Strategic Relationships in Over-the-Counter Markets

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Abstract

This paper provides a theory of dynamic formation of relationship in over-the-counter markets. Trading in OTC markets entails transaction costs that are less evident. For instance, counterparties search for each other or transactions involve counterparty risk, as it is the case in markets for credit derivatives or bank loans. When repeated interactions re-enforce the effectiveness of such mechanisms, transaction costs may be reduced and agents may find it beneficial to develop a network of relationships. Indeed, evidence on borrowing and lending in interbank markets, which function over-the-counter, suggests that banks form networks with a tiered, core-periphery structure. We identify two forces that drive the formation of such networks. On the one hand, decreasing transaction costs encourage the formation of bilateral relationships. On the other hand, additional benefits acquired from intermediating transactions between others can explain the emergence of a central counterparty for trade.

Keywords: over-the-counter trading; bargaining; network formation;
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1 Introduction

Over-the-counter (OTC) markets have been at the core of the financial system for decades, their growth being halted only during the financial crisis of 2008-2009. Distinctively, in these markets trade is conducted through bilateral negotiations, rather than a Walrasian auction. Counterparties meet and set prices through a bargaining process that reflects the strategic nature of the interactions. Moreover, in some cases, such as the market for credit derivatives or bank loans, trading involves counterparty risk. These frictions can introduce transaction costs that are less evident, as when counterparties search for each other or transactions require monitoring. When repeated interactions re-enforce the effectivness of such mechanisms, transaction costs may be reduced and agents may find it beneficial to develop a network of relationships.

This paper provides a theory of dynamic formation of relationships in over-the-counter markets. We develop a model that explains how a central counterparty can emerge endogenously in an OTC market when trading generates a surplus against a transaction cost. For the scope of this paper, we consider that transaction costs arise from the necessity to monitor trade (or, alternatively, to pledge collateral). Monitoring is costly, but decreasingly so with the frequency of interactions between counterparties. In essence, the monitoring cost captures the riskiness of the transaction, while the decrease in the monitoring cost reflects the extent to which repeated interatactions may lower counterparty risk. Decreasing monitoring costs provides incentives for agents to trade repeatedly with the same counterparty, and hence, establish relationships. However, this creates a trade-off: developing relationships implies that much of the trade will be undertaken through intermediaries. When agents trade with each other, they negotiate bilaterally the terms of trade. This implies that agents that act as intermediaries will bargain to acquire a share of the surplus. Thus, individuals can extract rents from intermediation, in addition to the benefits they receive from trading.

We consider a dynamic setting where every period pairs of agents are randomly selected to trade and generate a surplus. Parties bargain over the division of the surplus, making alternating offers. If two individuals interact directly, then the Rubinstein (1982) bargaining procedure will determine how the surplus is shared. If agents trade indirectly, the division of the surplus depends also on the number of intermediaries that facilitate the transaction. The more intermediaries are involved in the transaction, the lower will be the share of the surplus that each party receives.

In this setting, there are two types of incentives for individuals to decide on whom they interact with. The first incentive is related to the rewards from intermediation: players would like to place themselves between others in order to acquire benefits from intermediation. The second incentive arises out of the desire to avoid sharing surpluses with intermediaries; in other words, individuals will try to circumvent intermediate players to retain more of the surplus for themselves.

We show that agents interact strategically to develop long-run relationships, provided that monitoring costs decrease sufficiently to compensate those that pay intermediation rents. Moreover, one agent will become central and act as intermediary and trading counterparty for all other agents. The network of relationships will thus be shaped in a structure that we define as core-periphery. Inefficiencies arise when agents are no longer willing to forego a share of the surplus in order to mantain the relationship with an intermediary. Thus, even if there are efficiency gains when repeated interactions lower counterpaty risk, as captured by the decrease in monitoring costs, at least some agents will not benefit from the trade-off.

The interest in relationships in over-the-counter markets is motivated by the implications they may carry for both the price setting mechanism and for the pattern of interactions. For instance, the effect on prices is shown in a study of the Portuguese interbank market by Cocco et al. (2008) who find that banks with a larger reserve imbalance are more likely to borrow funds from banks with whom they have a relationship, and to pay a lower interest rate than otherwise. Further evidence suggests that social networks may be an important mechanism for information flow into asset prices. In Cohen et al. (2007) connections between mutual fund managers and corporate board members via shared education institutions proxy the social network. They find that portfolio managers place larger bets on firms they are connected to through their network, and perform significantly better on these holdings relative to their non-connected holdings. Colla and Mele (2009) and Ozsoylev and Walden (2009) provide theoretical models of how the information transmitted in a network of traders is embeded in prices.

The structure of interactions bares non-negligible effects as well. In the same study Cocco et al. (2008) show that smaller banks and banks with more nonperforming loans tend to have limited access to international markets, and rely more on relationships. An earlier work by Upper and Worms (2004) maps the German interbank market and identifies a two-tiered banking system. Banks belonging to the upper tier have lending relationships with a variety of other banks belonging to the same tier, and banks in the lower tier transact with banks in the upper tier. When banks establish relationships, the pattern of their interactions becomes stable over time. Moreover, as it has been shown (Allen and Gale, 2000, Freixas et al., 2000), the structure of mutual exposures that connects the banking system directly impacts the resilience of the system to contagion, posing a problem to financial stability.

We discuss how our results contribute to understanding interactions in over-the-counter markets. A series of studies on over-the-counter markets has been concerned with explaining asset pricing through trading frictions. Acharya and Pedersen (2005) study the effect on asset prices of an exogenously specified trading cost. Duffie, Garleanu and Pedersen (2005, 2007) endogenize the trading frictions arising through search and bargaining, and show their effects on asset prices. Vayanos and Wang (2007) extend this framework in order to treat multiple assets in the same economy, while introducing heterogeneity in investors' horizon. Weill (2008) considers a setting with multiple assets as well, and he explains different expected returns through the cross-sectional variation in tradeable shares. Complementary to this literature, we propose a model where trading assets drives the formation of strategic relationships. Although we do not model explicitly prices, agents bargain over the division of the surplus. We can identify how the division of the returns depends on the position players have in the network.

