit's output. Directly, for given beliefs and thus given effort levels, as when it is optimal that, say, the more able employee undertakes the more difficult task for given effort levels. Indirectly, because the specific way in which tasks are assigned induces the employees to update their beliefs about their abilities, and, as a consequence, to exert a particular level of effort rather than another. In what follows, we ignore any possible direct effect and abstract from the particular practice used.

In our model, the manager first observes the realized abilities of the employees whereas these only know the distribution of abilities. Then the manager has to decide which cheap talk message to send publicly to both employees. Depending on the message sent (and on the whole equilibrium strategy of the manager), employees may be in a position to update their prior about their own ability. Because of the complementarity between ability and effort, posterior beliefs about ability impact on the effort provision decision of the employees. In the type of cheap talk game we consider, the only credible type of message the manager can send is comparative in nature (see Chakraborty and Harbaugh 2007). Messages that contain statements about the absolute level of a single employee's ability are never believed in equilibrium.

The manager has to decide between sending two broad kinds of messages. A message of the first kind compares the two employees' abilities. This boosts the self-image of the favored employee but hurts that of the unfavored one. A message of the second kind does not compare abilities. This message guarantees that the two employees' posterior beliefs are identical (because of identical priors), even though, as we explain below, they may be different from the prior. Of course, because of the complementarity between ability and effort, if the manager chooses a comparative message, she will do so in favour of the more able employee.

Turning to our results, we characterize first sufficient conditions for the manager to abstain from differentiation with positive probability. We show that the manager avoids differentiation when employees are of similar abilities and absolute levels of abilities are high. An important implication is that avoiding differentiation boosts the self-images of the employees. This in turn encourages the manager to abstain from differentiation.

Second, we provide conditions that guarantee that the differentiation strategy is characterized by a single parameter, implying that the manager differentiates to a limited extent. If the ratio of observed abilities is lower than the value of this parameter, the manager refrains from differentiation; if the ratio exceeds the value, she does differentiate in favour of the employee with the higher ability. Had she used, say, two threshold values, she could have indicated degrees to which one employee is better than the other. The reason that the manager often avoids introducing higher degrees of differentiation is that it hardly, if at all, motivates the more able employee, but severely demotivates the other.

Third, we identify two factors mediating a manager's incentive to differentiate. We show that the existence of synergies between employees decreases the likelihood of differentiation. Furthermore, we find that the more convex the costs of effort are, the more inclined the manager is to abstain from differentiation.

Finally, we show that, if the manager could commit to a differentiation strategy before observing the realized abilities of her employees, she would refrain from differentiation less often than she does under the case of no commitment. The intuition for this result is that when the manager commits ex-ante, she takes into account the morale repercussions of her strategy for *all* possible realized abilities of her employees, and not for the observed realized abilities only.

There are various other papers that analyze the effects of disclosing information concerning ability (or performance). Ederer (2004) and Ertac (2005) focus on the effect of revealing interim information on relative ability to the contestants in a tournament.[4] Ederer devotes attention to the role played by the cost of effort function in the choice to disclose information on ability or not, while Ertac pays attention to the degree of synergy. Ray (2007) argues that firms may strategically withhold performance information to retain employees. Fang and Moscarini (2005) analyze the effect on morale stemming from wage differentiation. We adopt their individualistic approach to morale: "A worker's morale is interpreted as her confidence in her own ability" (Fang and Moscarini, 2005, pp. 750-51). An important assumption of their analysis is that in general agents are overconfident. Workers think they are more able than they really are. Because of the assumed overconfidence, a firm may refrain from differentiating. Gervais and Goldstein (2007) also analyze the effect of a worker's overconfidence about own ability on a firm's output and its workers' welfare. They show that, when there are synergies between workers, the presence of an overconfident worker can lead to an overall Pareto improvement in welfare through the impact of overconfidence on all workers' effort provision. We do not assume overconfidence. One implication of our model is that if a manager decides to refrain from differentiation employees are on average more confident about their abilities than if she were to differentiate. Thus, in our model a positive self-concept of employees is not the reason why managers do not differentiate. Rather, it is one of the implications of abstaining from differentiation.

The above papers share a number of features that make them different from ours. The most important one is the timing of decisions. Specifically, they assume that the manager commits to a disclosure policy before she observes actual ability levels. Furthermore –but this does not apply to Ray (2007)– the manager is constrained to choose between either revealing truthfully any and all difference in realized ability levels or providing no information at all. All considerations of incentive problems on the part of the manager are thus suppressed. Such considerations are central to our paper.

An important related paper is Chakraborty and Harbaugh (2007, CH for short). Translating their analysis to our setup, they show that allowing the manager to address publicly various employees makes comparative statements possible, thus widening the scope for cheap talk communication. Suppose, for example, that a

[4] Aoyagi (2003) and Lizzeri et al. (1999) study the effects of interim evaluations in tournaments if agents do not differ in ability levels.

manager supervises two employees, Peter and John, and has private information about their abilities. CH show that the manager can credibly convey information about the employees' abilities by publicly announcing that either Peter is more able than John, or John is more able than Peter. The reason that a ranking of employees contains credible information is that announcing that John is better than Peter is good news for John but bad news for Peter. In CH the manager cannot decide to publicly compare employees for some observed ability levels and avoid such comparisons for other levels. In our model we allow for this possibility. In fact, the decision to differentiate or not as part of an equilibrium is the main focus of the present paper. In CH, avoiding comparisons only exists as a pooling equilibrium.

There is mixed evidence from field (quasi-)experiments about the impact of the provision of relative performance information on workers' effort. Azmat and Iriberri (2009), Blanes i Vidal and Nossol (2009), and Delfgaauw et al. (2009) find that providing such information boosts performance while Bandiera et al. (2009) find the opposite effect.

The rest of the paper is organized as follows. The next section presents the model. Sections 3 presents three equilibria of a simple version of this model. In Section 4 we derive our main result that in a wide class of games managers abstain from differentiation with positive probability. In Section 5 we discuss two factors mediating managers' incentives to eschew differentiation. In Section 6 we show that the manager could benefit from committing to a strategy implying a higher probability of differentiations. Section 7 concludes. An appendix contains omitted proofs and omitted derivations.

2 The Model

Consider an organizational unit that is made up of an experienced manager and two employees, $i = 1, 2$. Each employee produces by exerting effort, e_i. It is the responsibility of the manager to maximize her unit's output. Output will depend on her employees' effort levels, on their ability levels a_i, and on the way the manager motivates them. We assume that the manager, based on years of experience with similar subordinates has a more accurate knowledge of her employees' abilities – contribution to the unit, really – than they have themselves. The simplest way of capturing this asymmetry is by assuming that she knows a_1 and a_2, whereas the employees only know that their abilities a_1 and a_2 are iid random variables, with continuous density functions $f(\cdot)$ on $[0,1]$, and associated distribution functions $F(\cdot)$. Let $a^e = \int_0^1 a_i f(a_i)\, da_i$ denote the prior expected ability level.

The objective function of employee i equals $V(e_i; a_i)$, with $V_e, V_a \geq 0$, $V_{ee} < 0$. The key assumption we make is that the cross-partial derivative satisfies $V_{ea} > 0$. This complementarity between effort and ability implies that $e_i^*(a_i)$, the unique value of e_i that maximizes V, is increasing in a_i. A common way of writing $V(e_i; a_i)$ is

$$V(e_i; a_i) = U(e_i; a_i) - C(e_i), \tag{1}$$

with, besides the standard assumptions on U and C, the assumption that $U_{ea} > 0$. As we will explain shortly, we think of (1) as a reduced form representation of preferences. It may actually be the case that V depends on the effort and ability of the other employee. What is important for our results is that, in equilibrium, the total derivative of e_i^* with respect to a_i is positive. An employee benefits from an accurate estimate of his own ability. As the manager is better informed about his ability then he is himself, he has an interest in deducing information about his level from whatever the manager says. This is the looking-glass self.

