Optimal taxation in the extensive model

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Abstract

We study optimal taxation in the extensive framework: agents decide whether to stay inactive or to work. Starting from a general participation model, we derive a two-dimensional reduced form involving productivity and work opportunity cost. Allowing for general distributions of these parameters and for income effects, we characterize optimal, incentive-compatible tax schedules. We then give sufficient conditions for the social weight of the employed workers to decrease with both income and productivity. When these conditions hold, upwards distortions of the financial incentives to work can occur for low-skilled workers only. Under a simple specification of utility, we show that upwards distortions indeed arise for a positive mass of agents.

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1 Introduction

Since Mirrlees (1971), the theory of optimal taxation has largely been developed in the intensive model setup, where labor supply is continuous. Comparatively, much less attention has been devoted to the extensive model, where hours of work are imposed to the worker whose only decision is to work or not work. This is not because the extensive model lacks empirical relevance (see Heckman (1993)), nor because the properties derived in the Mirrlees setup apply to the extensive model: they typically do not. Indeed the informational structures of the two models are fundamentally different. In the intensive model, the tax authority observes total income, the product of productivity by the number of hours worked, and cannot a priori separate the two components: a redistributive government wants to tax the high productivity types to subsidize the poor, but is limited in his actions by the fact that the rich hide their status by reducing their labor supply. In the extensive model, workers have little opportunity to evade the tax: provided that after tax income is increasing with gross income, they work at their full productivity. Their only choice is between working or not working. The informational asymmetry which prevents the fiscal authority from getting to the first best comes from the determinants of the participation decision (taking care of children, enjoying leisure, laziness, etc.) which it does not know of.

The early work of Diamond (1980) on optimal taxation in the extensive model stresses non convexity issues in a simple example. It had no following until Saez (2002), Homburg (2002), Choné and Laroque (2005), Laroque (2005) and Homburg (2002). In this paper, we set a general model of labor force participation. We provide a comprehensive approach to deriving the properties of optimal taxation. With an appropriate definition of a redistributive objective, we present the qualitative properties of optimal schedules and show how they depend on the agents’ heterogeneity in work opportunity cost. These results allow us to describe a large class of economies where a utilitarian government would distort upwards the labor supply of the low productivity workers in comparison with laissez-faire, in stark contrast with the received wisdom from the Mirrlees model.

First a word of caution is in order. In the intensive model, attention is focussed on the marginal tax rates which there determine labor supply. In the extensive model, workers are concerned with average or participation tax rates. The average tax rate is equal to 1 minus the ratio (net income at work - subsistence income out of work)/(labor cost). Here ‘net income at work’ denotes disposable income when taking a full time job, ‘subsistence income out of work’ is disposable income when not working, including, say in the USA, food stamps and temporary assistance for needy families, and ‘labor cost’ is the cost of labor to the employer, a proxy for productivity in a competitive environment.

Subsidizing work, compared
with laissez-faire, is equivalent to have negative average tax rates\(^2\) this means a disposable income larger than the sum of productivity and of the subsistence income one would collect if unemployed.

We set a general structural model of discrete choice, which allows for differences in productivity and multidimensional heterogeneity in the participation factors. We derive a convenient reduced form from the structural model, which summarizes the various determinants of participation into a unidimensional work opportunity cost. We investigate the relationships between the structural and reduced forms. It turns out that the distribution of work opportunity costs, conditional on productivity and on income out of work, inherits properties from the structural model which are important for the analysis.

The government is supposed to maximize a utilitarian objective under a budget constraint and an incentive constraint. The latter constraint arises because we allow agents to work less than their full time productivity. Accordingly, after tax income schedules must be nondecreasing. We study the second best program, from general to specific, first looking for basic features of the optimal tax scheme, then deriving properties under more restrictive assumptions of economic interest. The program fundamentally is not concave: the proportion of agents at work, whose opportunity costs are smaller than their gains from working, enters the feasibility constraint and this typically is not a concave function of its arguments. Given a value of the cost of public funds and of the benefits served to persons out of work, the shape of the tax schedule is directly linked to the partition of the plane (productivity, work opportunity cost) into the two regions where the Lagrangian respectively increases and decreases in income.