Methodologically, our model draws from the literature on networks. The general concept of a network is quite intuitive: a network describes a collection of nodes and the links between them. A network representation of financial systems can introduce a conceptual framework within which the various patterns of connections between financial institutions can be described and analyzed in a meaningful way. Moreover the theory of networks provides tools rooted in game theory, that allows us to explain economic phenomena in terms of individual decisions made through a rational deliberation of costs and benefits.

Situations, such as the one we study, where agents form or severe connections depending on the benefits they bring are modeled through a game of network formation. A recent and rapidly growing literature on network formation games has developed in the past few years, introducing various approaches to model network formation and proposing several equilibrium concepts (Bala and Goyal, 2000, Bloch and Jackson, 2007, Jackson and Wolinsky, 1996). Particularly relevant for our framework is the dynamic network formation game analyzed in Dutta et. al, 2005. Conceptually, our model is related to the study of Goyal and Vega-Redondo (2007), who have recently considered a static model of network formation where intermediation benefits are generated when the connection between two players is mediated by other players.

Although there are numerous applications of these models in the social science context, the research on financial networks is still at an early stage. Allen and Babus (2008) provide a comprehensive survey of this literature. Most of the existing research using network theory concentrates on issues such as financial stability and contagion. For instance, Leitner (2007) investigates the possibility of private bail-outs organized by a social planner, while Babus (2008) endogenizes the bail-out mechanism. The role of intermediaries in financial networks has been analyzed thus far only by Gale and Kariv (2007, 2009). They study both theoretically and experimentally how the presence of intermediaries affect the efficient allocation of assets or lead to market breakdowns. Our contribution is to show that the role of intermediaries will be endogenously assumed by certain agents out of the necessity to establish relationships.

In our model we endogenize the decision on whom to interact with and form a relationship. The choice of monitoring is simply associated to the choice of linking, and is not modeled independently. Since monitoring is costly, it becomes expensive to develop a relationship with every other possible counterparty. This, together with the benefits agents extract from being in a network, shapes the pattern of relationships as a core-periphery structure.

The paper is organized as follows. The first section introduces the basic model. We elaborate the trade-offs implied by the cost-benefit structure in Section 3. The equilibrium analysis and the main results are developed in section 4. Section 5 discusses the role of decreasing monitoring costs. Section 6 concludes.

2 The Model

In this section we introduce the basic framework, explain the trading mechanism and the surplus-sharing procedure, and discuss the monitoring costs implied by interactions.

Consider $N = \{1, 2, ..., n\}$ as the set of agents who participate in the market, and n is an even number. Agents are risk-neutral, and discount the future at the constant rate γ. Time is discrete, and agents are infinitely lived. Every period each agent has with probability 1/2 either a liquidity surplus or an investment opportunity. Every time an agent with a liquidity surplus finances an agent with an investment opportunity, a surplus of one unit is generated from the transaction. The surplus will be shared by the counterparties through a bargaining procedure, as described in Section 2.2. The share each agent receives following the division of the surplus will represent their gain from the transaction in the respective period. When an agent with a liquidity surplus does not finance an agent that has an investment opportunity, no surplus will be realized and both agents receive 0 in that period.

In addition, we assume that transactions require monitoring as described in Section 2.3. Although it is costly, monitoring is implicit. That is, transactions are necessarily monitored when agents engage in trade. Trading, thus, entails both cost and benefits.

2.1 The Matching Technology

Particular to over-the-counter markets is that the terms of trades can be individualized to meet traders' various requirements. Transactions are, thus, specific to counterparties' needs. We incorporate this feature of OTC markets and assume that, in any period, each agent meets another agent to trade and generate surplus with probablity 1. However, the meeting process is determined by a matching technology. In particular, the matching technology controls two aspects. First, agents will be paired at random in each period. Second, each agent in a pair is assigned, with probability 1/2, either a liquidity surplus or an investment opportunity.

The matching technology implies that trading opportunities are randomly distributed across agents in every period t. At the same time, the shocks incurred by each agent i are independently drawn over time. Hence, an agent will not necessarily have neither the same counterparty nor the same shock, over time. The matching technology insures this way that no relationships are ex-ante built between agents.

Formally, a matching is a simply collection of disjoint pairs formed on the set of agents N.

$$M(N) = \{(i,j)|\ (i,j) \cap (k,l) = \emptyset \text{ with } i,j,k,l \in N\}.$$

The set of all possible matchings on N is denoted by $\mathcal{M}(N)$. At any time t, a matching $M_t(N)$ is a random draw from $\mathcal{M}(N)$. The cardinality of $\mathcal{M}(N)$ is given by $|\mathcal{M}(N)| = (n-1)(n-3)...1$. It is straightforward to calculate the probability that a pair of agents (i,j) is matched to generate a surplus:

$$\Pr[(i,j) \in M_t(N)] = \frac{1}{n-1}.$$

This probability reflects the frequency with which a pair of agents will create one unit of surplus, and will drive agents' decision to form relationships.

The matching technology randomizes which agents will be paired to generate a surplus at every date t. In a simpler setup, we could assume that only one pair is active and trades at a time. Although conceptually the same, the current framework insures that the total surplus, as well as the expected individual gains would increase by a magnitude of n. In essence, we assume that all agents are trading in an "orderly" fashion.

2.2 The Division of the Surplus

Once the counterparties are paired through the matching technology, the agent with an investment opportunity can borrow funds from the agent with a liquidity surplus. We assume that a surplus of one unit is generated from the transaction if transactions are properly monitored. Monitoring is implicit and insures that the loan is repaid. Agents will bilaterally negotiate how to share the surplus. We assume that they bargain by making alternating offers, as in the Rubinstein (1982) protocol. The bargaining takes place within one period, and the economy does not move forward to the next period until an agreement is reached. For instance, a period can be considered to be a trading day at the beginning of which agents start bargaining over the terms of trade. The trading day ends when agents reach an agreement.

We analyze two bargaining procedures: direct negotiations and indirect negotiations.

Direct Financing

Under the direct financing procedure, an agent with a liquidity surplus finances her counterparty set by the matching technology through a direct transaction. A matched pair of agents bargains directly over the division of one unit of surplus. An agreement is a pair (x_1, x_2), in which x_i is agent's i share of the surplus. Agents take turns in proposing agreements. For simplicity, we assume that the agent that has the liquidity surplus makes the first proposal. An offer may be accepted or rejected. If the offer is accepted, then the bargaining ends and, the agreement is implemented. If the offer is rejected, then the negotiation continues and the agent with the investment opportunity takes the turn to make a counter-proposal. Any delay in reaching an agreement is, however, costly for both agents. We assume that agents use the same discount factor, δ, to value earnings depending on whether the agreement is reached sooner or later within one period.

