Egalitarianism and utilitarianism in committees of representatives*

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Abstract

We address the issue of the choice of a voting rule in a committee in which each member acts on behalf of a group of individuals or a constituency of a different size. We assume that the committee of representatives makes dichotomous choices (acceptance/rejection) by vote. Given the size of each group, what is the most adequate voting rule for the committee? We provide answers based on each of the two principles commonly used to make normative assessments in different contexts: egalitarianism and utilitarianism. To that end, we introduce utilities into the model and adopt a normative approach.

Keys words: Collective decision-making, voting, utilitarianism, egalitarianism.

JEL codes: D63, D70, D71.

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1 Introduction

We consider voting situations in which a set of voters or committee handles collective decision-making by means of a voting rule, and is entitled only to vote for or against proposals submitted to it by an external agency. In such a context (that we refer to as a "take-it-or-leave-it committee"[1]), what is the most adequate voting rule for the committee? We answer this question by adopting a welfarist approach, using two principles commonly used to make normative assessments in different contexts: egalitarianism and utilitarianism. The former states that equals should be given equal treatment or, in utility terms, the same utility level. The latter prescribes maximizing the sum of voters' utilities.

Now, consider a committee in which each member acts on behalf of a group of individuals (or a constituency) of a different size. At the design stage for the choice of a voting rule often the only data available are the number groups and their sizes. Thus a well-founded answer for the implementation of either principle requires the model to be enriched beyond these objective data. More precisely, some assumptions about preferences within the groups represented and about their relationship with the votes of their representatives are necessary. In addition, some assumptions concerning the utilities of the people represented when they obtain or fail to obtain their preferred outcome in a committee's decision are necessary. This is handled by adopting the simplest and most "neutral" assumptions concerning individuals' actual preferences and utilities. Once all these ingredients are put into the model the question of the choice of a voting rule in the committee is addressed.

Our approach is resolutely normatively-oriented at the design stage: individuals' actual utilities and actual preferences are ignored, as unknowable or subject to change through time. This point of view leads us to assume that all (so restricted) preference profiles are equally probable instead of dealing with arbitrary probability distributions over arbitrary utility profiles or with profiles of probability distributions over personal utilities.

When voters only represent themselves (i.e. in direct committees) egalitarianism suggests to use any anonymous or symmetric rule, that is, any rule where each voter has one vote. The utilitarian principle determines the choice of the quota, which depends exclusively on the ratio of the marginal utility of winning a negative vote (w.r.t. losing) and the marginal utility of winning a positive vote (w.r.t. losing)[2]. The higher this ratio, the higher the quota, with extreme cases being unanimity and the simple majority. When the groups represented are sufficiently large, it turns out that egalitarianism does not significantly restrict the choice of rule in the committee. Utilitarianism is more demanding, but again can be implemented quite accurately: the representatives weights should be proportional to the square root of the population

[1]See Laruelle and Valenciano (2008) for a distinction between this type of committee and a 'bargaining committee' with capacity to modify proposals and bargain in search of agreement.

[2]For a more precise definition of this ratio, see Section 3.

they represent, and the quota depends on the ratio of marginal utilities, as in direct committees.

Some related work deserves to be mentioned. Barberà and Jackson (2006) consider a very general setup where each representative's vote on the committee is a function of the preferences within his/her group and a probability distribution over these preference profiles is known. They address the question of the decision in the committee that maximizes the expected aggregate utility. Fleurbaey (2009) also addresses the choice of voting rule from a utilitarian approach, where utilities are interpreted rather as "measuring the social value of i's situation for the observer". In these papers, the optimal voting rule depends on the particular configuration of voters' preferences (i.e. utilities) about the issue at stake. Here we deliberately ignore the information about the utilities that voters draw from issues. In this sense we follow an approach similar to Beisbart et al. (2005) or Beisbart and Bovens (2007) who claim that the rule should not depend on the issue which is at stake. Indeed our probabilistic model concerning the individuals' behavior correspond to their "aggregate model". The closest work to ours can be found in Chapter 3 of Laruelle and Valenciano (2008), where these issues are addressed in less general conditions. When the conditions on preferences and decision-making are further conveniently specified some of the conclusions of the papers mentioned are very close to some of those obtained here, as commented later in more detail.

The rest of the paper is organized as follows. We first introduce the basic notation concerning voting rules (Section 2) and the assumptions concerning the utilities of individuals in a vote (Section 3), and the choice of a voting rule in direct committees is discussed in Section 4. In Section 5 expected utilities of individuals represented for decisions made by a committee of representatives are calculated. The choice of a voting rule in such committees is addressed from the egalitarian (Section 6) and utilitarian (Section 7) points of view. Section 8 contains some concluding remarks. Finally, an Appendix contains the proofs of lemmas and propositions.

2 Voting rules

Let $N = \{1, 2, .., n\}$ be the set of *seat* labels. Voters are labelled by their seat labels. A *vote configuration* is a possible result of a vote that lists the vote cast by the voter occupying each seat. If voters only vote "yes" or "no", there are 2^n possible vote configurations. We represent any *vote configuration* by the set of the yes-voters $S \subseteq N$ (the voters in $N \backslash S$ voting "no"). The cardinality of S is denoted by s or $\#S$. For $i \in S$, we write $S \backslash i$ instead of $S \backslash \{i\}$.

An N-*voting rule* is specified by the set $\mathcal{W} \subseteq 2^N$ of vote configurations that would lead to the passage of a proposal. This set of *winning configurations* is assumed to satisfy the following requirements: (i) $N \in \mathcal{W}$; (ii) $\emptyset \notin \mathcal{W}$; (iii) If $S \in \mathcal{W}$, then $T \in \mathcal{W}$ for any T containing S; and (iv) If $S \in \mathcal{W}$ then $N \backslash S \notin \mathcal{W}$. The first two conditions are Pareto efficiency conditions[3], while the third is a condition of monotonicity. The last condition prevents the passage of a proposal and its negation if it was supported by S and $N\backslash S$, respectively. When only the first three conditions are satisfied the rule is said to be *improper*. If the rule is improper it may be the case that depending on the way an issue is submitted to the committee, both its acceptance and its rejection are possible for a given preference profile. For this reason all rules to make decisions of substance to be found in real world committees are proper. Given that our purpose is exploring the possibility of founding a recommendation for the choice of a voting rule in real world contexts we constrain our attention to proper rules[4].

