This paper studies the asymptotic properties of the quasi-maximum likelihood estimator of ARCH(1) models without strict stationarity constraints, and considers applications to testing problems. This estimator is unrestricted in the sense that the value of the intercept, which cannot be consistently estimated in the explosive case, is not fixed. A specific behavior of the estimator of the ARCH coefficient is obtained at the boundary of the stationarity region, but this estimator remains consistent and asymptotically normal in every situation.

The asymptotic variance is different in the stationary and non stationary situations, but is consistently estimated, with the same estimator, in both cases. Tests of strict stationarity and non stationarity are proposed. Their behaviour is studied under the null assumption and under local alternatives. The tests developed for the ARCH(1) model are able to detect non-stationarity in more general GARCH models. A numerical illustration based on stock indices is proposed.