We assume that the manager maximizes the sum of individual outputs:

$$U_M(e_1, e_2, a_1, a_2) = a_1 e_1 + a_2 e_2 \tag{2}$$

In Section 4, we analyze a model in which the manager is also concerned about synergies between the employees.

The manager's decision whether or not to differentiate employees is modelled as follows. After the manager has observed the employees' abilities, she makes a public, cheap talk statement about these abilities.[5] Let M be the set of messages, and let $\mathcal{A} = [0,1] \times [0,1]$ be the space of abilities (a_1, a_2) of employees 1 and 2. Let m_l be the unique message the manager sends if $(a_1, a_2) \in \mathcal{A}_l$, such that $\cup_l \mathcal{A}_l = \mathcal{A}$. We call the mapping from observed abilities to messages the *differentiation strategy*

[5] Due to the fact that an employee's effort and ability are complements in the objective functions of both the manager and the employee, any cheap talk communication between the manager and a *single* employee is void.

of the manager. A manager abstains from differentiation if she sends a message m', for which $E(a_1|m') = E(a_2|m')$.

The timing is as follows. (1) Nature draws a_i, $i = 1, 2$, and reveals (a_1, a_2) to the manager, but not to the employees; (2) the manager sends a cheap talk message m_l to the employees; (3) having received the message, each employee decides how much effort to exert; (4) payoffs are realized.

To solve the game, we look for Perfect Bayesian Equilibria in pure strategies (a PBE), in which players' strategies are optimal responses to each other, given the beliefs about abilities, and beliefs are updated according to Bayes' rule wherever possible. In the present type of game babbling equilibria always exists.[6] We will ignore such equilibria.

3 Benchmark

The benchmark is characterized by two assumptions concerning preferences and abilities: the objective function of employee i can be written as $V(e_i; a_i) = a_i e_i - e_i^2/2$, and abilities are uniformly distributed on $[0, 1]$. Given the objective function of the employee, the optimal level of effort conditional on a message m_l satisfies $e_i(m_l) = E(a_i|m_l)$.

Let the manager be able to send at most three messages m_l, $l \in \{1, 2, 3\}$, where m_1 (m_3) favours employee 1 (2), and m_2 means that the manager does not differentiate. In the next section, we discuss the implications of a richer message set. We now discuss three equilibria of the benchmark model.

Equilibrium 1: Let $M = \{m_1, m_3\}$. The manager sends m_1 if $a_1 \geq a_2$, and m_3 if $a_2 > a_1$. The posteriors are $E(a_1|m_1) = E(a_2|m_3) = \frac{2}{3}$, and $E(a_1|m_3) = E(a_2|m_1) = \frac{1}{3}$. The employees choose $e_i = E(a_i|m_l)$.
Proof: Given the posteriors and $e_i(m_l) = E(a_i|m_l)$, the manager prefers sending m_1 to sending m_3 if $\frac{2}{3}a_1 + \frac{1}{3}a_2 \geq \frac{1}{3}a_1 + \frac{2}{3}a_3$, implying $a_1 \geq a_2$. Likewise, sending m_3 yields a higher payoff to the manager than sending m_1 if $a_2 > a_1$. The posteriors corresponding to m_1 and m_3 directly follow from the manager's differentiation

[6] In such an equilibrium, the manager's message does not contain information about the employees' abilities, the employees ignore the manager's message, and posterior beliefs equal prior beliefs.

strategy. ■

Equilibrium 1 is the comparative cheap talk equilibrium of Chakraborty and Harbaugh (2007) in Theorem 1.

Equilibrium 2: Let $M = \{m_1, m_2, m_3\}$. The manager sends m_1 if $a_1 > a_2$, m_2 if $a_1 = a_2$ and m_3 if $a_2 > a_1$. The posteriors are $E(a_1|m_1) = E(a_2|m_3) = \frac{2}{3}$, $E(a_1|m_3) = E(a_2|m_1) = \frac{1}{3}$ and $E(a_1|m_2) = E(a_2|m_2) = \frac{1}{2}$. The employees choose $e_i = E(a_i|m_l)$.

Proof: The proof is as the proof of Equilibrium 1. Given the posteriors and employees strategies, it is a best-reply for the manager to send m_2 if $a_1 = a_2$.■

As $a_1 = a_2$ is a zero probability event, the observed outcomes of equilibrium 1 and 2 are the same, see Figure 1, panel a. However, the nature of these equilibria is different: in Equilibrium 2 m_2 is part of the message set of the manager whereas it is not in Equilibrium 1.

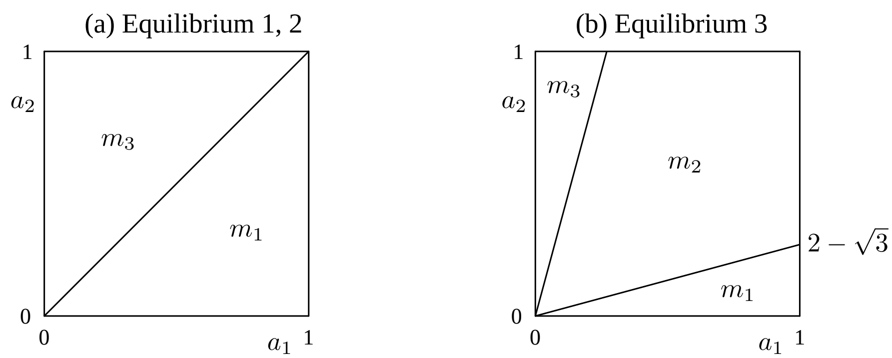

Figure 1: Equilibria in the benchmark case.

Equilibrium 3: Let $M = \{m_1, m_2, m_3\}$. The manager sends m_1 if $a_1 \geq \frac{1}{t}a_2$, m_2 if $a_1 \in \left(\frac{1}{t}a_2, ta_2\right)$, and m_3 if $a_1 < ta_2$, where $t = 2 - \sqrt{3} \approx 0.27$. The posteriors are $E(a_1|m_1) = E(a_2|m_3) = \frac{2}{3}$, $E(a_1|m_3) = E(a_2|m_1) = \frac{t}{3}$ and $E(a_1|m_2) = E(a_2|m_2) = \frac{1}{2} + \frac{t}{6}$. The employees choose $e_i = E(a_i|m_l)$.