A *weighted majority rule,* denoted by $\mathcal{W}^{(w,Q)}$*,* is specified by a system of positive *weights* $w = (w_1, .., w_n)$, and a *quota* $Q > 0$*,* so that the final result is "yes" if the sum of the weights in favor of the proposal exceeds the quota, that is:

$$\mathcal{W}^{(w,Q)} = \{S \subseteq N : \sum_{i \in S} w_i > Q\},$$

which can be specified alternatively in terms of the *relative quota* $q := \frac{Q}{\sum_{j \in N} w_j}$, as

$$\mathcal{W}^{(w,q)} = \{S \subseteq N : \frac{\sum_{i \in S} w_i}{\sum_{j \in N} w_j} > q\}.$$

If all weights are equal the corresponding voting rule is *symmetric*[5], and referred to as a *q-majority rule*. The *simple majority* rule, denoted $\mathcal{W}^{SM}$, corresponds to the case $q = \frac{1}{2}$.

3 Utility of a vote

We deal here with what can be called a take-it-or-leave-it committee to which proposals are submitted which the committee can only accept or reject by a vote. In particular no room is left for negotiating or modifying the proposals. We assume that no voter is indifferent between acceptance and rejection on any issue, and voting is not costly. In these conditions, we assume that voters are rational in the sense that they vote "yes" or "not" on an issue only depending on whether their utility is higher if the proposal is accepted or higher if it is rejected.

Thus, if S denotes the resulting vote configuration on a given issue, voter i's vote is "yes" (i.e. $i \in S$) if and only if voter i's utility in case of acceptance, denoted by $u_{i^+}(Acc) = A^{i+}$, is greater than in case of rejection, denoted by $u_{i^+}(Rej) = R^{i+}$. As the

[3]When preferences are strict in a take-it-or-leave-it voters' rational behaviour follows preferences trivially, so that for the extreme configurations of votes, N and $\varnothing$, acceptance and rejection are Pareto efficient decisions in either case respectively.

[4]Barberà and Jackson (2006) and Fleurbaey (2009) do not impose this constraint.

[5]A rule is symmetric or anonymous if the permutation of any winning vote configuration is also winning. In other words a configuration is winning or not depending on its number of "yes"-voters.

proposal is accepted if $S \in \mathcal{W}$, and rejected if $S \notin \mathcal{W}$, we use the following notation, the superscript "+" indicating that i is a "yes"-voter (i.e. $i \in S$):

$$u_{i^+}(\mathcal{W}, S) = \begin{cases} u_{i^+}(Acc) = A^{i+} \text{ if } S \in \mathcal{W}, \\ u_{i^+}(Rej) = R^{i+} \text{ if } S \notin \mathcal{W}, \end{cases}$$

with $A^{i+} > R^{i+}$. Similarly if a voter j is against the proposal ($j \notin S$), her or his utility depends on whether the proposal is accepted or rejected. In this case we write

$$u_{j^-}(\mathcal{W}, S) = \begin{cases} u_{j^-}(Acc) = A^{j-} \text{ if } S \in \mathcal{W}, \\ u_{j^-}(Rej) = R^{j-} \text{ if } S \notin \mathcal{W}, \end{cases}$$

with $A^{j-} < R^{j-}$. We assume two forms of "anonymity". First, concerning the issues, we assume that the four possible utilities for a voter (i.e. $A^{i+}, R^{i+}, A^{j-}, R^{j-}$) are the same for every issue. Second, "anonymity" concerning the voters: we assume that the intensity of preferences is identical for all voters. This sort of "anonymity of issues" or "neutrality" as well as the "anonymity of voters" seems the less biased assumption, and can be justified from a normative point of view at the design stage if it is not known in advance what degree of importance each voter will give to each issue. We can then drop "i" and "j" in the superindices and write just A^+ instead of A^{i+}, R^+ instead of R^{i+}, etc.

We can summarize the above assumptions like this: If a decision is made by means of voting rule $\mathcal{W}$ and the resulting vote configuration is S the utility of voter i is given by

$$u_i(\mathcal{W}, S) = \begin{cases} A^+ \text{ if } i \in S \in \mathcal{W}, \\ R^+ \text{ if } i \in S \notin \mathcal{W}, \\ R^- \text{ if } i \notin S \notin \mathcal{W}, \\ A^- \text{ if } i \notin S \in \mathcal{W}, \end{cases} \tag{1}$$

with $A^+ > R^+$ and $A^- < R^-$.

Observe that from the point of view of any voter, comparing A^+ or R^+ (his/her utilities in case of acceptance or rejection of *a proposal that he/she supports*) with A^- or R^- (his/her utilities in case of acceptance or rejection of *a proposal that he/she rejects*) requires *the utility to be compared for different issues*: one that the voter favors and one that the voter is against. Although various assumptions can be made in this respect[6], no further assumption is necessary. As shown below, in the model under consideration here the relevant data are the differences

$$\Delta^+ := A^+ - R^+ > 0 \quad \text{and} \quad \Delta^- := R^- - A^- > 0, \tag{2}$$

that is to say, the marginal utility of winning a vote (w.r.t. losing) for an affirmative voter (Δ^+), and the marginal utility of winning a vote (w.r.t. losing) for a negative voter (Δ^-).

[6] For instance Barberà and Jackson (2006) assume (in our notation) $R^{i-} = R^{i+}$, Beisbart and Bovens (2007) assume $R^{i-} = R^{i+} = 0$, Fleurbaey (2009) assumes instead $A^{i+} = R^{i-}$ and $R^{i+} = A^{i-}$, while Laruelle and Valenciano (2008) assume $R^+ = A^-$ and $R^- + A^+ = 1$.

We assume that each voter belongs to one of two possible "types": either a "yes"-voter (i.e. one whose utilities in case of acceptance and rejection are given respectively by A^+ and R^+), or a "no"-voter (i.e. one whose utilities in case of acceptance and rejection are given respectively by A^- and R^-). In incomplete information terms, each voter "learns her/his type" once the issue is known. We assume that each voter is of one type or the other with probability 1/2 independently of the others' types (or equivalently, all type-profiles are equiprobable). That is, if S denotes the resulting vote configuration,

$$P(i \in S) = P(i \notin S) = \frac{1}{2} \quad \text{for all } i \in N,$$

where $P(i \in S)$ denotes the probability of i voting "yes", and $P(i \notin S)$ the probability of i voting "no". Thus, if $p(S)$ denotes the probability of occurrence of vote configuration S, we have:

$$p(S) = \frac{1}{2^n} \quad \text{for all } S \subseteq N, \tag{3}$$

(i.e. all preference/vote configurations are equally probable).

All the calculations and results that follow with a normative recommendation purpose are based on the model summarized by (1) and the "prior" (3).

The following Lemma gives the expected utility of a voter under the above assumptions. Its proof is given in the Appendix.