This setting corresponds exactly to the two-player bargaining procedure as analyzed in Rubinstein (1982).

Indirect Financing

In the second situation, one or more agents mediate the transaction between an agent with a liquidity surplus and an agent with an investment opportunity. Agents that are involved this way in the creation of the surplus are called intermediaries. Any intermediary will also be paired with another agent to generate a surplus, but not all agents will act as intermediaries. In this case, a matched pair of agents will negotiate indirectly and divide the one unit of the surplus they generate with the other agents involved in the transaction. Intermediaries will bargain to acquire a share of the surplus as follows.

We propose the following bargaining procedure. Suppose that (i, j) is a pair matched to generate the surplus and the sequence $(i_1, i_2, ..., i_k)$ are the intermediaries that facilitate the transaction in this order. The sequence $(i_1, i_2, ..., i_k)$ forms a *path* between i and j. The *distance*, d, between i and j is the number of intermediaries on the path between i and j: $d(i, j) = k$. Agents negotiate how to split the surplus via successive bilateral bargaining sessions. More precisely, agents bargain in the following order: (i_k, j), (i_{k-1}, i_k), ..., (i_1, i_2), (i, i_1) if j is the agent with the investment opportunity in the pair (i, j), and in reversed order (i_1, i), (i_2, i_1), ..., (i_k, i_{k-1}), (j, i_k) if if i is the agent with the investment opportunity in the pair (i, j).

In each bilateral bargaining session, two players negotiate a partial agreement via the alternating-proposal framework of Rubinstein (1982). In each session, one agent, the proposer, makes an offer that the other agent, the receiver, either accepts or rejects. A partial agreement specifies the share for the receiver to exit the game. After a partial agreement, the other agent continues to negotiate the remaining surplus in one subsequent session, when she becomes the receiver. In other words, an intermediary i_l on the path between (i, j) will bargain as a receiver with an agent i_{l-1} before him on the path over the surplus she acquired as a proposer from the agent i_{l+1} that follows him on the path. As above, the agent with the investment opportunity in the pair (i, j) is also the receiver.

A full agreement is reached when all bargaining sessions end in a partial agreement. That is, a full agreements is reached after $(k+1)$ successful bargaining sessions.[1] An outcome consists of $(k+1)$ partial agreements that specify player's l share of the surplus, denoted as $x_l \in [0,1]$, for $l \in \{i, i_1, i_2, ..., i_k, j\}$ such that $\sum_l x_l = 1$. All bargaining sessions take place within one period, for a given matching $M_t(N)$. However, delay in reaching an (partial) agreement is penalized, as earnings are discounted at rate δ depending on whether the agreement is reached sooner or later within one period. We assume that there is no discounting between two consecutive bargaining sessions.

2.3 Monitoring Costs

An implicit assumption of our model is that the risk of a transaction is entirely priced in through the monitoring cost. Moreover, monitoring is necessarily undertaken whenever agents engage in trade.

Monitoring is usually costly for the lender, and in our model the cost is born by the agent that has a liquidity surplus in the matched pair (i, j) when negotiations are direct. If trading is facilitated by intermediaries, then each agent i_l will monitor her successor i_{l+1} on the path between i and j and incur the monitoring cost. Crucial in our model is that we assume that there are economies of scale from monitoring the same party over time. Formally, the cost structure is described as follows:

- For any agent i that monitors agent j for the first time, $c_{ij} = C$
- For any agent i that monitors agent j in period t, $c_{ij} = d$ if i and j have interacted in period $t-1$, and $c_{ij} = C$ otherwise.

[1] Thus, the distance between i and j, $d(i, j)$, represents as well the number of partial agreements necessary to reach a full agreement between i and j.

Since monitoring the same counterparty becomes cheaper over time, agents have incentive to develop relationships. While the matching is exogenous and agents do not choose with whom they generate the surplus, they choose whom they monitor. In fact, the choice of whom to monitor becomes a choice of whom to interact with. Agents' decisions to establish relationships creates an endogenous network structure and is modelled explicitly in Section 4.

3 Benefits and Network Externalities

We have explained above how the surplus is divided if intermediaries facilitate trade. However, that trade is mediated by intermediaries is not merely an assumption. In fact, trade will be undertaken through intermediaries only if agents decide to interact repeatedly and establish relationships. In this section we illustrate the incentives that may drive agents to trade through intermediaries. The intuition is as follows. On the one hand, economies of scale from monitoring the same counterparty over time creates incentive for agents to form relationships. On the other hand, forming relationships implies that the surplus is split among more parties, which will disfavor some agents. Nevertheless, if monitoring costs decrease sufficiently to compensate those agents that receive lower shares of the surplus, relationships are established. To understand how this trade-off settles we study two situations.

3.1 The Market Setup

The market setup simply describes a situation where agents bargain in every period with the counterparty set by the matching technology. Bargaining directly over the division of the surplus insures both parties a higher share of the surplus than otherwise. Proposition 1 characterizes the players' share of the surplus.

Proposition 1 *(Rubinstein, 1982) Under the direct financing procedure, the alternating-offers bargaining game has a unique subgame perfect equilibrium. Moreover, for any pair $(i,j)_t \in M_t(N)$, where i has a liquidity surplus and j has an investment opportunity, the shares players receives at each date t are $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$.*

Each period, an agent gains in expectation a benefits from direct financing of $\frac{1}{2}\left[\frac{1}{1+\delta} + \frac{\delta}{1+\delta}\right]$.

When every period agents bargain directly with the counterparty they are matched to generate the surplus, they no longer benefit from the economies of scale of monitoring the same party over time. Since for any given pair (i,j) the probability of being matched two consecutive periods is $\frac{1}{n-1}$, the expected cost involved in a transaction between i and j is given by $\frac{1}{2}\left[\frac{n-2}{n-1}C + \frac{1}{n-1}d\right]$ in each period.