Proof: Derivations of the posteriors can be found in the Appendix. The manager is indifferent between sending m_1 and m_2 if and only if

$$\begin{aligned} a_1 E(a_1|m_1) + a_2 E(a_2|m_1) &= a_1 E(a_1|m_2) + a_2 E(a_2|m_2) \text{ or} \\ a_2 &= t a_1 \text{ with } t = \frac{E(a_1|m_1) - E(a_1|m_2)}{E(a_2|m_2) - E(a_2|m_1)}. \end{aligned} \quad (3)$$

Substituting the posteriors into (3), and solving for t yields $t = 2 - \sqrt{3}$. Hence, the locus of pairs (a_1, a_2) for which the manager is indifferent between sending m_1 and m_2 is given by the line $a_2 = \left(2 - \sqrt{3}\right) a_1$. Because of symmetry, the line $a_2 = \frac{1}{2-\sqrt{3}} a_1$ describes the locus of pairs (a_1, a_2) for which the manager is indifferent between sending m_2 and m_3.■

Figure 1 panel b depicts the differentiation strategy the manager follows in Equilibrium 3. The figure demonstrates that the manager abstains from differentiation with a high probability, $\Pr(m_2) = (1 - t) \approx 0.73$. In this respect, Equilibrium 3 is very different from Equilibrium 1 and 2. An important feature of Equilibrium 3 is that abstaining from differentiation boosts employees self-images: $E(a_i|m_2)$ exceeds the prior value of a_i. The reason is that the manager abstains from differentiation in particular for relatively high values of observed abilities. This in turn implies that m_2 indicates relatively high abilities, and employees' self-images improve accordingly.[7]

The existence of multiple equilibria raises the question of which equilibrium is the more plausible one. The principle difference between Equilibrium 1 on the one hand and Equilibrium 2 and 3 on the other, is the possibility to abstain from differentiation (m_2). If the manager is in a situation where differentiation is the rule, as in tournaments, Equilibrium 1 is the more plausible outcome. If, by contrast, abstaining from differentiation is a natural option, Equilibria 2 and 3 seem more plausible. An appealing feature of Equilibrium 3 is that it can explain why managers abstain from differentiation (see introduction). Moreover, this explanation is consistent with Bewley in that differentiation is in expected terms bad for morale. In Section 5, we will discuss two extensions of the basic model. These extensions point to another appealing feature of Equilibrium 3 relative to Equilibrium 2. Equilibrium 3 is robust to small changes in the assumptions. For instance, once we allow for (very small) synergies between the employees, an equilibrium like Equilibrium 2 does not exist. Equilibrium 3, by contrast, survives.

[7] CH use the term ranking or categorization. We here apply their terminology to Equilibria 1-3. In Equilibrium 1, the manager uses two ranks or categories: the employee with the higher ability is put in the higher category, and the employee with the lower rank in the lower category. With two employees this gives rise to two possible messages, m_1 and m_3. Note that the same number of ranks is used for all observed ability levels. In Equilibria 2 and 3, for some ability levels the manager uses two ranks, and for others she uses one rank. In the latter case, the employees are ranked the same. As a result, there are three possible messages, m_1, m_2, and m_3.

4 Equilibria in the More General Model

In the previous section, we have shown that in a simple version of the model of Section 2 an equilibrium exists in which the manager avoids differentiation with a high probability. In this section, we investigate whether in the more general model presented in Section 2 such an equilibrium survives. So, in comparison with the model of the previous section we relax the assumption that $f(x) = 1$, and that the employees' payoff functions are linear-quadratic in effort. We derive two main results. First, we show that for a class of density functions, the manager uses in equilibrium at most three words to describe employees' abilities. Our first finding provides a sufficient condition such that managers differentiate to a limited extent. Second, we derive sufficient conditions for the existence of an equilibrium in which the manager avoids differentiation with positive probability. Our second finding is consistent with observations discussed in the introduction that managers are reluctant to differentiate.

Let us return to the general model as specified in Section 2. For an equilibrium message m_l, let $f(a_i|m_l)$ denote the density of a_i conditional on this message. The employee uses this updated belief to determine his optimal effort level,

$$e_i(m_l) = \arg\max_{e_i} \int_0^1 V(e_i; a_i) f(a_i|m_l)\, da_i. \tag{4}$$

Consider any two messages m' and m'' that are used in equilibrium. As the objective function (2) of the manager is increasing in both effort levels, it must be the case that if, say, $e_1(m') > e_1(m'')$, then $e_2(m') < e_2(m'')$ as otherwise the manager would never send m'' in equilibrium: message m' would lead to more effort for 1 and at least as much effort for 2. This means that in any equilibrium in which n messages are being used, the following is true:

Lemma 1 *In any equilibrium with $n \geq 2$ messages, the messages can be ordered such that $e_1(m_1) > e_1(m_2) > \cdots > e_1(m_n)$ and $e_2(m_1) < e_2(m_2) < \cdots < e_2(m_n)$.*

For $(a_1, a_2) = (a, 0)$, with $a > 0$, the manager wants employee 1 to exert the highest level of effort possible and does not care about the effort of employee 2. On the other hand, for $(a_1, a_2) = (0, a)$ with $a > 0$, the reverse holds. This means that there exists an equilibrium in which the manager uses at least two messages. In such an equilibrium, the a_1-axis is a subset of $\mathcal{A}_1$, the subspace inducing the highest effort level for employee 1 (and the lowest for 2), whereas the a_2-axis is a subset of $\mathcal{A}_n$, the subspace inducing the highest effort level for employee 1 (and the lowest for 2). We have proved the following Lemma.

Lemma 2 *For the class of games described in Section 2, a non-babbling PBE with some degree of differentiation on the basis of ability exists. In this equilibrium, when n messages are used, the pairs $(a_1, a_2) = (a, 0)$, with $a > 0$, lead to message m_1 (the message inducing 1 to exert the highest level of effort); the pairs $(a_1, a_2) = (0, a)$ with $a > 0$ lead to message m_n (the message inducing 2 to exert the highest level of effort).*

As $e_1(m_l) \geq e_1(m_{l+1})$ for $l = 1, \ldots, n-1$, there are restrictions that should hold on the subspaces $\mathcal{A}_l$ for which messages are sent. With effort and ability complements in the objective function of the employee, these subspaces should be such that $\mathcal{A}_l$ contains "more favorable" information about a_1 than $\mathcal{A}_{l+1}$, and "less favorable" information about a_2. Hence, one can say that message 1 (and n) differentiate the employees more than message 2 (and $n-1$) etc. In case of an even number of messages, the messages that differentiate the least are messages $\frac{n}{2}$ and $\frac{n}{2}+1$. Thanks to the symmetry of the model, these messages are the ones closest to the main diagonal in the (a_1, a_2) space. With an even number of messages, differentiation cannot be avoided. In case of an odd number of messages, the manager can avoid comparisons by using message $(n+1)/2$. Of course, ability pairs on the main diagonal are part of the set for which this message is sent. In this case, both employees hold the same posterior beliefs. Recall from Equilibrium 3 in Section 3 that the posterior may be different from the prior.

Is there a limit to the degree of differentiation the manager wants to use in equilibrium? The next Proposition gives a condition that is sufficient to guarantee that three is the maximum number of messages.

Proposition 1 *Suppose that abilities have a power function distribution, $f(a_i; d) = da_i^{d-1}$ on $[0, 1]$ for some $d > 0$. Then, the maximum number of messages the manager uses in equilibrium equals three.*

Proof: Suppose the manager uses $n > 3$ messages. The manager is indifferent between sending m_{l-1} and sending m_l (where $l \leq \frac{1}{2}(n+1)$) if

$$\begin{aligned} a_1 e_1 (m_{l-1}) + a_2 e_2 (m_{l-1}) &= a_1 e_1 (m_l) + a_2 e_2 (m_l) \text{ or} \\ a_1 &= \frac{1}{\beta_l} a_2 \text{ with } \frac{1}{\beta_l} = \frac{e_2 (m_l) - e_2 (m_{l-1})}{e_1 (m_{l-1}) - e_1 (m_l)}. \end{aligned} \quad (5)$$