Lemma 1 *Assuming (1) and (3), the expected utility of voter i is given by*

$$\begin{aligned} E_p\left[u_i(\mathcal{W}, S)\right] &= \frac{1}{2}(R^+ + R^-) + \frac{1}{2}(\Delta^+ - \Delta^-) \sum_{S:S\in\mathcal{W}} \frac{1}{2^n} \\ &\quad + \frac{1}{2}(\Delta^+ + \Delta^-) \sum_{\substack{S:i\in S\in\mathcal{W} \\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^n}. \end{aligned} \tag{4}$$

This result deserves some comment. According to formula (4) the expected utility of each voter is the sum of three terms: one which is *independent of the rule* ($\frac{1}{2}(R^+ + R^-)$), one which depends on the rule but is *common to all voters*, and a third which also depends on the rule but is the *only one that may differ from voter to voter*. In the second term Coleman's (1971) *a priori* probability of a proposal being accepted appears[7]. We denote this as:

$$\alpha(\mathcal{W}) := \sum_{S:S\in\mathcal{W}} \frac{1}{2^n}, \tag{5}$$

[7] It is usually interpreted as a measure of the ease to pass a proposal with a given rule.

while in the third term, specific to each voter i, voter i's Banzhaf (1965) index[8] for rule $\mathcal{W}$ appears. We denote this as:

$$Bz_i(\mathcal{W}) := \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^{n-1}}. \tag{6}$$

Voter i's expected utility given by (4) can thus be rewritten as:

$$E_p\left[u_i(\mathcal{W},S)\right] = \frac{1}{2}(R^+ + R^-) + \frac{1}{2}(\Delta^+ - \Delta^-)\alpha(\mathcal{W}) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_i(\mathcal{W}). \tag{7}$$

Note that setting $\Delta^+ = \Delta^-$ and $R^+ = R^- = 0$ in (7) yields Beisbart and Bovens' (2007) equation (12) in their "aggregate" default model.

We close this section by recalling some inequalities for these indices that will be used later:

$$Bz_i(\mathcal{W}) = \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}} \frac{1}{2^{n-1}} \leq \sum_{S:S\in\mathcal{W}} \frac{1}{2^{n-1}} = 2\sum_{S:S\in\mathcal{W}} \frac{1}{2^n} = 2\alpha(\mathcal{W}), \tag{8}$$

and

$$\alpha(\mathcal{W}) = \sum_{S:S\in\mathcal{W}} \frac{1}{2^n} = \frac{\#\mathcal{W}}{2^n} \leq \frac{1}{2}. \tag{9}$$

The last inequality arises because $\#\mathcal{W} \leq 2^{n-1}$ (which would not necessarily hold if $\mathcal{W}$ were improper).

4 Egalitarianism and utilitarianism in a direct committee

In the current framework a voting rule $\mathcal{W}$ implements the egalitarian principle if all voters have the same expected utilities, that is, if

$$E_p\left[u_i(\mathcal{W},S)\right] = E_p\left[u_j(\mathcal{W},S)\right], \text{ for all } i,j. \tag{10}$$

In view of (7), this holds if and only if for all i,j, $Bz_i(\mathcal{W}) = Bz_j(\mathcal{W})$. Therefore we have the following

Proposition 2 *Any q-majority rule implements the egalitarian principle.*

[8] It gives the likelihood of i's vote being decisive in a vote under voting rule $\mathcal{W}$ assuming all vote configurations equiprobable. In the voting power literature it is interpreted as a measure of "a priori voting power".

Note that q-majority rules are not the only possible rules for implementing egalitarianism. Choose for instance $N = A \cup B$, with $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8, 9, 10\}$, and let $\mathcal{W}$ be the rule

$$\mathcal{W} = \{S \subseteq N : \#(A \cap S) > 2 \ \& \ \#(B \cap S) > 2\}.$$

This rule satisfies condition (10) in spite of not being symmetric.

Utilitarianism prescribes the choice of a voting rule that maximizes the aggregate expected utility, that is, the rule that solves the maximization problem

$$Max \sum_{i \in N} E_p \left[u_i(\mathcal{W}, S)\right]. \tag{11}$$

In the Appendix we prove the following equivalence result:

Lemma 3 *The maximization problem (11) is equivalent to*

$$Max \sum_{S \in \mathcal{W}} (s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n). \tag{12}$$

In view of this equivalence the best rule seems to be one that considers as winning a vote configuration S as far as $s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n$ is positive, but this is equivalent to consider S winning if its size s is such that $\frac{s}{n} > \frac{\Delta^-}{\Delta^+ + \Delta^-}$. If $\frac{\Delta^-}{\Delta^+ + \Delta^-} \geq \frac{1}{2}$ (or, equivalently, $\Delta^- \geq \Delta^+$) this condition defines a voting rule and we thus have the following result:

Proposition 4 *If $\Delta^- \geq \Delta^+$ the q-majority rule implements the utilitarian principle, with*

$$q = \frac{\Delta^-}{\Delta^+ + \Delta^-}.$$

Note that what matters is the marginal utility of winning (w.r.t. losing) an affirmative vote (Δ^+), and that of winning a negative vote (Δ^-). Also note that when they are equal, then the simple majority is optimal. Related results are those of Rae (1969) and Taylor (1969), who show that the majority rule minimizes the average dissatisfaction (which corresponds to the case where $\Delta^+ = \Delta^-$ in our framework) over the class of symmetric rules. Later Curtis (1972) extends this result allowing for different probability distributions, while Straffin (1977) considers the whole class of voting rules. We recover Proposition 4 by restricting Barberà and Jackson's (2006) Corollary 1 to our framework and assuming that each country consists of a single citizen (coefficient γ in Barberà and Jackson (2006) is the ratio Δ^-/Δ^+).

A problem appears when $\Delta^- < \Delta^+$. In this case condition $s > \frac{\Delta^-}{\Delta^+ + \Delta^-} n$ may define an improper rule. The solution seems to be to keep the quota as low as possible, i.e. at 1/2. Thus the simple majority seems to be the best of *all* proper rules. Nevertheless the simple majority rule may fail to be the utilitarian optimum. The reason is that as long as it is possible to add vote configurations whose size is greater than $\frac{\Delta^-}{\Delta^+ + \Delta^-}n$ to the set of winning configurations without making the rule improper, the aggregate expected utility increases. The following result proves, though, that the simple majority is either optimal or "almost" optimal.