The one-period payoff of an agent i from participating in the market setup is given by:[2]

$$\pi_i^M = \frac{1}{2}\left[\frac{1}{1+\delta} + \frac{\delta}{1+\delta} - \left(\frac{n-2}{n-1}C + \frac{1}{n-1}d\right)\right], \tag{3.1}$$

while the aggregate payoff of agent i over time is

$$V_i^M = \sum_t \gamma^t \pi_i^M. \tag{3.2}$$

3.2 The Network Setup

The second situation we consider is a case where all agents are embedded in a network g. A network g specifies a set of links between agents. The network structure determines the intermediaries involved in the generation of the surplus in every period t for a given matching $M_t(N)$. Depending on the matching realized each period, a link in the network g connects either two agents that are matched to generate a surplus or connects intermediaries on a path between a matched pair.

At the same time, the network structure enforces a monitoring pattern. Any two agents that are linked in the network are subject to mutual monitoring and will pay the associated

[2] i's expected payoff is the same every period.

cost. In a network agents may benefit from decreasing monitoring costs, depending on whether they revise their linking strategy. We model this decision explicitly in the next section, when we study what networks emerge endogenously, in a dynamic framework. For the moment though, we need to characterize the benefits and the costs agents incur if they are embedded in such a network.

The benefits agents receive in a network g depend on the financing procedure that applies. If there exists a link between a pair of agents that is matched to trade, then they can use the *direct financing* procedure and Proposition 1 characterizes each agent share of the surplus. If a matched pair of agents needs to trade through intermediaries, then they will divide the surplus according to the *indirect financing* procedure. The solution of this bargaining protocol clearly depends on the number of agents that facilitate the transaction between i and j, $d_t(i,j)$, at date t. The following proposition summarizes the outcome of the bargaining process.

Proposition 2 *Let $(i,j)_t \in M_t(N)$ be a pair matched to trade and a sequence of intermediaries $(i_1, i_2, ..., i_k)_t$ between i and j in the network g_t that facilitate the generation of the surplus at date t. Then, under the indirect financing procedure, the alternating-offers bargaining game has a unique subgame perfect equilibrium. Moreover, if i has a liquidity surplus and j has an investment opportunity, then the shares players $(i, i_1, i_2, ..., i_k, j)$ receive at date t are*
$(\frac{1}{(1+\delta)^{d_t(i,j)+1}}, \frac{\delta}{(1+\delta)^{d_t(i_1,j)+2}}, \frac{\delta}{(1+\delta)^{d_t(i_2,j)+2}}, ..., \frac{\delta}{(1+\delta)^{d_t(i_k,j)+2}}, \frac{\delta}{1+\delta})$ *respectively.*

Proof. The proof is provided in the appendix. ■

For a matched pair (i,j) and a sequence of intermediaries $(i_1, i_2, ..., i_k)$ that facilitate trade between i and j in a given network g_t, the indirect financing bargaining procedure unravels in $(k+1)$ bargaining sessions and is solvable by backward induction. The last session is effectively the same as the Rubinstein (1982) model, where the total share to be allocated between the two agents is the remainder of the surplus given by the k partial agreements from the first k sessions. Moreover, all the previous agreements will be

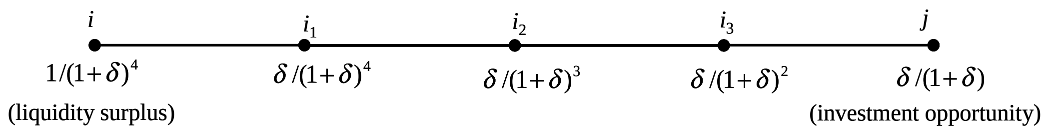

Figure 3.1: Indirect financing bargaining procedure

implemented only if the two players in the last session reach an agreement. Figure 3.1 illustrates the division of the surplus between 5 agents.

The two bargaining procedures, direct and indirect, describe the benefits agents receive in each period t. All players receive benefits from being matched to generate a surplus. However, some players may act as intermediaries and acquire additional gains, at the expense of others. This crucially affects agents' decision on whether to form strategic relationships. If forming relationships implies trading through intermediaries, then agents matched to generate surplus need to forego part of their share. The more intermediaries are involved in a transaction between a pair of agents (i, j), the lower will be the share i and j will receive in the division of the surplus.

Our findings are robust to the choice of the bargaining procedure. Basically, our results hold under any rule for the division of the surplus that depends on the number of intermediaries involved in a transaction. For instance, equal division of the surplus between all agents involved in a transaction is not an equilibrium of our bargaining game. However, the same incentives affect players' decision to form relationships.

Thus, in a network, an agent can derive two types of benefits. On the one hand, she gains benefits from generating the surplus. However, her share of the surplus may be reduced accordingly, depending on the number of intermediaries that facilitate the trade. On the other hand, an agent may gain rents from intermediating transactions between other agents. The cost associated to monitoring can be C or d, depending on whether agents have been linked in the previous period. Hence, the net payoff of an agent i from being embedded in a network g at time t is given by:

$$\pi_i(g_t) = \sum_{k\in N\setminus\{i\}} \frac{1}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)^{d_t(i,k)+1}} + \frac{\delta}{1+\delta}\right] + \sum_{\substack{(j,k)\in M_t(N)\\ i\in P_t(j,k)}} \frac{1}{n-1}\frac{1}{2}\left[\frac{\delta}{(1+\delta)^{d_t(i,j)+2}} + \frac{\delta}{(1+\delta)^{d_t(i,k)+2}}\right] - \frac{1}{2}\sum_{j\in N_{g_t}(i)} c_{ij}, \tag{3.3}$$

where $P_t(j,k)$ represents the set of agents $(i_1, i_2, ..., i_l)$ that act as intermediaries between j and k at time t, c_{ij} is the cost incurred by i for monitoring j, and $N_{g_t}(i)$ is the set of agents i monitors in period t.

The first part of the summation represents the expected benefit agent i derives from being matched to generate the surplus. Although she will be matched with probability one and generate a surplus with certainty, the probability of being matched with another agent j at a distance $d(i,j)$ is $\frac{1}{n-1}$. In addition, for any given match, i will start the bargaining with probability half, when she has the liquidity surplus. The second part of the summation represents the intermediation rents an agent can acquire from the network g. If an agent i acts as an intermediary between between a pair (j,k), her share of the surplus will depend on how many agents are between her and the receiver: $\frac{1}{2}\left[\frac{\delta}{(1+\delta)^{d(i,j)+2}} + \frac{\delta}{(1+\delta)^{d(i,k)+2}}\right]$, where either j and k can be the receiver with probability half. Clearly, not all agents will derive benefits from intermediation. Even when an agent i lies on a path between two agents j and k, she will receive a share of the surplus only if j and k are matched to generate a surplus which happens with probability $\frac{1}{n-1}$.