It follows that the area $\mathcal{A}_l$ for which m_l is sent is a two-dimensional cone through the origin: $\mathcal{A}_l = \{(a_1, a_2) : a_1 \in [0,1], a_2 \in [\beta_{l-1} a_1, \beta_l a_1]\}$ with $0 \leq \beta_{l-1} < \beta_l \leq 1$. Then,

$$\mathsf{E}\left[a_1 | a_2 \in [\beta_{l-1} a_1, \beta_l a_1]\right] = \frac{\int_0^1 \int_{\beta_{l-1} a_1}^{\beta_l a_1} a_1 f(a_1) f(a_2)\, da_2 da_1}{\int_0^1 \int_{\beta_{l-1} a_1}^{\beta_l a_1} f(a_1) f(a_2)\, da_2 da_1} = \frac{2d}{2d+1},$$

where the last equality holds as $f(a_i; d) = d a_i^{d-1}$. In particular, it is independent of β_{l-1} and β_l. This means that any message that is sent for values of (a_1, a_2) below the diagonal induces employee 1 to exert the same level of effort: $e_1(m_1) = e_1(m_2) = \cdots = e_1(m_n)$. This conflicts with Lemma 1, which states that in any equilibrium with $n \geq 2$, we have that $e_1(m_1) > e_1(m_2) > \cdots > e_1(m_n)$.■

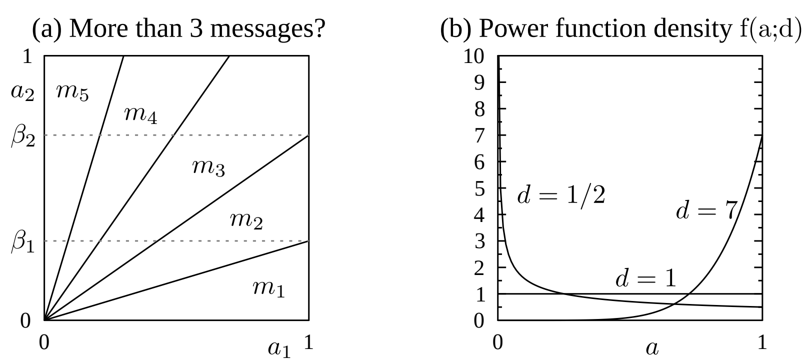

Figure 2: Panel (a): more than three messages hurts morale of worse agent and may leave self-esteem of better agent unaffected. Panel (b): Power function densities for various values of d.

With the help of Figure 2, panel a, one can grasp the intuition behind Proposition 1. It depicts a situation where the manager wants to use five messages. Notice that sending m_1 rather than m_2 contains much information about a_2: m_1 shows that $a_2 < \beta_1$ while m_2 shows that $a_2 < \beta_2$. For the uniform distribution, we have that

$E(a_2|m_1) = \beta_1/3 < E(a_2|m_2) = (\beta_1+\beta_2)/3$. Sending m_1 rather than m_2 thus demotivates employee 2 ($\beta_1 < \beta_1 + \beta_2$). By contrast, the effect of sending m_1 rather than m_2 on the expected value of a_1 is less clear. Irrespective of whether the manager sends m_1 or m_2, a_1 can lie in the range $[0,1]$. In fact, if $f(a_i;d) = da_i^{d-1}$, then $E(a_1|m_1) = E(a_1|m_2)$. Hence, if $f(a_i;d) = da_i^{d-1}$, sending m_1 rather than m_2 discourages employee 2, but does not encourage employee 1. The manager is thus better off by reducing the number of messages used.

Note that the power function densisty is decreasing for $d \in (0,1)$, equals the uniform for $d = 1$, and is increasing for $d > 1$, see panel (b) of Figure 2.

We now provide sufficient conditions such that an equilibrium exists in which the manager avoids differentiation with positive probability.

Proposition 2 *Suppose $n = 3$. An equilibrium exists in which the manager avoids differentiation with strictly positive probability if*
(i) $af(a)$ is non-decreasing, or $f(a)$ is symmetric; and
(ii) $V_{eee} \leq 0$ and $V_{eea} \leq 0$.

The proof can be found in the Appendix. Recall that if the manager avoids differentiation, it follows from the incentive compatibility constraint (5) that she does so in particular for high values of (a_1, a_2). Condition (i) guarantees that such values are sufficiently likely relative to low values of (a_1, a_2) such that not differentiating indeed boosts an employee's self-image relative to what it is a priori. As a result, $e_1(m_2) > e^p$, where e^p is the optimal effort level based on the prior distribution, and $e_1(m_2)$ the effort level in case the manager refrains from differentiation. Condition (ii) guarantees that in equilibrium $e_1(m_1) + e_2(m_1) \leq 2e^p$. Together with $e_1(m_2) > e^p$, it then follows that $e_1(m_1) + e_2(m_1) < 2e_1(m_2)$. As a result, the manager does not want to differentiate if employees are of comparable ability level.

Note that in case V is written as $V(e_i;a_i) = U(e_i;a_i) - C(e_i)$, then $V_{eee} \leq 0$ means that not only the costs are convex, but so are the *marginal* costs C' (i.e., $C'' > 0$ by $V_{ee} < 0$ and $C''' \geq 0$ by $V_{eee} \leq 0$). This condition plays an important role in the literature on tournaments with interim performance feedback (see Ederer 2004). In that literature if this condition holds, then a principal who can choose whether or not the commit to credibly revealing any and all differences between employees decides not to reveal any information.

5 Factors Mediating the Incentive to Differentiate

In the previous section, we have presented a fairly general model that can explain why managers are reluctant to differentiate employees by ability. In this section, we identify two factors that mediate the manager's incentives to differentiate. To this end, we extend the benchmark model of Section 3 in two directions. First, we allow for a more general cost of effort function $C(e_i)$. We show that the shape of the cost function influences the probability with which the manager differentiates. Second, we analyze a situation where there are synergies between employees. We show that such synergies discourage the manager to differentiate. In this section, we focus on the equilibrium in which the manager can both differentiate and abstain from it. However, in both games an equilibrium *à la* Chakraborty and Harbaugh (2007) exists. In such an equilibrium, the manager differentiates with probability one (like Equilibrium 1 in Section 3).

5.1 Convexity of the Cost of Effort Function

Consider the benchmark model, but write the employee's objective function as

$$V(e_i; a_i) = a_i e_i - \frac{1}{n}(e_i)^n. \tag{6}$$

The family of cost functions $C(e_i; n) = \frac{1}{n}(e_i)^n$ is parametrized by $n \in \mathbb{R}$, with $n > 1$. Within this family, n can be interpreted as a degree of convexity. The higher is n, the more convex the cost function is. It follows from (6) that

$$e_i^* = (\mathsf{E}(a_i|m))^{\frac{1}{n-1}}. \tag{7}$$

The quadratic case, $n = 2$, was discussed in Section 3.

Given (7), the manager is indifferent between sending m_1 and m_2 if $a_1 e_1(m_1) + a_2 e_2(m_1) = a_1 e_1(m_2) + a_2 e_2(m_2)$ or for

$$a_1 = \frac{1}{\beta_2} a_2 \text{ with } \frac{1}{\beta_2} = \frac{e_2(m_2) - e_2(m_1)}{e_1(m_1) - e_1(m_2)} \tag{8}$$

Using (7) and the fact that $E(a_1|m_1) = \frac{2}{3}$, $E(a_2|m_1) = \frac{\beta_2}{3}$, and $E(a_1|m_2) = E(a_2|m_2) = \frac{1}{2} + \frac{\beta_2}{6}$, β_2 can be written as

$$\beta_2 = \frac{\left(\frac{2}{3}\right)^{\frac{1}{n-1}} - \left(\frac{1}{2} + \frac{\beta_2}{6}\right)^{\frac{1}{n-1}}}{\left(\frac{1}{2} + \frac{\beta_2}{6}\right)^{\frac{1}{n-1}} - \left(\frac{\beta_2}{3}\right)^{\frac{1}{n-1}}}, \tag{9}$$

Equation (9) implicitly defines β_2. Using (9), we have calculated the probability that the manager abstains from differentiation (i.e., $\Pr(m_2) = (1 - \beta_2)$) as a function of n. The dotted line in Figure 3 depicts the results.