Proposition 5 *With $\Delta^- < \Delta^+$, if $\mathcal{W}$ is a voting rule that best implements the utilitarian principle, then for all $S \in \mathcal{W}$ it holds that $s \geq \frac{n}{2}$.*

Observe that *if n is odd* the necessary condition of optimality established by this proposition (whose proof is given in the Appendix) defines simple majority. So that this rule implements the utilitarian principle also with $\Delta^- < \Delta^+$ if n is odd, while for n even it only *almost* implements it, because in this case the necessary condition does *not* define a rule as some configuration(s) S with $s = \frac{n}{2}$ may enter an optimal rule. But still in this case the only *symmetric* rule that satisfies the necessary condition is the simple majority. Then we have:

Proposition 6 *If $\Delta^- < \Delta^+$, then if n is odd the simple majority is the voting rule that best implements the utilitarian principle. If n is even the simple majority is the utilitarian-best of all symmetric rules.*

In short, egalitarianism and utilitarianism are compatible[9] and easy to implement in a direct committee. Egalitarianism suggests q-majority rules, while the utilitarian principle determines the choice of the quota, which varies with the ratio $\frac{\Delta^-}{\Delta^+}$. The higher this ratio, the higher the quota, with extreme cases being unanimity and the simple majority.

5 Utilities in a committee of representatives

Now consider the case of a committee of representatives in which each member acts on behalf of a group of a different size[10]. Let $N = \{1, 2, ..., n\}$ be the members of the committee, which makes decisions by means of a voting rule $\mathcal{W}_N$. Each $i \in N$ represents the m_i individuals of group M_i. Groups are assumed to be pairwise disjoint. Let $M := \cup_{i \in N} M_i$ be the set of all individuals represented, and m their number. If each representative i is assumed to follow the majority opinion in group M_i on every issue, the situation may be modelled as a decision made by the vote of the individuals in M by means of an M-voting rule that we denote by $\mathcal{W}_M$ in the following way. For

[9] If, unlike in the model, different intensities of preferences were assumed, the utilitarian principle would not yield any more symmetric rules and different weights should be given to different voters (Fleurbaey, 2009).

[10] Among others the reader my keep in mind the Council of Ministers of the European Union as a good example.

any vote configuration among the individuals $S \subseteq M$, and each $i \in N$, let S_i the set of individuals in S that belong to M_i, that is

$$S_i = S \cap M_i,$$

which determines representative i's vote on the committee of representatives. That is, $i \in N$ votes "yes" if

$$\#S_i > \frac{m_i}{2}.$$

Thus, if $S \subseteq M$ is the vote configuration in M, the resulting vote configuration in the committee of representatives is

$$S_N = \{i \in N : \#S_i > \frac{m_i}{2}\},$$

and vote configuration $S \subseteq M$ is winning if $S_N \in \mathcal{W}_N$. Thus the decision-making procedure by M can be described by means of M-voting rule

$$\mathcal{W}_M = \{S \subseteq M : S_N \in \mathcal{W}_N\}.$$

What follows is a discussion about the choice of the voting rule $\mathcal{W}_N$ in the committee from the points of view of egalitarianism and utilitarianism, *but focused on the utilities of the individuals represented* (i.e. in M). Two interpretations are possible. First, the conclusions apply to a situation in which actual decision-making follows these two steps, so that each representative acts like a mechanical transmitter of the choice of the majority in his/her group. In this case, rule $\mathcal{W}_M$ is an exact description of the actual decision-making, and the usual notation is that of a "composite voting rule":

$$\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}_{M_1}^{SM}, ..., \mathcal{W}_{M_n}^{SM}].$$

Second, such a two-step decision-making procedure can also be taken as a model of a situation in which only the rule in the committee of representatives actually exists, and their following the majority in their respective group is an approximation that allows the effects of the chosen voting rule to be examined from the point of view of the people represented[11].

We assume that individuals *in M* have expected utility preferences with utilities given by (1) with $\mathcal{W} = \mathcal{W}_M$. We also assume that individuals have the same probability of being of one type or the other, so that all preference/vote configurations *in M* are equally probable, and denote by p the probability distribution such that $p(S) = \frac{1}{2^m}$, for all $S \subseteq M$. Thus we can apply Lemma 1 to the $\mathcal{W}_M$ rule as we do below.

In the following, two approximations concerning the composite rule $\mathcal{W}_M$ will be used (see e.g. Laruelle and Valenciano (2008)). First, if all m_i are large enough, then for all $i \in N$ and $k \in M_i$:

$$Bz_k(\mathcal{W}_M) \simeq \sqrt{\frac{2}{\pi m_i}} Bz_i(\mathcal{W}_N). \tag{13}$$

[11] Other two-stage models are considered in Barberà and Jackson (2006) or Beisbart and Bovens (2007).

It is also easy to see (in fact equality holds if all m_i are odd) that

$$\alpha(\mathcal{W}_M) \simeq \alpha(\mathcal{W}_N). \tag{14}$$

6 Egalitarianism in a committee of representatives

In terms of the above two-stage model, the egalitarian principle is satisfied if the expected utility of any two individuals in M is the same, irrespective of what group they belong to. That is, if

$$E_p\left[u_k(\mathcal{W}_M, S)\right] = E_p\left[u_l(\mathcal{W}_M, S)\right], \quad \text{for all } k, l \in M.$$

Given (7), egalitarianism is satisfied if and only if for all $k, l \in M$:

$$Bz_k(\mathcal{W}_M) = Bz_l(\mathcal{W}_M). \tag{15}$$

This is a requirement frequently made in voting power literature, but *on completely different grounds*. Equality (15) is usually supported on grounds of fairness by the double claim that what matters is "voting power" and that this is measured by the Banzhaf index. Hence the motivation of (15) in that literature: achieving equal "voting power" for all citizens. Here (15) is just a necessary and sufficient condition for egalitarianism in terms of utilities[12]. As we show below, this difference in logical basis entails important consequences.

In general, there is no rule in the committee $\mathcal{W}_N$ such that (15) holds for the composite rule $\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}^{SM}_{M_1}, ..., \mathcal{W}^{SM}_{M_n}]$. A natural question that arises then is how far one is from egalitarianism for a given rule. Here the difference with the traditional voting power approach becomes apparent. In the present model comparisons are to be made between expected utilities ($E_p\left[u_k(\mathcal{W}_M, S)\right]$), and *not* between Banzhaf indices interpreted as absolute measures of "a priori voting power".