The aggregate payoff of agent i over time is

$$V_i = \sum_t \gamma^t \pi_i(g_t), \tag{3.4}$$

where we allow the possibility for the network g to change over time.

4 Network Dynamics

We analyze the formation of strategic relationships in a dynamic framework, where each period one agent is allowed to revise her linking strategy. In doing so, agents seek to improve their discounted future payoff stream given by (3.4), while taking into account the effect of their actions in the current period on the actions of others in the future. Thus, agents are not taken to be atomistic, but to behave strategically. In particular, at any date, one agent i is selected and endowed with the capacity to unilaterally severe existing links with any other agents and/or propose one link to another agent if the link doesn't exist to begin with. The agent that has been proposed a link can accept or reject. These actions create a new network, and then one-period payoffs are received according to the payoff structure described above. The current period then ends, and the whole process begins again ad infinitum.

In this context, we ask questions related to stable outcomes and convergence of this dynamic network formation process. We aim to identify networks that may be absorbing (in the sense that the process, once there, remains there), and show that they attract the process at least from some initial conditions. More precisely, a network is absorbing if there is no agent that can improve her discounted future payoff stream given by (3.4) when she has the possibility to change her current relationships and anticipates the consequences of her choice on other agents actions. The notion of convergence is more encompassing as it captures the agents' incentives to change their relationships such that their choices generates a sequence of networks converging to an absorbing network in finite time. In evaluating whether their choices are profitable, agents are willing to incur small temporary losses provided they receive sufficient gains in the future to compensate them.

We briefly discuss the forces that drive the network formation process. On the one hand, the costs associated to linking prevent agents to form too many connections. Despite that linking to more agents is desirable, it will be expensive to do so. Moreover, decreasing monitoring costs over time incentivize agents to interact repeatedly with the same counterparties. On the other hand, the benefits scheme carries two implications. The first implication is related to the rewards from intermediation: agents would like to place themselves between others in order to acquire benefits from intermediation. The second implication arises out of the incentive to avoid sharing surpluses with intermediaries; in other words, individuals will try to circumvent intermediate players to retain more of the surplus for themselves.

These three forces will drive the network formation process towards a star network structure, as shown in fig. 4.1. We show this in two stages: First we prove that the star is an absorbing state of the network formation process. Second we show that the dynamic process reaches a star from some initial conditions in finite time. The order of play in which agents act is essential in determining who becomes the unique intermediary for all trade.

To formalize these ideas we formulate a dynamic game of network formation (with perfect information), for which the appropriate equilibrium concept to use is Markov Perfect Equilibrium. For this, we assume that players decisions to form links are governed by Markov strategies; i.e. traders' actions are assumed to depend only on the existing payoff-relevant state. The payoff-relevant state s is simply the pair $s = (g, i)$, where g is the historically given network and i is the agent selected to be active. The matching $M(N)$ is not included in the payoff relevant state, since agents revise their linking strategy before the matching at time t is realized.

A strategy profile for a player prescribes a set of actions at any state s. Formally, at any state s with active player i, the set of actions for player i is simply a vector $\mu_i(s) = (\mu_{ij_1}, \mu_{ij_2}, ..., \mu_{ij_p})$ of revised linkages between i and a subset of players $\{j_1, j_2, ..., j_p\}$, where $\mu_{ij_l} \in \{0, 1\}$ where $\mu_{ij_l} = 1$ implies that i proposes a link to j_l, and $\mu_{ij_l} = 0$ implies that i severes a link with j_l. Clearly, the strategy prescribes as well the choice of the subset of players $\{j_1, j_2, ..., j_p\}$, which can be any subset of N, including the empty set. For a player j_l inactive at state s, such that $\mu_{ij_l} = 1$, the set of actions is $\mu_{j_l}(s) = \{Yes, No\}$, depending on whether she accepts or rejects the link.

Let $\boldsymbol{\mu}$ stand for the entire profile $\mu(s) = (\mu_i(s), \mu_j(s))$ over all states and refer to as

$\boldsymbol{\mu}$ a strategy profile. A strategy profile $\boldsymbol{\mu}$ precipitates, for each state s, some probability measure q_s over the feasible set $F(s)$ of future networks starting from s. In particular, a Markov process is induced on the set S of states: at any state s, q_s describes the movement to a new network, and the given choice of the active player moves the system to a new active player.

The process creates values for each player. The overall payoff to any agent i (under the strategy profile $\boldsymbol{\mu}$) is the unique solution to the functional equation:

$$V_i(s, \boldsymbol{\mu}) = \sum_{g' \in F(s)} q_s(g') \left\{ \pi_i(g') + \gamma \sum_{i'} p(i') V_i(s', \boldsymbol{\mu}) \right\}, \tag{4.1}$$

where $p(i')$ is the probability that agent i' will be active "tomorrow" and s' stands for the state (g', i').

An *equilibrium process of network formation*, as introduced in Dutta et al. (2005), is a strategy profile $\boldsymbol{\mu}$ with the property that there is no active player at any state s which can benefit by departing from $\mu(s)$. The benefit is evaluated according to the benefit structure introduced above. Profitable deviations need to be forward looking: individuals take the ongoing process as given and evaluate the entire stream of consequences arising from a single action.

In essence a strategy profile $\boldsymbol{\mu}$ is a Markov perfect equilibrium of the dynamic network formation process if for any state s and any active player i, there is no move $\mu'(s)$ that i finds profitable: $V_i(s, \boldsymbol{\mu}') < V_i(s, \boldsymbol{\mu}')$, where $\boldsymbol{\mu}'$ is the strategy profile induced by i's move to μ'.

The following two propositions collect the two main results.