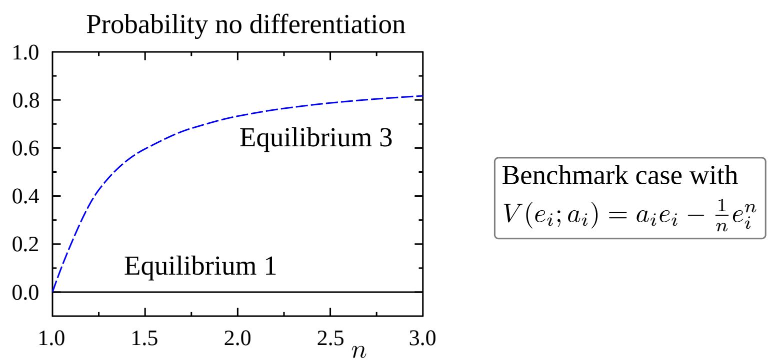

Figure 3: The probability with which the manager avoids differentiation as a function of the degree n of convexity of the employee's cost function in Equilibria 1 and 3.

Clearly, the more convex are the costs, the larger is the area for which the manager avoids differentiation. The intuition behind this result is straightforward. The advantage of differentiating is a higher effort level of one employee. The drawback is a lower effort level of the other employee. The more convex are the costs, the smaller is the advantage relative to the drawback.

5.2 Synergies between Employees

In this section, we extend the benchmark model by allowing for synergies between the employees. Specifically, we assume that the manager's payoff function is given by

$$U_M(e_1, e_2, a_1, a_2) = a_1 e_1 + a_2 e_2 + k a_1 e_1 a_2 e_2, \tag{10}$$

where $k \geq 0$ represents the strength of the synergy between the two employees. As an example of a situation the present model describes, think of the manager as being responsible for the development of a new car. This development requires the specialist contributions of employees, say, a paint expert, a body expert, and an engine expert. Each specialist cares about or is responsible for his own, identifiable, contribution, whereas the manager is responsible for the end product. The whole may be more than the sum of its parts, and the parameter k captures this difference.[8] The main result of this section is that synergies reinforce the manager's incentives to refrain from differentiating between employees when they are of similar (enough) ability, and that these incentives are strongest for very able employees.

Suppose the manager observes (a_1, a_2). If the manager sends m_1, she obtains $a_1 e_1(m_1) + a_2 e_2(m_1) + k a_1 a_2 e_1(m_1) e_2(m_1)$, while she gets $a_1 e_1(m_2) + a_2 e_2(m_2) + k a_1 a_2 e_1(m_2) e_2(m_2)$ if she sends m_2. The manager prefers to differentiate rather than to avoid it if and only if

$$a_2 < h_-(a_1; k) := a_1 \frac{e_1(m_1) - e_1(m_2)}{e_2(m_2) - e_2(m_1) + a_1 k \left[e_1(m_2) e_2(m_2) - e_1(m_1) e_2(m_1)\right]}.$$

Indeed, the equilibrium differentiation strategy of the manager when $k > 0$ becomes:

$$D = \begin{cases} m_1 & \text{if } a_2 < h_-(a_1; k) \\ m_2 & \text{if } a_2 \in [h_-(a_1; k), h_+(a_1; k)] \\ m_3 & \text{if } a_2 > h_+(a_1; k), \end{cases}$$

where h_+ is the inverse of h_-. It is easy to check that $h_-(0; k) = 0$, and that h_- and h_+ are increasing in a_1.[9]

Panel (a) in Figure 4 illustrates the two key features of the differentiation strategy. First, the more a_1 deviates from a_2, the more inclined is the manager to differentiate. Second, the more able both employees are, the more the manager tends to eschew comparisons. Both features highlight that the manager faces a dilemma when deciding whether or not to differentiate. On the one hand, a manager wants to differentiate in order to properly match effort and ability. This relates to the first feature. On the other hand, the manager wants to exploit synergy between the employees. This gives an incentive to the manager to abstain from differentiation.

[8] For simplicity, we assume that the synergy term does not enter into the payoff functions of the employees. However, we can show that adding those terms would not alter our main results.

[9] See the Appendix.

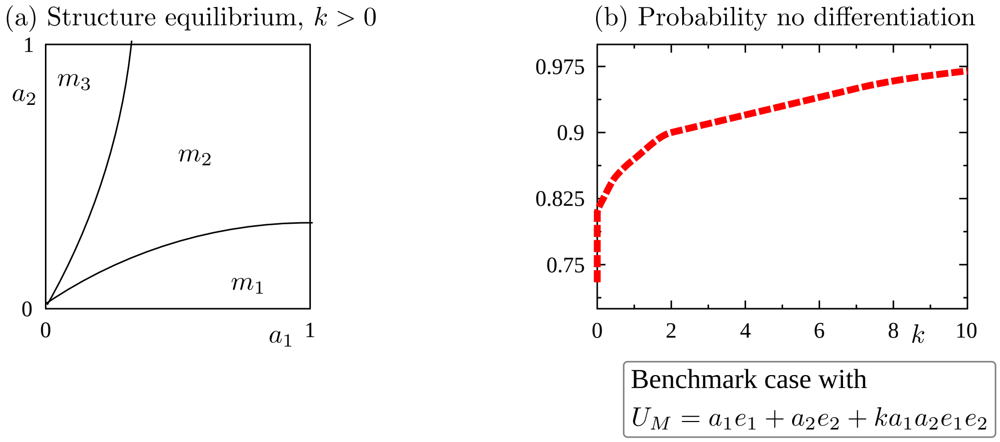

Figure 4: Panel (a) shows the structure of equilibrium in case of synergy. Panel (b) shows that the probability with which the manager avoids differentiation increases steeply for low values of k. For $k = 0.001$, the probability equals 0.81.

As the synergetic value increases in a_1 and a_2, higher values of a_1 and a_2 weaken the manager's incentive to differentiate. This explains the concavity of $h_-(a_1; k)$ and relates to the second feature of panel a.

This panel is also helpful in understanding how the manager's differentiation strategy affects the two employees' beliefs about their abilities. Since the manager abstains from differentiation for high values of a_1 and a_2, not differentiating, rather than leaving an employee's belief about his ability unaffected, *boosts* employees' confidence in their abilities. The flip side of the coin is that m_1 severely damages employee 2's perception of his ability.

The exact location of $h_-(a_1; k)$, and therefore of $h_+(a_1; k)$, depends on k. The higher is k, the more important is synergy between the employees for the manager, and the weaker is the incentive to differentiate. Panel b in Figure 4 shows the likelihood that the manager refrains from differentiation as a function of k.[10] As k tends to zero, this likelihood tends to $2 - \sqrt{3}$, the probability that we obtained in Equilibrium in Section 3, and $h_-(a_1; k)$ becomes a straight line.