When all the groups are sufficiently large, the following proposition (Prop. 20 in Laruelle and Valenciano (2008) for less general conditions) gives some significant bounds for absolute and relative pairwise comparisons of expected utilities. The proof is given in the Appendix. In our model, we have

$$u_{Max} = Max\{A^+, R^+, A^-, R^-\} = Max\{A^+, R^-\}$$

[12] When all m_i's are large enough, it turns out using approximation (13) that equality (15) is (with great approximation) achieved if

$$\frac{Bz_i(\mathcal{W}_N)}{\sqrt{m_i}} = \frac{Bz_j(\mathcal{W}_N)}{\sqrt{m_j}} \quad \text{for any } i, j \in N,$$

a result which is known in the literature as the (first) "square root rule". Several papers address the problem of obtaining the rule that satisfies this condition (see e.g. Laruelle and Widgrén (1998), Leech (2002), Slomczynski and Zyczkowski (2006)).

and

$$u_{Min} = Min\{A^+, R^+, A^-, R^-\} = Min\{A^-, R^+\}.$$

Proposition 7 *Let $\mathcal{W}_M$ be the composite rule $\mathcal{W}_M = \mathcal{W}_N[\mathcal{W}^{SM}_{M_1}, ..., \mathcal{W}^{SM}_{M_n}]$, then, if all m_i are large enough to consider (13) and (14) as good approximations, for all k, l we have*

$$\frac{|E_p[u_k(\mathcal{W}_M, S)] - E_p[u_l(\mathcal{W}_M, S)]|}{u_{Max} - u_{Min}} \leq \frac{1}{2}\xi \tag{16}$$

and

$$\frac{E_p[u_k(\mathcal{W}_M, S)] - u_{Min}}{E_p[u_l(\mathcal{W}_M, S)] - u_{Min}} \leq 1 + \xi, \tag{17}$$

where ξ is given by

$$\xi := \sqrt{\frac{2}{\pi Min_{i\in N} m_i}}. \tag{18}$$

Thus, in the light of the present model, the egalitarian principle seems of little consequence after all for the choice of the voting rule in a take-it-or-leave-it committee of representatives when the groups are sufficiently large. This is because according to the underlying model in which each individual independently is equally likely to be a supporter or a rejecter, even the expectations of winning a vote of an individual from a group whose representative was a null voter in the committee are rather close to that of one from a group whose representative was a dictator. This conclusion is substantively different from the claim of the voting power approach due to the fact that we base our analysis on voters' utilities. The reader interested in the critical implications on traditional voting power approach is addressed to Laruelle and Valenciano (2008), where this is discussed in more detail. Of course this conclusion would not hold for different probabilistic assumption on voters' behavior (as in Beisbart and Bovens' (2007) "interest model").

7 Utilitarianism in a committee of representatives

In the current model implementing utilitarianism means choosing a voting rule in the committee $\mathcal{W}_N$ that maximizes the aggregate expected utility in M, i.e. solving the problem:

$$Max \sum_{k\in M} E_p[u_k(\mathcal{W}_M, S)].$$

Or equivalently, as aggregation and expectation permute, a voting rule that maximizes the expected aggregate utility, i.e. solves the problem:

$$Max\ E_p[\sum_{k\in M} u_k(\mathcal{W}_M, S)].$$

Therefore the point is to choose $\mathcal{W}_N$ so as to maximize the latter expectation. Note that while each M-vote configuration determines an N-vote configuration, the same N-vote configuration may result from different M-vote configurations. Thus the utilitarian best rule consists of making the decision for which this expectation is the highest for each vote configuration in the committee. More precisely, if the vote configuration in the committee is $C \subseteq N$, the best decision is to accept the proposal if

$$E_p[\sum_{k\in M} u_k \mid C \ \& \ Acc] > E_p[\sum_{k\in M} u_k \mid C \ \& \ Rej]. \tag{19}$$

The proof of the following equivalence is given in the Appendix:

Proposition 8 *Assuming that all the groups represented are large enough, inequality (19) is equivalent to*

$$\sum_{i\in C} \sqrt{m_i} > \frac{1}{2}\sum_{i\in N} \sqrt{m_i} + \frac{m}{2}\frac{\Delta^- - \Delta^+}{\Delta^+ + \Delta^-}\sqrt{\frac{\pi}{2}}. \tag{20}$$

If $\Delta^- \geq \Delta^+$ condition (20) defines a weighted majority rule with weights $w_i = \sqrt{m_i}$, and a quota that depends on the population figures and the ratio between the marginal utility of winning a negative vote w.r.t. that of winning a positive one (i.e. $\frac{\Delta^-}{\Delta^+}$), given by

$$Q(\tfrac{\Delta^-}{\Delta^+}) = \frac{1}{2}\sum_{i\in N} \sqrt{m_i} + \frac{1}{2}\frac{\frac{\Delta^-}{\Delta^+} - 1}{\frac{\Delta^-}{\Delta^+} + 1} m\sqrt{\frac{\pi}{2}}, \tag{21}$$

or a relative quota given by

$$q(\tfrac{\Delta^-}{\Delta^+}) = \frac{1}{2} + \frac{1}{2}\frac{\frac{\Delta^-}{\Delta^+} - 1}{\frac{\Delta^-}{\Delta^+} + 1}\frac{m\sqrt{\frac{\pi}{2}}}{\sum_{i\in N}\sqrt{m_i}}. \tag{22}$$

Observe that when $\Delta^+ = \Delta^-$ the relative quota is $\frac{1}{2}$, and increases when $\frac{\Delta^-}{\Delta^+}$ increases. We then have the following result:

Proposition 9 *Assuming that all the groups represented are large enough, if the marginal utility of winning a negative vote is not smaller than the marginal utility of winning a positive vote (i.e. $\Delta^- \geq \Delta^+$), the weighted majority rule $\mathcal{W}_N = \mathcal{W}_N^{(w,q)}$ in the committee for weights $w_i = \sqrt{m_i}$ and the relative quota q given by (22) implements the utilitarian principle very closely. In particular, when the marginal utility of winning a negative vote is the same as that of winning a positive vote, i.e. $\Delta^- = \Delta^+$, the optimal quota is one half.*

Observe that in the case $\Delta^- = \Delta^+$, the recommendation coincides with that of Beisbart and Bovens' (2007) in their "aggregate default model". This recommendation is also well-known as the "second square root rule", which is usually derived from other than utilitarian considerations (see Moriss (1987, 2002) and Felsenthal and Machover (1999)). In a sense our approach is complementary of that of Barberà and Jackson (2006). They approach the issue of the optimal decision in a very abstract framework commented in the introduction that leads to a very general conclusion. When sufficiently further specified, this conclusion "endogenously" yields a concrete voting rule in the usual sense. We instead address the issue directly for and in terms of voting rules, with the restrictions that this imposes. For instance they allow for ties (tossing a coin to deal with them), while we restrict our attention to real-world voting rules (that are proper for decisions of substance), which complicates the detailed answer, but permits us to provide an explicit formulation of the quota as a function of the available data of our framework.