First, we examine whether optimal tax schemes exhibit pooling. We give sufficient conditions for pooling to be ruled out. Next, we restrict our attention to governments who have redistributive objectives. The social preference from redistribution follows from the properties of the cardinal utilities. In the intensive model, when agents have the same utility function that does not depend on productivity, the mere concavity of the utility functions gives society a taste for equality. Here, in the extensive model with multidimensional heterogeneity, there is no such simple characterization of preference for redistribution. Instead we define a redistributive society as one where cardinal utilities induce a marginal utility of income of the workers that decreases with both income and productivity. We give a set of conditions on the economic fundamentals which warrant these properties: they include the distribution of work opportunity costs.

\(^2\)In the standard Mirrlees setup, with only an intensive margin, the marginal tax rate is everywhere nonnegative, so that the average tax rate is also non negative. Some of the models of the literature have both intensive and extensive components. Then subsidizing labor may take place at the extensive margin (negative average tax rate), at the intensive margin when leaving the Mirrlees framework (negative marginal tax rate), at no margins, or at both. The literature is not explicit on the topic.
being log-concave and stochastically decreasing in productivity and the marginal utility of income being nondecreasing in work opportunity cost. Under these conditions, optimal tax schedules turn out to take two possible shapes: either at all productivity levels the financial incentive to work is smaller at the second best than at laissez-faire, i.e. smaller than productivity; or there is a productivity threshold below which financial incentives to work are distorted upwards, and above which they are distorted downwards compared to laissez-faire. After tax income is a continuous function of productivity in the region where financial incentives to work are distorted downwards, but discontinuities can occur for low productivities when financial incentives to work are distorted upwards.

From an economic perspective, the foregoing analysis may leave the reader unsatisfied. If the optimal tax schedule has the second shape, negative taxes are in order, but there is no indication whether this is the likely case, or whether labor supply downward distortions, as in the Mirrlees setup, are more common. From Choné and Laroque (2005) we know that a Rawlsian government always implement a tax schedule of the second type, with everywhere positive participation tax rates. Are negative tax rates a curiosum? We believe that this is not the case, based on the example of an economy where the agents have a simple concave increasing utility function \( U \): the persons that do not work get utility \( U(s) \) where \( s \) is subsistence income, while the workers get \( U(R - \delta) \), with \( R \) their after-tax income and \( \delta \) their work opportunity cost. We show that with this specification of utility functions, a redistributive government always sets negative participation tax rates in a non-negligible interval at the bottom of the productivity distribution.

Relative to the prior literature, our contribution is threefold. First, while in the extensive model of Saez (2002) each individual can work only in one occupation or be unemployed, we allow an agent to work in any occupation which requires a skill below her type. The incentives to mimic agents with lower productivities give rise to monotonicity constraints and, possibly, pooling. Under simple assumptions, we are able to characterize optimal, incentive compatible schedules. Second, starting from a full-fledged structural model with multidimensional heterogeneity, we introduce the opportunity cost of work to summarize the agents’ incentives, and derive a reduced-form model with one-dimensional heterogeneity. We explore the links between the structural and reduced-form models, thereby shedding light on the role of multidimensional heterogeneity. Third, we allow for income effects and consider general distributions of productivities and work opportunity costs, the two variables possibly being correlated\(^3\). In sum, our analysis makes explicit the primitives of the structural problem, emphasizes the qualitative features of optimal, incentive compatible tax schedules, and derives a reduced-form model that is suitable for econometric specifications and empirical

\(^3\)Saez (2002) mentions income effects only in passing. Also, he has a discrete model while we consider a continuum of productivities.
work.