Let us finish this section with a remark on the robustness of Equilibrium 2 in Section 3. The above analysis shows that when k is infinitesimally small, the equilibrium of the model with synergies becomes like Equilibrium 3 of the benchmark model, and not like Equilibrium 2. Likewise, in the model of the more general cost

[10] We have to rely on numerical simulations as it is not possible to obtain an analytical solution for the equilibrium of this game.

of effort function, if n is just higher than 2, the equilibrium is similar to Equilibrium 3, not Equilibrium 2. From this point of view, Equilibrium 2 of the benchmark model is not robust to small changes in the assumptions.

6 Differentiation under commitment

So far, we have assumed that the manager cannot commit to a message strategy. In this section, we return to the benchmark case of Section 3, and show the importance of the presence or absence of commitment to a specific strategy for the likelihood of differentiation.

Consider the benchmark situation, and the differentiation strategy m_1 if $a_1 \geq \frac{1}{t}a_2$; m_2 if $a_1 \in \left(\frac{1}{t}a_2, ta_2\right)$; and m_3 if $a_1 \leq ta_2$, where $t \in (0,1]$. Assume that the manager can commit to a value of t. The manager can then ignore incentive compatibility considerations. This value is simply chosen to maximize the manager's expected payoff $W(t)$,

$$
\begin{aligned}
W(t) &= 2\int_0^1 \int_0^{ta_1} \left(\frac{2}{3}a_1 + \frac{1}{3}ta_2\right) da_2 da_1 + \\
&\quad \int_0^t \int_{ta_1}^{\frac{a_1}{t}} \left(\frac{1}{2} + \frac{1}{6}t\right)(a_1 + a_2)\, da_2 da_1 + \int_t^1 \int_{ta_1}^1 \left(\frac{1}{2} + \frac{1}{6}t\right)(a_1 + a_2)\, da_2 da_1 \\
&= \frac{1}{18}t^3 - \frac{5}{18}t^2 + \frac{5}{18}t + \frac{1}{2}. \qquad (11)
\end{aligned}
$$

This expression is maximized for $t^C = \frac{5-\sqrt{10}}{3} \approx 0.61$, which is larger than $t^{NC} = 2 - \sqrt{3} \approx 0.27$, the equilibrium value in the absence of commitment. It is straightforward to check that the manager benefits from a commitment to $t = t^C$, $W\left(t^C\right) > W\left(2 - \sqrt{3}\right)$. The reason is that when the manager decides *ex post* she only considers whether differentiation or no differentiation yields more for a *given* pair of observed ability levels and *given* the inferences that the employees draw from the messages. If the manager can commit *ex ante* to a differentiation strategy, she takes into account what the repercussions are for the self-images of the employees of a choice to differentiate or not. Moreover, she considers the effect of this change of self-image not only for a given pair of ability levels but for all possible pairs for which she contemplates sending a particular message. In particular, the larger is the area around the 45° degree line for which the manager does not differentiate, the lower is the boost in self-image in case of no differentiation for all ability pairs in that area. Hence, the area in which the manager refrains from differentiation in case of $t = t^{NC}$ (see Equilibrium 3 in Section 3) is too large from an ex ante point of view. Hence, $t^C > t^{NC}$. Similarly, the fact that the manager differentiates for all pairs (a_1, a_2) in case of $t = 1$ (see Equilibria 1 and 2 in Section 3) means that there is too much differentiation from an ex ante point of view, $t^C < 1$.

In the benchmark model, the manager can send at most three messages. Proposition 1 shows that when the manager cannot commit himself, this assumption is not restrictive for a wide class of situations. Things are completely different when the manager can commit himself. One can verify that by committing to always revealing the employees' abilities the manager receives the highest expected payoff ($2\int_0^1 a_i^2 da_1 = \frac{2}{3} > W\left(t^C\right)$). This implies that rather than three messages, the manager wants to use infinite messages.

The next proposition summarizes these findings.

Proposition 3 *If the manager can commit to a differentiation strategy in the benchmark model, then* $t^C = \frac{5-\sqrt{10}}{3}$. *Relative to the case without commitment, commitment increases the probability of differentiation. Moreover, if we relax the assumption of three messages, the manager would use the largest possible message set when he can commit to a differentiation strategy.*

7 Conclusion

This paper studies the pros and cons of differentiating employees by ability. The model developed here focuses on situations in which the effort an employee exerts depends positively on his perception of his ability. A key aspect of our model is that inter-personal comparisons lead to higher effort levels by the more able, but to lower effort levels by the less able. We identify three features of the environment that may affect an employer's decision whether or not to differentiate on the basis of ability: realized abilities, synergies between the employees, and the convexity of the cost of effort function. Our findings provide a theoretical rationale for the reluctance of managers to differentiate employees because of morale management considerations, as documented by Bewley (1999).

A higher degree of synergies weakens the incentive for the employer to differentiate. One implication of this result is that employers are reluctant to differentiate when total performance depends on the "weakest link" in the team. In such a situation, the benefit of boosting the morale of the more able is unlikely to exceed the cost of undermining the morale of the weakest link. A higher degree of convexity of the cost of effort function (more precisely, the presence of convex marginal costs) also reduces the employer's inclination to differentiate.

One finding of the paper is that the more the realized abilities of the employees differ, the more the employer is inclined to differentiate. We have argued that the driving force behind this result is that differentiating leads to a better matching of abilities and effort levels. Notice that the nature of this last feature deviates from the nature of the first two. The first two may help us to explain why differentiation varies across different types of organizations. Realized abilities are important for understanding variation of differentiation for a given type of organization.

By introducing the looking-glass self concept into principal-agent models, we have highlighted that incentive mechanisms often have direct and indirect effects. A tournament is the prototype of an incentive mechanism leading to differentiation. Until not so long ago, the focus of studies on tournament theory was on the direct effects of tournaments, that is on the effects on effort exerted by agents before the prize is given. Our analysis suggests that tournaments also have indirect effects, that is, incentive effects after the prize is given. The reason is that the outcome of a tournament provides information about the agents' abilities: after a tournament the winner is more motivated than the losers. More specifically, if in our model differentiating employees on the basis of ability is desirable, a tournament has a double dividend. It motivates before the prize is given, and the balance of motivation and frustration in the aftermath of the contest is positive. If, in contrast, differentiating is undesirable, a tournament is a double-edged sword, as the balance of motivation and demotivation after the prize is given is negative.[11] As discussed in the introduction, demotivation of losers is far from a remote possibility: in Bewley (1999), fear of undermining morale was one of the main reasons managers mentioned to justify their reluctance to differentiate employees on the basis of ability.

[11] Casas-Arce and Martínez-Jerez (forthcoming) find empirical evidence that extreme past performance at either end of the performance distribution attenuates incentives to exert effort.

In this paper, we have analyzed a situation in which effort and ability are complements in the objective function of both the employees and the manager. This assumption accurately describes many real life situations. Of course, there is no denying that effort and ability could be substitutes in the employees' objective functions: they may just care about achieving a set goal. Then, communication problems that exist in an interaction between the manager and one employee cannot be overcome by treating two employees differently.

Further, we have limited attention to the two-employee case. Increasing the number of employees the manager can differentiate between will increase the manager's ability to tailor effort levels to each employee's realized ability. This suggests that different organizations may be associated to teams of different optimal sizes, as the management may have to trade off the comparative statement benefits of increasing team size with other costs, such as those associated to team coordination. We leave the analysis of this issue to future work.