Now consider the case in which $\Delta^- < \Delta^+$, so that (20) may define an *improper* voting rule. In this case the best that can be done seems to be to lower the quota Q as much as possible in such a way that $\mathcal{W}_N^{(w,Q)}$, for weights $w_i = \sqrt{m_i}$, is a proper rule. Namely, take

$$\bar{Q} := Min\ \{Q : \sum_{i\in C} w_i > Q \Rightarrow \sum_{j\in N\setminus C} w_j \leq Q \quad (\forall C \subseteq N)\}. \tag{23}$$

The following proposition, whose proof is given in the Appendix, shows how $W^{(w,\bar{Q})}$ is *almost* the utilitarian optimum.

Proposition 10 *If $\Delta^- < \Delta^+$ and a voting rule $\mathcal{W}_N$ implements the utilitarian optimum according to the approximation based on (25) and (26), then for all $C \in \mathcal{W}_N$, $\sum_{i\in C} w_i \geq \bar{Q}$.*

Therefore a utilitarian-optimal voting rule (according to approximations (25) and (26)) should contain the winning configurations in $W^{(w,\bar{Q})}$ *plus some* configurations whose weight equals the quota $\bar{Q}$ if such a thing is possible. Then we have the following

Corollary 11 *Assuming that all the groups represented are large enough, if the marginal utility of winning a negative vote is smaller than the marginal utility of winning a positive vote (i.e. $\Delta^- < \Delta^+$), the weighted majority rule $\mathcal{W}_N = \mathcal{W}_N^{(w,\bar{Q})}$ in the committee for weights $w_i = \sqrt{m_i}$ and quota $\bar{Q}$ given by (23) implements the utilitarian principle with great approximation.*

8 Concluding remarks

Weighted majority rules are often used in real-world committees of different types. We have examined the rationale of their choice in committees of representatives based on a simple model. In this type of committee, when voters represent groups of different sizes egalitarianism may suggest that different numbers of votes be assigned to different voters. This intuition proves not to be true if the sizes of the groups represented are large enough. It is utilitarianism that motivates the choice of different weights and quotas depending on the sizes of groups. If all groups are large, utilitarianism recommends weights proportional to the square root of the size of the group represented, while the quota depends on the ratio between the marginal utility of winning a negative vote w.r.t. the marginal utility of winning a positive vote. When this ratio is one, i.e. those marginal utilities are equal, the relative quota recommended is $\frac{1}{2}$ (this is the well-known "second square root rule"). As this ratio increases the recommended quota increases and is calculated in terms of this ratio, while for a ratio below $\frac{1}{2}$ keeping the quota at $\frac{1}{2}$ "almost" implements the utilitarian optimum. This gives a rationale for the use of different sizes of majority: the more important it is not to have a proposal that one does not like imposed on one, the higher the quota should be. This setting also allows for a clear comparison with some well-known results in voting power literature.

Most of the results in this paper are anticipated in Laruelle and Valenciano (2008) under less general conditions. It is worth remarking that the conclusions presented here are established in a more general model and only depend on the marginal utilities of winning a positive vote and winning a negative vote or, more strictly speaking, on a single parameter: their ratio.

Finally it is important to stress that the conclusions do not apply to bargaining committees where negotiation about the proposal prior to the vote is a crucial feature of the voting situation (see Laruelle and Valenciano 2007 and 2008).

9 Appendix

9.1 Proof of Lemma 1

By (1), the expected utility of voter i is given by

$$E_p\left[u_i(\mathcal{W},S)\right] = A^+P(i\in S\in\mathcal{W}) + R^+P(i\in S\notin\mathcal{W})$$

$$+A^-P(i\notin S\in\mathcal{W}) + R^-P(i\notin S\notin\mathcal{W}),$$

where $P(i\in S\in\mathcal{W})$ is the probability that i votes "yes" and the proposal is accepted (i.e. "$i\in S$ & $S\in\mathcal{W}$"), etc. In view of the following equalities:

$$P(i\in S\notin\mathcal{W}) = P(i\in S) - P(i\in S\in\mathcal{W})$$

$$P(i\notin S\in\mathcal{W}) = P(S\in\mathcal{W}) - P(i\in S\in\mathcal{W}),$$

$$P(i\notin S\notin\mathcal{W}) = P(i\notin S) - P(i\notin S\in\mathcal{W})$$

$$= P(i\notin S) - (P(S\in\mathcal{W}) - P(i\in S\in\mathcal{W})),$$

the above expression can be rewritten as

$$E_p\left[u_i(\mathcal{W},S)\right] = (A^+ - R^+ + R^- - A^-)P(i \in S \in \mathcal{W}) + R^+P(i \in S)$$
$$-(R^- - A^-)P(S \in \mathcal{W}) + R^-P(i \notin S).$$

By (3), using notation (2), we have:

$$E_p\left[u_i(\mathcal{W},S)\right] = (\Delta^+ + \Delta^-)\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n} - \Delta^-\sum_{S:S\in\mathcal{W}}\frac{1}{2^n} + \frac{1}{2}(R^+ + R^-). \tag{24}$$

Note that

$$\begin{aligned}\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n} &= \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n} + \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\in\mathcal{W}}}\frac{1}{2^n} = \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n} + \sum_{T:i\notin T\in\mathcal{W}}\frac{1}{2^n}\\ &= \sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n} + \sum_{S:S\in\mathcal{W}}\frac{1}{2^n} - \sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n}.\end{aligned}$$

Therefore

$$\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n} = \frac{1}{2}\sum_{\substack{S:i\in S\in\mathcal{W}\\ S\setminus i\notin\mathcal{W}}}\frac{1}{2^n} + \frac{1}{2}\sum_{S:S\in\mathcal{W}}\frac{1}{2^n}.$$

Then substituting this in (24) we have (4).

9.2 Proof of Lemma 3

In view of (24), the aggregate expected utility is

$$\sum_{i\in N} E_p\left[u_i(\mathcal{W},S)\right]$$

$$\begin{aligned}&= \sum_{i\in N}\left[(\Delta^+ + \Delta^-)\sum_{S:i\in S\in\mathcal{W}}\frac{1}{2^n} - \Delta^-\sum_{S:S\in\mathcal{W}}\frac{1}{2^n} + \frac{1}{2}(R^+ + R^-)\right]\\ &= \frac{1}{2^n}\sum_{S:S\in\mathcal{W}}\left[s(\Delta^+ + \Delta^-) - n\Delta^-\right] + \frac{1}{2}n(R^- + R^+)\\ &= \frac{1}{2^n}(\Delta^+ + \Delta^-)\sum_{S:S\in\mathcal{W}}(s - \frac{\Delta^-}{\Delta^+ + \Delta^-}n) + \frac{1}{2}n(R^- + R^+)\end{aligned}$$

As $\Delta^+ > 0$ and $\Delta^- > 0$, and the second term does not depend on the rule, the result follows.