The paper is organized as follows. Section 2 describes the labor supply model, while Section 3 presents the tax instruments and government objective. The first order conditions satisfied by the tax schedule are derived in Section 4. Section 5 presents in turn the properties of general second best tax schedules and the restrictions that they satisfy under a redistributive government. Section 6 presents the full set of equations satisfied by an optimum, and Section 7 discusses the role of heterogeneity. Proofs are gathered in the appendix.

2 Heterogeneity and the description of the private economy

We consider an economy with a continuum of participants of measure 1. The agents’ only decision is whether to work or not. The agents differ along several dimensions. First, they differ by their productivity levels $\omega$, the before tax income that they generate when they work. Second, there is a possibly multidimensional heterogeneity parameter $\alpha$ that describes other idiosyncratic characteristics, such as the costs of going to work. Allowing for a lot of heterogeneity (i.e. a multidimensional parameter $\alpha$) accords with empirical studies where marital status, family composition, human capital and culture often appear to be determinants of the labor supply decision.

Let $c_E$ be the income of someone who decides to work, $c_U$ her income when unemployed. Utility is described by

\[
\begin{align*}
    u(c_E; \alpha) & \quad \text{if she participates}, \\
    v(c_U; \alpha) & \quad \text{if she does not work},
\end{align*}
\]

and the decision is the one that yields the highest utility. The utility functions $u$ and $v$ are assumed to be twice continuously differentiable. They are increasing and concave in (nonnegative) consumption.

The work opportunity cost is the (possibly negative) sum of money which, given to an agent if she works on top of the subsistence income she has while unemployed, makes her indifferent between working or not. Formally, from the monotonicity of $u$, for each agent ($\alpha$) and for each level of consumption when not working $c_U$, there exists a number $\delta = \Delta(\alpha, c_U)$ such that the agent wants to work ($\hat{u} > \hat{v}$) if $c_E > c_U + \delta$, does not want to work if $c_E < c_U + \delta$ and is indifferent if $c_E = c_U + \delta$, i.e. $\delta$ is solution to the equation

\[
\hat{u}(c_U + \delta, \alpha) = \hat{v}(c_U, \alpha).
\]  

We let $\delta$ equal to $-c_U$ for persons who always want to work, and $\delta$ equal $+\infty$ for persons that never work. We do not rule out $\delta < 0$: some agents may be ready to pay to go to work.
The threshold $\delta$ is the work opportunity cost of the agent. In the terminology of labor supply, the financial incentive to work is the difference $c_E - c_U$, and an agent works whenever her net gain exceeds her opportunity cost of work. The subsistence income or other income unrelated to work is $c_U$. Labor supply is subject to an income effect when the work opportunity cost $\Delta(\alpha, c_U)$ depends on other income $c_U$. If leisure is a normal good, labor supply weakly decreases with other income, i.e. $\Delta(\alpha, c_U)$ is nondecreasing in $c_U$.

An economy is defined by a pair of utility functions $(\hat{u}, \hat{v})$ and a distribution of characteristics $(\alpha, \omega)$. It is convenient to derive a reduced form by substituting the work opportunity cost $\delta$ for the multidimensional parameter $\alpha$. We define the average utilities of the agents who have the same work opportunity cost $\delta$ as

$$
\begin{align*}
&\{ u(c_E; \delta, c_U) = \mathbb{E}_\alpha [\hat{u}(c_E; \alpha) \mid \delta = \Delta(\alpha, c_U), c_U], \\
&v(c_U; \delta) = \mathbb{E}_\alpha [\hat{v}(c_U; \alpha) \mid \delta = \Delta(\alpha, c_U), c_U].
\end{align*}
$$

By construction, the reduced-form model satisfies:

$$
u(c_E; \delta, c_U) \geq v(c_U; \delta) \iff c_E \geq c_U + \delta.
$$

Note that the function $u$ generally depends on the subsistence level $c_U$, except when there are no income effects ($\Delta$ independent of $c_U$). The function $u$ inherits from $\hat{u}$ its monotonicity and concavity in $c_E$. Moreover, when the idiosyncratic heterogeneity is unidimensional (the dimension of $\alpha$ is one), the marginal rate of substitution $u_\delta / u_\alpha$ of the utility of the employee in the reduced form is independent of the level of income when employed $c_E$. The latter result, however, does not extend to higher dimensions of heterogeneity. We conjecture that the structural model does not restrict the shape of the reduced form utilities. Proposition A.1 in Appendix states these properties in a formal way.