8 Appendix

Derivation of the beliefs in Equilibrium 3, page 9:

$$\mathsf{E}\left(a_1|m_1\right)=\frac{\int_0^1\int_0^{a_1 t}a_1 da_2 da_1}{\int_0^1\int_0^{a_1\frac{1}{\gamma}}da_2 da_1}=\frac{2}{3},\ \mathsf{E}\left(a_1|m_3\right)=\frac{\int_0^t\int_{a_1\frac{1}{t}}^1 a_1 da_2 da_1}{\int_0^t\int_{a_1\frac{1}{t}}^1 da_2 da_1}=\frac{t}{3}$$

and

$$\mathsf{E}\left(a_1|m_2\right)=\frac{\int_0^t\int_{a_1 t}^{a_1\frac{1}{t}}a_1 da_2 da_1+\int_t^1\int_{a_1 t}^1 a_1 da_2 da_1}{\int_0^t\int_{a_1 t}^{a_1\frac{1}{t}}da_2 da_1+\int_t^1\int_{a_1 t}^1 da_2 da_1}=\frac{1}{2}+\frac{t}{6}.$$

Proof of Proposition 2: Recall that we assume $V_{ee}<0,\ V_{ea}>0$ and $V_e\left(0,a\right)\geq 0$. An equilibrium in which the manager avoids differentiation with strictly positive probability exists if and only if there exists a $t\in(0,1)$ such that she sends m_1 if $a_1\geq\frac{1}{t}a_2$, m_2 if $a_1\in\left(\frac{1}{t}a_2,ta_2\right)$, and m_3 if $a_1<ta_2$. For any t, this strategy defines the updated beliefs $E\left(a_i|m\right)$ as functions of t, and effort levels are best-replies given these beliefs. Consider employee 1 (2). Let $e^H\left(t\right)$ be his best reply to m_1 (m_3); let $e^M\left(t\right)$ be his best reply to m_2 (m_2); and let $e^L\left(t\right)$ be his best reply to m_3 (m_1). The manager is indifferent between m_1 and m_2 if and only if

$a_1 e^H(t) + a_2 e^L(t) = a_1 e^M(t) + a_2 e^M(t)$. For the desired equilibrium to exist the fixed point condition

$$t = G(t), \text{ with } G(t) = \frac{e^H(t) - e^M(t)}{e^M(t) - e^L(t)}$$

must allow for a solution $t \in (0,1)$.

The expression for the objective functions of the employee conditional on a message m, and the associated first order condition for an interior solution are

$$\begin{aligned} E(V(e,a)|m_1) &= \frac{\int_0^1 \int_0^{ta} V(e,a)\,dF(b)\,dF(a)}{\int_0^1 \int_0^{ta} dF(b)\,dF(a)} = \frac{\int_0^1 V(e,a)\,F(ta)\,dF(a)}{\int_0^1 F(ta)\,dF(a)} \\ &\rightarrow \frac{\int_0^1 V_e(e,a)\,F(ta)\,dF(a)}{\int_0^1 F(ta)\,dF(a)} = 0 \end{aligned}$$

$$\begin{aligned} E(V(e,a)|m_2) &= \frac{\int_0^t \int_{ta}^{\frac{1}{t}a} V(e,a)\,dF(b)\,dF(a) + \int_t^1 \int_{ta}^1 V(e,a)\,dF(b)\,dF(a)}{\int_0^t \int_{ta}^{\frac{1}{t}a} dF(b)\,dF(a) + \int_t^1 \int_{ta}^1 dF(b)\,dF(a)} \\ &= \frac{\int_0^t V(e,a)\left[F\left(\frac{1}{t}a\right) - F(ta)\right] dF(a) + \int_t^1 V(e,a)\left[1 - F(ta)\right] dF(a)}{\int_0^t \left[F\left(\frac{1}{t}a\right) - F(ta)\right] dF(a) + \int_t^1 \left[1 - F(ta)\right] dF(a)} \\ &\rightarrow \frac{\int_0^t V_e(e,a)\left[F\left(\frac{1}{t}a\right) - F(ta)\right] dF(a) + \int_t^1 V_e(e,a)\left[1 - F(ta)\right] dF(a)}{\int_0^t \left[F\left(\frac{1}{t}a\right) - F(ta)\right] dF(a) + \int_t^1 \left[1 - F(ta)\right] dF(a)} = 0 \end{aligned}$$

$$\begin{aligned} E(V(e,a)|m_3) &= \frac{\int_0^t \int_{\frac{1}{t}a}^1 V(e,a)\,dF(b)\,dF(a)}{\int_0^t \int_{\frac{1}{t}a}^1 dF(b)\,dF(a)} = \frac{\int_0^t V(e,a)\left[1 - F\left(\frac{1}{t}a\right)\right] dF(a)}{\int_0^t \left[1 - F\left(\frac{1}{t}a\right)\right] dF(a)} \\ &\rightarrow \frac{\int_0^t V_e(e,a)\left[1 - F\left(\frac{1}{t}a\right)\right] dF(a)}{\int_0^t \left[1 - F\left(\frac{1}{t}a\right)\right] dF(a)} = 0 \end{aligned}$$

Because of the continuity of $G(t)$ in t, to establish that the fixed point condition has an interior solution, it suffices to show that $\lim_{t\to 0} G(t) > 0$ and $\lim_{t\to 1} G(t) < 1$. For $G(0) > 0$, we need to show that $e^H(0) > e^M(0) > e^L(0)$. Taking limits of the above first-order conditions and using de L'Hôpital's rule, we obtain

$$\begin{aligned} \int_0^1 V_e\left(e^H(0), a\right) a dF(a) &= 0 \quad (12) \\ \int_0^1 V_e\left(e^M(0), a\right) dF(a) &= 0 \quad (13) \\ V_e\left(e^L(0), 0\right) &= 0. \quad (14) \end{aligned}$$

Note that

$$\begin{aligned} \int_0^1 V_e\left(e^L(0), a\right) dF(a) &= \int_0^1 \left[V_e\left(e^L(0), 0\right) + \int_0^a V_{ea}\left(e^L(0), x\right) dx\right] dF(a) \\ &= \int_0^1 \int_0^a V_{ea}\left(e^L(0), x\right) dx dF(a) > 0 = \int_0^1 V_e\left(e^M(0), a\right) dF(a), \end{aligned}$$

where the second equality holds because of (14), the inequality sign because of $V_{ea} > 0$, and the last equality by (13). Then, because of $V_{ee} < 0$, we know $e^M(0) > e^L(0)$.

To prove $e^H(0) > e^M(0)$, it follows from (13), in combination with $V_{ea} > 0$, that

$$\int_0^1 V_e\left(e^M(0), a\right) a dF(a) > 0 = \int_0^1 V_e\left(e^H(0), a\right) a dF(a),$$

where the equality follows from (12). Again, because of $V_{ea} > 0$, we then have $e^H(0) > e^M(0)$. Hence, we have shown that $\lim_{t \to 0} G(t) > 0$.

Turning to the proof of $\lim_{t \to 1} G(t) < 1$, this requires that we show that $e^M(1) > e^L(1)$ and $\left[e^H(1) + e^L(1)\right]/2 < e^M(1)$. From the first-order conditions we derive that

$$\begin{aligned} \int_0^1 V_e\left(e^H(1), a\right) dF^2(a) &= 0 \\ \int_0^1 V_e\left(e^M(1), a\right) a f(a) dF(a) &= 0 \\ \int_0^1 V_e\left(e^L(1), a\right) [1 - F(a)] dF(a) &= 0, \end{aligned}$$

where we rearranged the first condition to highlight that it rests on the distribution of the maximum of two i.i.d. variables.