Proposition 3 *Suppose that traders receive benefits from their relationships as described in 3.4. Then there exists a Markov Perfect Equilibrium linking strategy such that a star network that connects all traders is an absorbing state of the dynamic network formation game.*

Proof. A proof is provided in the appendix. ■

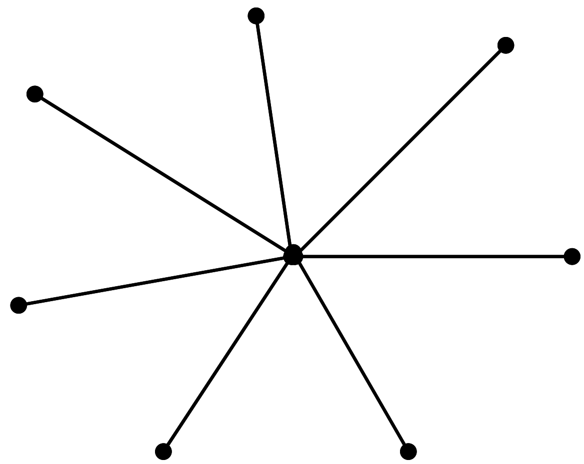

Figure 4.1: Star network architecture

In other words, we show that once traders are connected in a star network, no agent can revise her relationships without incurring losses. The star is in essence the network structure that minimizes the number of intermediaries between any two players, while it allows agents to benefit from decreasing monitoring costs. The central player extracts substantial benefits from intermediating transactions between other players. The periphery players will not want to change their linking behavior either. A periphery player can improve her payoff only if she becomes central. However, any player that links to a spoke, receives a lesser share of the surplus while paying a higher monitoring cost, at least for one period.

Proposition 3 illustrates how traders' incentives to avoid sharing the surplus with too many others materialize. To fully reveal the trade off that agents face between forming relationships in order to save on monitoring costs and accepting a lower share of the surplus, we study the convergence of the network formation process from some initial conditions. That is, we start by assuming that traders are connected in a network and aim to identify a linking strategy such that, if they all follow it, the outcome of their decisions drive the network formation process to a star-shaped network. Then we need to show that no trader has an incentive to deviate from her linking strategy. For tractability, we focus on the subclass of *minimally connected networks.* Such networks have the property that between any pair of players there exists a unique sequence of intermediaries. More importantly, minimally connected networks maximize aggregate welfare, as they generate the full surplus at the lowest monitoring cost. Therefore, we restrict the strategy space and allow the active player at state s to make only one linking proposal, conditional on severing at least one link.

Traders' linking actions induce a sequence of networks, such that each network in the sequence is obtained from the previous one by adding or severing links. We restrict our attention to consider only the addition and the severing of one link at a time. Formally, a sequence from network g to network g' is a finite series of adjacent networks $g_1, g_2, ..., g_T$ with $g = g_1$ and $g' = g_T$ such that at any step $t \in \{1, 2, ...T-1\}$ the transition from g_{t-1} to g_t is given by $g_t = g_{t-1} - i_t k + i_t l$, for some $i_t, l, k \in N$, $i_t k \in g_{t-1}$ and $i_t l \notin g_{t-1}$. When traders act in a predetermined order of play, the sequence of networks is deterrministic. A sequence of networks is supported as a Markov Perfect Equilibrium if there exists a strategy profile $\boldsymbol{\mu}$ that supports the sequence of network and $\boldsymbol{\mu}$ is an equilibrium.

Proposition 4 *From any minimally connected network that connects all traders there exists a sequence of networks to a star-shaped network that is supported as Markov Perfect Equilibrium.*

Proof. The proof is provided in the appendix. ■

The intuition for this result is as follows. In any network g, players have an incentive to reduce the number of intermediaries, in order to acquire a larger share of the surplus. Although some players may try to gain strategic positions that allows them to extract intermediation rents, there is always a sequence of players that will offset these strategic considerations and drive the number of intermediaries to one. Forward looking behavior enables players to anticipate the consequences of their own actions. Hence, in any minimally connected network, for a deterministic sequence of players, each player can rely on her successors to drive the network formation process towards a star.

5 The Role of Decreasing Monitoring Costs and Inefficiencies

While decreasing monitoring costs provide the main incentive for traders to form relationships, they are also the main source of innefficiences. An efficient outcome is one that maximizes traders' aggregate welfare:

$$W = \sum_{i=1}^{n} V_i.$$

Aggregate welfare is independent on how the surplus is divided, but is monotonically decreasing in monitoring costs. There are clear efficiency gains from trading with the same counterparties over time. A star-shaped network where all traders mantain relationships with one central counterparty provides such efficiency gains, even when the decrease in monitoring costs is incremental.

While a star is efficient, it is also a very unequal network. The trader that acts as a central counterparty for all trades extracts all the intermediation rents and gains substantially higher benefits than other traders. Moreover, except for the central trader, all other agents expect lower share of the surplus than they would receive in the market setup. Indeed, for any periphery player i, the following inequality holds:

$$\sum_{k \in N \setminus \{i\}} \frac{1}{n-1} \frac{1}{2} \left[\frac{1}{(1+\delta)^{d_t(i,k)+1}} + \frac{\delta}{1+\delta} \right] < \frac{1}{2} \left[\frac{1}{1+\delta} + \frac{\delta}{1+\delta} \right],$$

where in a star $d_t(i,k) = 0$ if k is the central player and $d_t(i,k) = 1$, otherwise. The left hand side of the inequality is the benefit a peripheral player expects in a star network (see eq. 3.3), while the right hand side is the benefit any player receives under the market setup (see eq. 3.1) at any given date.

Traders that occupy peripheral positions in the network may find the market setup more attractive, unless monitoring costs decrease sufficiently. In the market setup, players expect to pay a high cost every time they engage in trade, as the probability that a pair is matched in two consecutive periods is small. However, only when d is sufficiently lower than C, then the benefit of player i, net of cost, will be larger in a network than in a market setup.

$$\sum_{k\in N\setminus\{i\}} \frac{1}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)^{d_t(i,k)+1}}+\frac{\delta}{1+\delta}\right]-\frac{1}{2}d > \frac{1}{2}\left[\frac{1}{1+\delta}+\frac{\delta}{1+\delta}-\left(\frac{n-2}{n-1}C+\frac{1}{n-1}d\right)\right].$$

Thus, a tension between individual incentives and efficiency may arise when monitoring costs do not decrease with at least $\frac{\delta}{(1+\delta)^2}$. This further implies that if players are not forward looking, they will not foresee the advantages of developing relationships. When players cannot evaluate the long-run consequences of their own actions, they will always trade in the market setup and no networks will emerge.