While the structural model induces little restrictions on the shape of the utility functions, it has important consequences on the distribution of work opportunity costs. For each value of $c_U$, the distribution of the underlying parameters induces a distribution of $(\delta, \omega)$. We denote $F(|\omega, c_U)$ the cumulative distribution of work opportunity costs of persons of productivity $\omega$ when out of work income is equal to $c_U$: $F(c_E - c_U | \omega, c_U)$ is the proportion of agents of productivity $\omega$ that want to work when income at work is equal to $c_E$ and income out of work is $c_U$. Note that, from the monotonicity of $\hat{v}$, the function $F(c_E - c_U | \omega, c_U)$, seen as a function of $c_U$, is nonincreasing in $c_U$. Also, when leisure is a normal good, an increase in $c_U$, keeping constant the incentive to work $c_E - c_U$, decreases labor supply. To summarize, when $F$ is differentiable with respect to its arguments, we have

$$
\frac{d}{dc_U} F(c_E - c_U | \omega, c_U) = -\frac{\partial F(\delta | \omega, c_U)}{\partial \delta} + \frac{\partial F(\delta | \omega, c_U)}{\partial c_U} \leq 0,
$$

and, when leisure is a normal good,

$$
\frac{\partial F(\delta | \omega, c_U)}{\partial c_U} \leq 0.
$$
The structural and the reduced forms of the model are linked through two channels. The shape of the function \( \Delta(\alpha, c_U) \) is one of them. The implicit function theorem, when valid, gives
\[
\tilde{u}_1 \frac{\partial \Delta}{\partial \alpha} = \tilde{v}_\alpha' - \tilde{u}_\alpha'.
\]
The heterogeneity of the work opportunity cost \( \delta \), given the person’s productivity \( \omega \) which is observed by the government for the workers, is an important feature of the economy (see Section 7 for a discussion): for the model to be interesting, the utility functions must have a (generically) non zero \( \tilde{v}_\alpha' - \tilde{u}_\alpha' \). The other component that determines the reduced form is the distribution of \( \alpha \) conditional on \( \omega \).

Most of the results carry over to a situation where the distributions have mass points, provided that there is genuine heterogeneity in the work opportunity costs. To simplify the presentation, we focus on the case where distributions are continuous. Throughout the paper, except in Section 7, we maintain the following assumption.

**Assumption 1.** The marginal distribution of productivities \( \omega \) has support \( \Omega = [\omega, \bar{\omega}] \), an interval of the positive line. Its cumulative distribution function \( G \) has a continuous positive derivative \( g \) everywhere on the support.

The distribution of work opportunity costs \( \delta \), conditional on \( \omega \) and \( c_U \), is continuous with support \([\delta, \overline{\delta}], \delta < \overline{\delta}\), and cumulative distribution function \( F(.|\omega, c_U) \). Its probability distribution function \( f(.|\omega, c_U) \) is positive and continuous on \((\delta, \overline{\delta})\) and \( F/f(\delta|\omega, c_U) \) tends to 0 as \( \delta \) goes to \( \overline{\delta} \).

Assumption 1 plays a critical role in the analysis. Section 7 shows that our results are driven by the presence of genuine heterogeneity in work opportunity costs: for our results to hold, the inequality \( \delta < \overline{\delta} \) must be strict (as stated in the assumption).

**Benchmark case:** A simple specification of particular interest is the following. The parameter \( \alpha \) is unidimensional and the utility functions are defined through a concave twice differentiable increasing function \( U \), such that
\[
\tilde{u}(c_E; \alpha) = U(c_E - \alpha), \quad \tilde{v}(c_U; \alpha) = U(c_U).
\]

Then direct substitutions yield \( \Delta(\alpha, c_U) = \alpha \) and \( u(c_E; \delta, c_U) = U(c_E - \delta), \quad v(c_U; \delta) = U(c_U) \).