Let e^p denote the optimal level of effort in case of the *prior* density $f(a)$. It satisfies $\int_0^1 V_e(e^p, a) dF(a) = 0$. Using $V_{ee} < 0$ and $V_{ea} > 0$, we have that $e^p < e^M(1)$

if either $af(a)$ is non-decreasing or $f(a)$ is symmetric. Hence, if (i) in the statement of the Proposition holds, then to show $\left[e^H(1) + e^L(1)\right]/2 < e^M(1)$, it suffices to show that $e^H(1) + e^L(1) \leq 2e^p$. Then,

$$\begin{aligned}
0 &= \int_0^1 V_e(e^p, a) f(a)\,da = \int_0^1 V_e(e^p, a)(1 - F(a))\,dF(a) + \int_0^1 V_e(e^p, a) F(a)\,dF(a) \\
&= \int_0^1 V_e\left(e^H(1) - \left(e^H(1) - e^p\right), a\right) F(a)\,dF(a) + \\
&\quad \int_0^1 V_e\left(e^L(1) - \left(e^L(1) - e^p\right), a\right)(1 - F(a))\,dF(a) \\
&= -\left(e^H(1) - e^p\right)\int_0^1 V_{ee}(\bar{e}, a) F(a)\,dF(a) - \\
&\quad \left(e^L(1) - e^p\right)\int_0^1 V_{ee}(\underline{e}, a)(1 - F(a))\,dF(a),
\end{aligned}$$

where $\bar{e} \in \left(e^p, e^H(1)\right)$ and $\underline{e} \in \left(e^L(1), e^p\right)$ by the mean-value theorem for derivatives. Thus, for $e^H(1) + e^L(1) \leq 2e^p$ to hold, we need

$$\int_0^1 V_{ee}(\bar{e}, a) F(a)\,dF(a) < \int_0^1 V_{ee}(\underline{e}, a)(1 - F(a))\,dF(a).$$

This is indeed the case if $V_{eee} \leq 0$ and $V_{eea} \leq 0$.

Finally, note that $e^H(1) + e^L(1) = 2e^p$ if and only if V_{ee} is constant.■

Expressions for $h'_-(a_1; k)$ and $h''_-(a_1; k)$:

$$h'_-(a_1; k) = \frac{[e_1(m_1) - e_1(m_2)][e_2(m_2) - e_2(m_1)]}{(e_2(m_2) - e_2(m_1) + a_1 k [e_1(m_2) e_2(m_2) - e_1(m_1) e_2(m_1)])^2} > 0$$

and

$$\begin{aligned}
h''_-(a_1; k) &= -k[e_1(m_1) - e_1(m_2)][e_1(m_2) e_2(m_2) - e_1(m_1) e_2(m_1)] \\
&\quad \times \frac{2(e_2(m_2) - e_2(m_1))}{(e_2(m_2) - e_2(m_1) + a_1 k [e_1(m_2) e_2(m_2) - e_1(m_1) e_2(m_1)])^3}
\end{aligned}$$

which is smaller than zero for $e_1(m_2) e_2(m_2) - e_1(m_1) e_2(m_1) > 0$ and $k > 0$.■

References

Aoyagi, Masaki (2007), Information Feedback in a Dynamic Tournament, working paper Osaka University

Azmat, Ghazala and Nagore Iriberri (2009), The Importance of Relative Performance Feedback Information: Evidence from a Natural Experiment using High School Students. Unpublished manuscript.

Baker, George P., Michael C. Jensen and Kevin J. Murphy, (1988),"Compensation and Incentives: Practice vs. Theory", Journal of Finance 43: 593-616.

Baker, George, Michael Gibbs and Bengt Holmström (1994a), "The Internal Economics of the Firm: Evidence from Personal Data", Quarterly Journal of Economics 109: 881-919.

Baker, George, Michael Gibbs and Bengt Holmstrom (1994b). "The wage policy of a firm", Quarterly Journal of Economics, pp. 922-955

Baumeister, R. F. (1998). The self, in: Gilbert, D. T., Fiske, S. T. and Lindzey, G., eds, The Handbook of Social Psychology, New York: McGraw Hill, pp. 680-740.

Bandiera, Oriana, Iwan Barankay, and Imran Rasul (2009), Team Incentives: Evidence from a Field Experiment. Unpublished manuscript.

Bénabou, Roland and Jean Tirole (2003), Intrinsic and Extrinsic Motivation, Review of Economic Studies, 70, 489-520.

Bewley, Truman F. (1999), Why Wages Don't Fall During a Recession. Cambridge, MA: Harvard University Press.

Blanes i Vidal, Jordi and Mareike Nossol (2009), Tournaments without Prizes: Evidence from Personnel Records. Unpublished manuscript.

Brown, J. D. and Dutton, K. A. (1995). Truth and consequences: The costs and benefits of accurate self-knowledge, Personality and Social Psychology Bulletin, vol. 21 (12), pp. 1288-1296.

Casas-Arce, and F. Asís Martínez-Jerez (forthcoming), "Relative Performance Compensation, Contests and Dynamic Incentives", Management Science.

Chakraborty, Archishman and Rick Harbaugh (2007), "Comparative Cheap Talk", Journal of Economic Theory 132: 70-94.

Delfgaauw, Josse, Robert Dur, Joeri Sol, and Willem Verbeke (2009), Tournament Incentives in the Field: Gender Differences in the Workplace. Tinbergen Institute Discussion Paper TI 2009-069/1.

Doeringer, Peter B. and Michael J. Piore (1971), Internal Labor Markets and Manpower Analysis. Lexington, Mass.: Heath.

Ederer, Florian (2004). "Feedback and Motivation in Dynamic Tournaments", MIT.

Ertac, Seda (2005), Social Comparisons and Optimal Information Revelation: Theory and Experiments.

Fang, Hanming and Giuseppe Moscarini (2005), "Morale Hazard", Journal of Monetary Economics 52: 749-777.

Felson, R. B. (1993). The (somewhat) social self: How others affect self-appraisals, in: J. Suls (ed.), Psychological perspectives on the self, Volume 4: The Self in Social Perspective, Hillsdale, New Jersey: Lawrence Erlbaum Associates, pp. 1-27.

Gervais, Simon and Itay Goldstein (2007). "The Positive Effects of Biased Self-Perceptions in Firms", Review of Finance, p. 453-496

Felson, R. B. (1993). The (somewhat) social self: How others affect self-appraisals, in: J. Suls (ed.), Psychological perspectives on the self, Volume 4: The Self in Social Perspective, Hillsdale, New Jersey: Lawrence Erlbaum Associates, pp. 1-27.

Gibbs, Michael (1994) "An Economic Approach to Process in Pay and Performance Appraisals", working paper Graduate School of Business, Chicago.

Hannan, R. Lynn, Ranjani Krishnan, and Andrew H. Newman (2008), "The Effects of Disseminating Relative Performance Feedback in Tournament and Individual Performance Compensation Plans", The Accounting Review 83 (4): 893-913.

Ishida, Junichiro (2006). "Optimal Promotion Policies with the Looking-Glass Effect", Journal of Labor Economics 24: 857-878.

Lizzeri, Alessandro, Margaret Meyer and Nicola Persico (1999), Interim Evaluation in Dynamic Tournaments: The Effect of Midterm Exams, manuscripts.

Jones, E. E. and Wortman, C. (1973). Ingratiation: An attributional approach, Morristown, NJ: General Learning Press.

Ray, Korok, (2007), The Retention Effect of Withholding Performance Evaluation, The Accounting Review, Vol. 82 (2), pp. 389–425.

Swank, O.H. and B. Visser, (2007), Motivating through Delegating Tasks or Giving Attention, Journal of Law, Economics and Organization.

Welch, John F. (2001), Jack. Straight from the Gut. New York: Warner Books.