6 Conclusions

We study a setting where transactions involve counterparty risk that can be perfectly eliminated through monitoring. Monitoring is costly, but there are economies of scale from monitoring the same counterparty over time. Moreover, when trade is facilitated by intermediaries, they will bargain to acquire a share of the surplus. We show that if the decrease in monitoring costs compensates those agents that pay intermediation rents, then relationships are established such that star-shaped network is formed.

Our results stylize features of over-the-counter markets: links are concentrated around a few players, and larger, richer players are central in the network.

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Appendix

Proposition 2 *Let* $(i,j)_t \in M_t(N)$ *be a pair for which the sequence of intermediaries* $(i_1, i_2, ..., i_k)_t$ *facilitate the generation of the surplus at date* t*. Then, under the indirect financing procedure, the alternating-offers bargaining game has a unique subgame perfect equilibrium. Moreover, if* i *has a liquidity surplus and* j *has an investment opportunity, then the shares players* $(i, i_1, i_2, ..., i_k, j)$ *receive at date* t *are*
$(\frac{1}{(1+\delta)^{d_t(i,j)+1}}, \frac{\delta}{(1+\delta)^{d_t(i_1,j)+2}}, \frac{\delta}{(1+\delta)^{d_t(i_2,j)+2}}, ..., \frac{\delta}{(1+\delta)^{d_t(i_k,j)+2}}, \frac{\delta}{1+\delta})$ *respectively.*

Proof. The indirect financing bargaining procedure has $(k+1)$ bargaining sessions and is solvable by backwards induction. The last session is effectively the same as the Rubinstein (1982) model, where the total share to be allocated between the two players is the remainder of the surplus given by the k partial agreements from the first k sessions.

Denote player i's offer to his opponent in a any bargaining session by x_i. Consider the last bargaining session between players i and i_1, and suppose that the history is such that a total share of Y_k has already been divided in the previous k bargaining sessions. Thus, the remaining surplus that players i and i_1 bargain over is $(1 - Y_k)$. Thus, in the last session there is a unique subgame perfect equilibrium for every Y_k. In equilibrium, player i offers x_i and accepts x_{i_1}, and player i_1 offers x_{i_1} and accepts x_i, where x_i and x_{i_1}satisfy the following two conditions:

$u(x_{i_1}) = \delta u(1 - Y_k - x_i)$

$u(x_i) = \delta u(1 - Y_k - x_{i_1})$

The same conditions must hold for intermediate bargaining session. Thus, any two players in the sequence, $(i_l,\ i_{l+1})$ that bargain over the surplus Y_{k-l} remaining from previous $(k-l-1)$ bargaining sessions offer in equilibrium x_{i_l} and $x_{i_{l+1}}$ such that

$u(x_{i_l}) = \delta u(1 - Y_{k-l} - x_{i_{l+1}})$

$u(x_{i_{l+1}}) = \delta u(1 - Y_{k-l} - x_{i_l})$

By backwards induction, the indirect financing bargaining procedure has a unique subgame perfect equilibrium. For linear utility functions the solution of any bargaining session between i_l and i_{l+1} is given by

$x_{i_l} = \frac{1}{1+\delta} Y_{k-l}$

$x_{i_l+1} = \frac{\delta}{1+\delta} Y_{k-l}$

By iterating forward, we have that $x_{i_l} = \frac{\delta}{(1+\delta)^{d(i_l,j)+2}} Y_0$ for any i_l, with $Y_0 = 1$. This gives the shares players $(i, i_1, i_2, ..., i_k, j)$ receive at date t. ■

Proposition 3 *Suppose that traders receive benefits from their relationships as described in (3.4). Then there exists a Markov Perfect Equilibrium linking strategy such that a star network that connects all traders is an absorbing state of the dynamic network formation game.*

Proof. We show that once the process of network formation reaches a star, it remains there. The proof requires two steps. First, we define a strategy that keeps players connected in a star network. Second, we show that the strategy is a Markov Perfect Equilibrium.

Let $\beta_i(g)$ be the gross benefit trader i obtains from network g: $\beta_i(g) = \pi_i(g) + \frac{1}{2} \sum_{j \in N_g(i)} c_{ij}$, where $\pi_i(g)$ has been introduced in eq. (3.3). Consider the strategy profile $\boldsymbol{\mu}$ such that at any state $s = (g, i)$

- i exchanges one link from a neighbor k ($ik \in g$) to a node j ($ij \notin g$) to maximize $\beta_i(g + ij - ik)$, if $\beta_i(g + ij - ik) > \beta_i(g)$; otherwise she does not change her links.
- j accepts the link, if proposed.

Under the strategy profile $\boldsymbol{\mu}$, a star network g is absorbing for any player i randomly selected to revise her actions at state s. Without loss of generality, we label the central player as i'. In a star network g, for any periphery player i and any player j such that

$ij \notin g$, we have

$$\begin{aligned}\beta_i(g+ij-ii')-\beta(g) \quad = \quad & \left\{\frac{1}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)}+\frac{\delta}{1+\delta}\right]+\frac{1}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)^2}+\frac{\delta}{1+\delta}\right]+\right.\\ & \left.\frac{n-3}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)^3}+\frac{\delta}{1+\delta}\right]\right\}-\\ & \left\{\frac{1}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)}+\frac{\delta}{1+\delta}\right]+\frac{n-2}{n-1}\frac{1}{2}\left[\frac{1}{(1+\delta)^2}+\frac{\delta}{1+\delta}\right]\right\}.\end{aligned}$$

It follows that $\beta_i(g+ij-ii')-\beta(g)<0$. Hence, if they follow the strategy μ, no player will change links (note that the central player is already linked with everyone else).

If we show that the strategy profile $\boldsymbol{\mu}$ is a subgame perfect equilibrium, the proof is complete. We first show that a strategy profile $\boldsymbol{\mu}$ is an equilibrium of the extensive game and then show that is an equilibrium in every subgame. To show that $\boldsymbol{\mu}$ is an equilibrium, we apply the one shot deviation principle in order to check that in a star network:

(i) No periphery player i exchanges the link to the central player i' for a link to another periphery j in a star network g. That is because a periphery trader will receive following such deviation at most

$$V_i(\mu, g+ij-ii') = \left[\beta_i(g+ij-ii')-\frac{1}{2}C\right]+\gamma\left[\beta_i(g+ii'-ij)-\frac{1}{2}C\right]+\sum_{t=2}\gamma^t\left[\beta_i(g)-\frac{1}{2}d\right],$$

as the trader is assumed to return to its strategy whenever given the chance. At the same time, the payoff of a periphery trader from staying in a star is

$$V_i(\mu, g) = \left[\beta_i(g)-\frac{1}{2}d\right]+\lambda\left[\beta_i(g)-\frac{1}{2}d\right]+\sum_{t=2}\gamma^t\left[\beta_i(g)-\frac{1}{2}d\right].$$

Since $\beta_i(g+ij-ii')-\beta_i(g)<0$, it follows that $V_i(\mu, g+ij-ii')<V_i(\mu, g)$.