The work opportunity cost is simply equal to \( \alpha \). It is independent of the value of the subsistence income \( c_U \): there are no income effects. Also the value attached by the government to the welfare of the unemployed agents only depend on their income, and does not vary with their characteristics \((\alpha, \omega)\).

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The last part of the assumption is satisfied in particular when \( f(\delta|\omega, c_U) > 0 \) and when \( f(\delta|\omega, c_U) = 0 \) and \( f \) is differentiable at \( \delta \) with \( f'(\delta|\omega, c_U) > 0 \).

5The last part of the assumption is satisfied in particular when \( f(\delta|\omega, c_U) > 0 \) and when \( f(\delta|\omega, c_U) = 0 \) and \( f \) is differentiable at \( \delta \) with \( f'(\delta|\omega, c_U) > 0 \).
3 Government behavior and tax schedules

In this section, we define tax schedules and discuss the link between average tax rates and labor supply. Next, we examine the incentive constraints faced by the government. Finally, we write the government objective and budget constraint in terms of the reduced-form model.

3.1 Tax schemes and labor supply

To redistribute income, the government announces an after tax income schedule $R(y)$, which associates to any before tax income $y$, $y > 0$, a nonnegative disposable income $R(y)$ and gives to the non workers a subsistence income $s$, $s \geq 0$. To the income schedule, one can associate the participation tax rate $\tau(y)$ faced by the worker

$$\tau(y) = \frac{y - (R(y) - s)}{y} = 1 - \frac{R(y) - s}{y}.$$

At laissez-faire, $(R(y) = y, s = 0)$, the tax rate $\tau(y)$ is equal to zero. A person of productivity $\omega$ decides to work when her financial incentive $\omega$ is larger than her work opportunity cost $\Delta(\alpha, 0)$, where the second argument of $\Delta$, the subsistence income, is zero in the absence of a government safety net. In a second best allocation, the person works when

$$R(\omega) - s > \Delta(\alpha, s).$$

The tax rate faced by a person of productivity $\omega$ is positive if and only if $\omega > R(\omega) - s$, i.e. when the financial incentive to work is smaller at the second best allocation than at laissez-faire. Then, if leisure is a normal good, that is $\Delta$ is nonincreasing in its second argument, labor supply is unambiguously distorted downwards compared to laissez-faire: any unemployed person at laissez-faire, with $\omega < \Delta(\alpha, 0)$, is still unemployed at second best, since $R(\omega) - s < \omega < \Delta(\alpha, 0) \leq \Delta(\alpha, s)$.

When the tax rate is negative and $R(y) > s + \delta$, the financial incentive to work is larger at the second best than at laissez-faire, $\omega < R(\omega) - s$. In this circumstance, every agent of productivity $\omega$, with or without a job, prefers the second best to laissez-faire: $\max[\tilde{u}(R(\omega); \alpha), \tilde{v}(s; \alpha)] \geq \max[\tilde{u}(\omega; \alpha), \tilde{v}(0; \alpha)]$. Absent income effects, i.e. when work opportunity costs are independent of the level of subsistence income, labor supply distortion is of the opposite sign of the tax rate, compared with laissez-faire. But this is not true in general, even when leisure is a normal good: if income effects are large enough, so that utility at home and work opportunity cost strongly increase with subsistence income, labor supply may be smaller at the second best allocation.