9.3 Proof of Proposition 5

Let $\mathcal{W}$ be a voting rule. We prove that if $t < \frac{n}{2}$ holds for some $T \in \mathcal{W}$ then $\mathcal{W}$ does not solve (12). Assume this and take

$$\mathcal{W}' = (\mathcal{W} \backslash \{T\}) \cup \{S \subseteq N : N \backslash T \subseteq S\},$$

that is, $\mathcal{W}'$ is obtained from $\mathcal{W}$ by making T losing and adding as winning all configurations containing $N \backslash T$. As $N \backslash T$ intersects all $S \in \mathcal{W} \backslash \{T\}$, $\mathcal{W}'$ is a (proper) voting rule. Then we have

$$\sum_{S \in \mathcal{W}'} (s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n)$$

$$\geqslant \sum_{S \in \mathcal{W}} (s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n) - (t - \frac{\Delta^-}{\Delta^+ + \Delta^-} n) + ((n-t) - \frac{\Delta^-}{\Delta^+ + \Delta^-} n)$$

$$= \sum_{S \in \mathcal{W}} (s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n) + (n - 2t) > \sum_{S \in \mathcal{W}} (s - \frac{\Delta^-}{\Delta^+ + \Delta^-} n),$$

where the last inequality holds because $t < \frac{n}{2}$. Thus $\mathcal{W}$ does not solve (12) and consequently is not utilitarian-optimal.

9.4 Proof of Proposition 7

Assume $Bz_k(\mathcal{W}_M) \neq Bz_l(\mathcal{W}_M)$ (if they are equal the result is trivial from (7)). Then, assuming for instance that $Bz_k(\mathcal{W}_M) > Bz_l(\mathcal{W}_M)$, in view of (7) we have:

$$\begin{aligned}
&\frac{|E_p[u_k(\mathcal{W}_M, S)] - E_p[u_l(\mathcal{W}_M, S)]|}{u_{Max} - u_{Min}} \\
= \;& \frac{1}{4} \frac{\Delta^+ + \Delta^-}{Max\{A^+, R^-\} - Min\{R^+, A^-\}} (Bz_k(\mathcal{W}_M) - Bz_l(\mathcal{W}_M)) \\
\leq \;& \frac{1}{4} \frac{\Delta^+ + \Delta^-}{Max\{\Delta^+, \Delta^-\}} (Bz_k(\mathcal{W}_M) - Bz_l(\mathcal{W}_M)) \\
\leq \;& \frac{1}{2} Bz_k(\mathcal{W}_M).
\end{aligned}$$

Now if $k \in m_j$, assuming that all m_i are large enough to consider that (13) is a good approximation, the last term can be rewritten

$$\frac{1}{2} Bz_k(\mathcal{W}_M) \simeq \frac{1}{2} \sqrt{\frac{2}{\pi m_j}} Bz_j(\mathcal{W}_N) \leq \frac{1}{2} \xi,$$

which leads to (16).

Now, again assuming $Bz_k(\mathcal{W}_M) > Bz_l(\mathcal{W}_M)$, we have

$$\begin{aligned}
&\frac{E_p\left[u_k(\mathcal{W}_M, S)\right] - u_{Min}}{E_p\left[u_l(\mathcal{W}_M, S)\right] - u_{Min}} \\
= \;& \frac{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M) - Min\{R^+, A^-\}}{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_l(\mathcal{W}_M) - Min\{R^+, A^-\}} \\
\leq \;& \frac{\bar{u}(\mathcal{W}_M) + \frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M) - Min\{R^+, A^-\}}{\bar{u}(\mathcal{W}_M) - Min\{R^+, A^-\}} \\
= \;& 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\bar{u}(\mathcal{W}_M) - Min\{R^+, A^-\}} \\
= \;& 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ - \Delta^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\}} \\
= \;& 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ + \Delta^-)\alpha(\mathcal{W}_M) + (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\}}.
\end{aligned}$$

In order to find an upper bound of this expression, let us show that

$$(A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\} \geq 0.$$

If $Min\{R^+, A^-\} = A^-$, then

$$\begin{aligned}
& (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\} \\
= \;& R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \frac{1}{2}R^+ - \frac{1}{2}A^- - A^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) \\
= \;& (R^- - A^-)(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \frac{1}{2}(R^+ - A^-) \geq 0,
\end{aligned}$$

the last inequality because of (9) and $R^- - A^- = \Delta^- > 0$.
If $Min\{R^+, A^-\} = R^+$, then

$$\begin{aligned}
& (A^- - R^-)\alpha(\mathcal{W}_M) + \frac{1}{2}(R^+ + R^-) - Min\{R^+, A^-\} \\
= \;& R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) - \frac{1}{2}R^+ + A^-\alpha(\mathcal{W}_M) \\
= \;& R^-(\frac{1}{2} - \alpha(\mathcal{W}_M)) - R^+(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \alpha(\mathcal{W}_M)(A^- - R^+) \\
= \;& (R^- - R^+)(\frac{1}{2} - \alpha(\mathcal{W}_M)) + \alpha(\mathcal{W}_M)(A^- - R^+) \geq 0,
\end{aligned}$$

again using (9) and given that $R^- > A^- \geq R^+$.

Thus, in both cases we have

$$\frac{E_p\left[u_k(\mathcal{W}_M, S)\right] - u_{Min}}{E_p\left[u_l(\mathcal{W}_M, S)\right] - u_{Min}} \leq 1 + \frac{\frac{1}{4}(\Delta^+ + \Delta^-)Bz_k(\mathcal{W}_M)}{\frac{1}{2}(\Delta^+ + \Delta^-)\alpha(\mathcal{W}_M)} = 1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_M)}{\alpha(\mathcal{W}_M)}.$$

Now if $k \in m_j$, assuming that all m_i are large enough to consider (13) and (14) good approximations, using (8) we have:

$$1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_M)}{\alpha(\mathcal{W}_M)} \simeq 1 + \frac{1}{2}\frac{Bz_k(\mathcal{W}_N)}{\alpha(\mathcal{W}_N)}\sqrt{\frac{2}{\pi m_i}} \leq 1 + \xi,$$

which yields (17).