(ii) No periphery player severes the link with the central player.

If a node is disconnected from the network, she engages in trade with the counterparty selected by the matching technology and her payoff will be at most

$$V_i(\mu, g - ii') = \frac{1}{2}\left[\left(\frac{1}{1+\delta} + \frac{\delta}{1+\delta}\right) - \left(\frac{n-2}{n-1}C + \frac{1}{n-1}d\right)\right] + \gamma\left[\beta_i(g) - \frac{1}{2}C\right] + \sum_{t=2}\gamma^t\left[\beta_i(g) - \frac{1}{2}d\right]$$

since the trader will seek to link to the central counterparty as soon as it is its turn to move. Thus, as long as

$$\beta_i(g) - \frac{1}{2}d \geq \frac{1}{2}\left[\left(\frac{1}{1+\delta} + \frac{\delta}{1+\delta}\right) - \left(\frac{n-2}{n-1}C + \frac{1}{n-1}d\right)\right]$$

we have that $V_i(\mu, g) > V_i(\mu, g - ii')$, and no player wants to be disconnected. This holds if $C - d \geq \frac{\delta}{(1+\delta)^2}$

(iii) The central player does not severe any link with periphery players.

The central trader receives in a star network receives at a given date:

$$V_{i'}(\mu, g) = \frac{1}{1-\gamma}\left\{\frac{1}{2}\left[\frac{1}{(1+\delta)} + \frac{\delta}{1+\delta}\right] + \frac{(n-2)(n-3)}{2}\frac{1}{n-1}\frac{1}{2}\left[\frac{\delta}{(1+\delta)^2} + \frac{\delta}{(1+\delta)^2}\right]\right\} - \frac{1}{1-\gamma}(n-1)\frac{1}{2}d$$

If she severes a link with the periphery, the central player receives at most

$$\begin{aligned} V_{i'}(\mu, g - i'i) = {} & \frac{n-2}{n-1}\left\{\frac{1}{2}\left(\frac{1}{(1+\delta)} + \frac{\delta}{1+\delta}\right) + \frac{(n-4)(n-5)}{2}\frac{1}{n-1}\frac{1}{2}\left(\frac{\delta}{(1+\delta)^2} + \frac{\delta}{(1+\delta)^2}\right)\right\} + \\ & \frac{1}{n-1}\frac{1}{2}\left\{\left(\frac{1}{(1+\delta)} + \frac{\delta}{1+\delta}\right) + \frac{(n-2)(n-3)}{2}\frac{1}{n-1}\frac{1}{2}\left(\frac{\delta}{(1+\delta)^2} + \frac{\delta}{(1+\delta)^2}\right)\right\} - \\ & \frac{n-2}{n-1}\frac{(n-2)}{2}d - \frac{1}{n-1}\frac{1}{2}\left(\frac{n-2}{n-1}C + \frac{1}{n-1}d\right) + \gamma V_{i'}(\mu, g) \end{aligned}$$

where we applied eq. (3.3), taking into account that with probability $\frac{1}{n-1}$ she receives $\frac{1}{2}\left[\left(\frac{1}{1+\delta} + \frac{\delta}{1+\delta}\right) - \left(\frac{n-2}{n-1}C + \frac{1}{n-1}d\right)\right]$ from the disconnected player in the deviation period and that the intermediation benefits are affected as well. As before, traders are assumed to return to their strategy as soon as given the possibility to move.

Thus if

$$d < \frac{n-2}{n-1}\frac{\delta}{(1+\delta)^2}$$

then the central player will not deviate by severing a link with the periphery.

Conditions (i), (ii) and (iii) insure that the profile $\boldsymbol{\mu}$ is an equilibrium. The proof for Proposition 4 establishes that $\boldsymbol{\mu}$ is subgame perfect as well. ■

Proposition 4 *From any minimally connected network there exists a network path to a star-shaped network that is supported as Markov Perfect Equilibrium.*

Proof. Let $\boldsymbol{\mu}$ be the strategy profile defined above.

We construct the following order of play. At any date t, suppose that the current state is g_t and the active player is i_t. If i_t wants to change her position in network g_t, let x be the node for which $\beta_{i_t}(g_t + i_t x - i_t k)$ is maximized (assume $ik \in g_t$). If for any player j $\beta_{i_t}(g_t + i_t j - i_t k) < \beta_i(g)$, let x be i_t's neighbor in the network g_t (by construction i_t will always be a periphery player). Then the active player at date $(t+1)$ is $i_{t+1} \in \{l | d(x, l; g_t) = \max\}$, where $d(x, l; g_t)$ is the distance between x and l in the network g_t. In other words, the active player selected at $(t+1)$ is the furthest away player from x in g_t.

We first show that if players follow the strategy profile $\boldsymbol{\mu}$ and act in the order of play defined above, then from any minimally connected network g_0 there exists a network path $g_1, g_2, ..., g_T$ to a star network. Second, we show that the strategy μ is a Markov Perfect equilibrium on the respective path.

Let i_0 be a randomly selected periphery player in g_0. Suppose that i_0 exchanges one link from a neighbor k_0 ($i_0 k_0 \in g_0$) to a node x ($i_0 x \notin g_0$) to maximize $\beta_{i_0}(g_0 + i_0 x - i_0 k_0)$. Then, in the network $g_1 = g_0 + i_0 x - i_0 k_0$, the player i_1 that follows in the order of play exchanges one link from a neighbor k_1 ($i_1 k_1 \in g_1$) to the same node x ($i_1 x \notin g_1$) to maximize $\beta_{i_1}(g_1 + i_1 x - i_1 k_1)$. The same argument holds at any transition between network g_t to network g_{t+1}. Hence x becomes the center of a star. ■