$^6$Formally, the labor supply of agents with productivity $\omega$ is (weakly) distorted upwards if
3.2 The incentive constraint

In the extensive model, the work opportunity cost is independent of the production level $y$: a worker of productivity $\omega$ is indifferent to generate any ex-ante income in the interval $[0, \omega]$. Depending on the income schedule, she can choose to work at a productivity below her type. If the government announces a (possibly sometimes decreasing) tax function $\tilde{R}(.)$, then a worker would choose the production level $y$ solution to $\max_{y \leq \omega} \tilde{R}(y)$. The nondecreasing schedule $R(.)$, defined by $R(y) = \max_{z \leq y} \tilde{R}(z)$, is as good as $\tilde{R}$, induces production at $\omega$: workers facing $R$ have no interest to produce $y < \omega$. In the language of mechanism design, when an agent can mimic any agent of lower productivity, incentive compatibility is equivalent to the monotonicity of the income schedule. Alternatively put, the allocation obtained under a general income scheme, with possibly decreasing parts, is also the outcome of a nondecreasing income scheme, which never pushes workers to work strictly below their full productivity.

3.3 Writing the government problem

It turns out that the government objective, as well as the feasibility constraint, can be written in terms of the reduced-form model. The utilitarian government maximizes the sum of the utilities of the participants in the economy. Its objective takes the form

$$
\mathbb{E}_{\omega, \alpha} \max \{ \tilde{u}(R(\omega); \alpha), \tilde{v}(s; \alpha) \} = \mathbb{E}_{\omega, \alpha} \{ \tilde{u} \mathbb{1}_{R(\omega) - s \geq \delta} + \tilde{v} \mathbb{1}_{R(\omega) - s < \delta} \}.
$$

Remarking that the participation decision only depends on $\delta$ and $\omega$ and applying the law of iterated expectations, we can replace the expectation over $(\omega, \alpha)$ with an expectation over $(\omega, \delta)$:

$$
\mathbb{E}_{\omega, \alpha} \max \{ \tilde{u}(R(\omega); \alpha), \tilde{v}(s; \alpha) \} = \mathbb{E}_{\omega, \delta} \{ u \mathbb{1}_{R(\omega) - s \geq \delta} + v \mathbb{1}_{R(\omega) - s < \delta} \}.
$$

The government objective thus is the expectation with respect to $\omega$ of

$$
\int_{\omega}^{R(\omega) - s} u(R(\omega); \delta, s) \mathrm{d}F(\delta|\omega, s) + \int_{R(\omega) - s}^{\delta} v(s; \delta) \mathrm{d}F(\delta|\omega, s). \quad (6)
$$

The feasibility constraint is

$$
\int_{\Omega} [\omega - R(\omega) + s] F(R(\omega) - s|\omega, s) \mathrm{d}G(\omega) = s. \quad (7)
$$

and only if

$$
\omega \geq \Delta(\alpha, 0) \implies R(\omega) \geq s + \Delta(\alpha, s).
$$

With a nondecreasing $R$, this is satisfied when

$$
R(\Delta(\alpha, 0)) \geq s + \Delta(\alpha, s).
$$

The tax rate is unambiguously linked with the distortion of labor supply only when there are no income effects, $\Delta$ independent of its second argument.
Let $\lambda$ be the multiplier associated with the feasibility constraint (7). The Lagrangian of the problem is $E(\omega) L(R(\omega), s; \omega)$, with

$$L(R, s; \omega) = \int_{\delta}^{R-s} u(R; \delta, s) dF(\delta|\omega, s) + \int_{R-s}^{\delta} v(s; \delta) dF(\delta|\omega, s)$$

$$+ \lambda[\omega - R + s] F(R - s|\omega, s) - \lambda s.$$  

The optimization takes place over $(R, s, \lambda)$, where $R$ is a nonnegative nondecreasing function and $s$ and $\lambda$ are nonnegative. The Lagrangian $L$ is not concave in its arguments, since the cdf of the work opportunity cost $F$ is typically not concave. Any utilitarian optimum however must satisfy first order necessary conditions, which turn out to be especially fruitful here. In the next two sections, we concentrate on the study of $R(\cdot)$, for given values of $s$ and $\lambda$, before commenting briefly on the full program in Section 6.