9.5 Proof of Proposition 8

In order to calculate the expectations in (19) we need the aggregate expected utility in each group in either case (acceptance or rejection) for each vote configuration in the committee. If the vote configuration in the committee is C, then $i \in C$ (i.e. M_i's representative votes "yes") when a majority in group M_i supports a "yes", while if $i \in N \setminus C$ (i.e. M_i's representative votes "no") when no majority in group M_i supports a "yes". Then from the possibility of permuting aggregation and expectation, the aggregate expected utility in group M_i, given that the majority in M_i votes "yes" and the proposal is accepted (rejected), is given, respectively, by

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Acc]$$

$$= A^+ E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] + A^- E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}],$$

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Rej]$$

$$= R^+ E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] + R^- E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}];$$

while the aggregate expected utility in group M_i, given that the majority in M_i does not vote "yes" and the proposal is accepted (rejected), is given, respectively, by

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i \leq \frac{m_i}{2} \& \ Acc]$$

$$= A^+ E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] + A^- E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}],$$

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i \leq \frac{m_i}{2} \& \ Rej]$$

$$= R^+ E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] + R^- E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}].$$

Lemma 23 in Laruelle and Valenciano (2008, p. 89) gives the following approximation:

$$E_p[\#S_i \mid \#S_i > \frac{m_i}{2}] = E_p[\#(M_i \setminus S_i) \mid \#S_i \leq \frac{m_i}{2}] \simeq \frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}},$$

$$E_p[\#(M_i \setminus S_i) \mid \#S_i > \frac{m_i}{2}] = E_p[\#S_i \mid \#S_i \leq \frac{m_i}{2}] \simeq \frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}}.$$

Thus when a majority in group M_i votes "yes" (and consequently M_i's representative votes "yes") the aggregate expected utility in this group *if the decision in the committee is to accept the proposal* is (with good approximation for large enough m_i's):

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Acc] \simeq A^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) + A^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right);$$

while *if the decision in the committee is to reject the proposal,* the aggregate expected utility in group M_i is,

$$E_p[\sum_{k \in M_i} u_k \mid \#S_i > \frac{m_i}{2} \ \& \ Rej] \simeq R^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) + R^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right).$$

Similar approximations can be drawn up for the case in which no majority in M_i supports the proposal.

Now, aggregating across all groups we have that for a given vote configuration in the committee $C \subseteq N$, the aggregate expected utility in M *if the committee accepts the proposal*, is (with close approximation for large enough m_i's)

$$E_p[\sum_{k \in M} u_k \mid C \ \& \ Acc] \simeq \sum_{i \in C} \left(A^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) + A^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right) \right)$$

$$+ \sum_{i \in N \setminus C} \left(A^+ \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right) + A^- \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) \right)$$

$$= (A^+ + A^-) \frac{m}{2} + \frac{A^+ - A^-}{\sqrt{2\pi}} \left(\sum_{i \in C} \sqrt{m_i} - \sum_{i \in N \setminus C} \sqrt{m_i} \right); \qquad (25)$$

while *if the proposal is rejected* the aggregate expected utility is

$$E_p[\sum_{k \in M} u_k \mid C \ \& \ Rej] \simeq \sum_{i \in C} \left(R^+ \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) + R^- \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right) \right)$$

$$+ \sum_{i \in N \setminus C} \left(R^+ \left(\frac{m_i}{2} - \sqrt{\frac{m_i}{2\pi}} \right) + R^- \left(\frac{m_i}{2} + \sqrt{\frac{m_i}{2\pi}} \right) \right)$$

$$= (R^+ + R^-)\frac{m}{2} + \frac{R^+ - R^-}{\sqrt{2\pi}} \left(\sum_{i \in C} \sqrt{m_i} - \sum_{i \in N \setminus C} \sqrt{m_i} \right). \quad (26)$$

Thus the best decision in the committee is to accept the proposal if (19) holds, that is, after substituting (25) and (26), and simplifying, if

$$\sum_{i \in C} \sqrt{m_i} - \sum_{i \in N \setminus C} \sqrt{m_i} > \frac{m(\Delta^- - \Delta^+)}{\Delta^+ + \Delta^-} \sqrt{\frac{\pi}{2}}.$$

This inequality holds if and only if (20).

9.6 Proof of Proposition 10

In the proof we use the notation $w(C) := \sum_{i \in C} w_i$. First note that, as $\Delta^- < \Delta^+$, and $\frac{w(N)}{2}$ is among the Q's that satisfy (23), we have $Q(\frac{\Delta^-}{\Delta^+}) \leq \bar{Q} \leq \frac{w(N)}{2}$, where $Q(\frac{\Delta^-}{\Delta^+})$ is given by (21). Then obviously

$$w(C) < \bar{Q} \Rightarrow w(N \setminus C) > \bar{Q} \geq Q(\tfrac{\Delta^-}{\Delta^+}).$$

Assume that for any $C \in \mathcal{W}_N$ we have $w(C) < \bar{Q}$. We can assume C to be minimal winning. Let

$$\mathcal{W}'_N := (\mathcal{W}_N \setminus \{C\}) \cup \mathcal{W}^{N \setminus C}.$$

That is, $\mathcal{W}'_N$ is the rule that results from $\mathcal{W}_N$ by eliminating C from the set of winning configurations, and adding $N \setminus C$ and all those containing it. As C is minimal, $N \setminus C$ intersects all $T \in \mathcal{W}_N \setminus \{C\}$, and $\mathcal{W}'_N$ is a proper rule. Let us see that $\mathcal{W}'_N$ is better than $\mathcal{W}_N$ from the utilitarian point of view. In order to compare the aggregate expected utility of a decision made by each rule, note that the decision differs only for the configuration C and for those T containing $N \setminus C$. For all the latter, as $w(T) \geq w(N \setminus C) > \bar{Q} \geq Q(\frac{\Delta^-}{\Delta^+})$, the decision by $\mathcal{W}'_N$ (acceptance) is utilitarian-better than that by $\mathcal{W}_N$ (rejection). The reverse only occurs for the configuration C. It thus suffices to show that what is lost by rejecting for configuration C, is outweighed by what is gained by accepting for the equally probable configuration $N \setminus C$. Again using (25) and (26), we have

$$E_p[\sum_{k \in M} u_k \mid C \ \& \ Acc] - E_p[\sum_{i \in M} u_i \mid C \ \& \ Rej]$$

$$= (\Delta^+ + \Delta^-)\frac{m}{2} + \frac{\Delta^+ - \Delta^-}{\sqrt{2\pi}} (w(C) - w(N \setminus C)) < (\Delta^+ + \Delta^-)\frac{m}{2}$$

while

$$E_p[\sum_{k \in M} u_k \mid N \setminus C \ \& \ Acc] - E_p[\sum_{i \in M} u_i \mid N \setminus C \ \& \ Rej]$$

$$= (\Delta^+ + \Delta^-)\frac{m}{2} + \frac{\Delta^+ - \Delta^-}{\sqrt{2\pi}}\left(w(N \setminus C) - w(C)\right) > (\Delta^+ + \Delta^-)\frac{m}{2}$$

Therefore, $\mathcal{W}_N$ does not implement the utilitarian optimum according to the approximation based on (25) and (26).

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