Without loss of generality, we impose the additional constraint: $R(\omega) - s \geq \delta$ for all $\omega \in \Omega$. Indeed, the objective does not depend on the precise value taken by $R(\omega)$ in the region where the set of workers is negligible, i.e. whenever $R(\omega) - s$ is smaller than or equal to the minimal work opportunity cost $\delta$. That is, if $R(w) - s < \delta$ is optimal, then $R(w) - s = \delta$ is optimal as well. Given this extra constraint, the financial incentives to work for agents of productivity $\omega$ are distorted downwards if and only if $R(\omega) < \omega + s$; they are distorted upwards if and only if $R(\omega) > \max(\omega + s, \delta + s)$; they are undistorted if and only if $R(\omega) = \max(\omega + s, \delta + s)$.

4 First order conditions for $R(\omega)$

The average social weight of the working agents of productivity $\omega$ is defined for $R > s + \delta$ by

$$p_E(R, s|\omega) = \mathbb{E}_\alpha \left[ \bar{u}'_1(R; \alpha) \mid \delta \leq R - s, \omega, s \right]$$

$$= \frac{1}{F(R - s|\omega, s)} \int_{\delta}^{R-s} u'_1(R; \delta, s) dF(\delta|\omega, s). \quad (8)$$

By continuity, we can set: $p_E(s + \delta, s|\omega) = u'_1(s + \delta; \delta, s)$. The derivative of the Lagrangian $L$ with respect to $R$ can be expressed in terms of the social weight $p_E$:

$$\frac{\partial L}{\partial R}(R, s; \omega) = \lambda[\omega - R + s] F(R - s|\omega, s) - F(R - s|\omega, s) [\lambda - p_E(R, s|\omega)]. \quad (9)$$

The expression of $\partial L/\partial R$ has a direct economic interpretation. The first term $\lambda[\omega - R + s] F(R - s|\omega, s)$ is the gain in government income obtained from the new
workers who participate following an increase in $R$: they produce $\omega$, they are paid $R(\omega)$ but do not receive the subsistence income $s$ anymore. The second term $F(R - s|\omega, s)[\lambda - p_E(R, s)]$ is the loss on the existing workers: the marginal cost is $\lambda$ per worker, while the social value of this distribution of income is equal to the average social weight of the employees of productivity $\omega$.

At any point $\omega$ where the income schedule $R(\omega)$ is strictly increasing ($R > s + \delta$ and no pooling), it satisfies the first order condition for a pointwise maximum:

$$\frac{\partial L}{\partial R}(R, s; \omega) = 0. \tag{10}$$

The average tax rate supported by the workers of productivity $\omega$ is $\tau(\omega) = [\omega - R(\omega) + s]/\omega$, so that the first order condition can be rewritten as

$$\omega - R + s = \omega \tau(\omega) = \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \left[ 1 - \frac{p_E(R, s|\omega)}{\lambda} \right]. \tag{11}$$

It may also help to rewrite this expression using labor supply elasticities, as in Proposition 1 of Saez (2002). The elasticity of the labor supply $F(R - s|\omega, s)$ of the agents of productivity $\omega$ with respect to their after tax income $R$, when subsistence income is $s$, is

$$\varepsilon_R(\omega, s) = \frac{f(R - s|\omega, s)}{F(R - s|\omega, s)},$$

so that (11) can be rewritten as

$$\omega \tau(\omega) = \frac{R}{\varepsilon_R(\omega, s)} \left[ 1 - \frac{p_E(R, s|\omega)}{\lambda} \right].$$

At a point $\omega$ where the tax schedule $R(\omega)$ is strictly increasing and $R - s$ is in the interval $(\delta, \delta)$, i.e. some, but not all, agents of productivity $\omega$ want to work, we have

- either $p_E(R(\omega), s|\omega) < \lambda$, the financial incentive to work $R(\omega) - s$ is smaller than before tax income $\omega$: it is distorted downwards compared to laissez-faire;
- or $p_E(R(\omega), s|\omega) = \lambda$, the financial incentive to work $R(\omega) - s$ equals before tax income $\omega$: financial incentives to work are the same as in laissez-faire and the workers are better off than at the laissez-faire;
- or $p_E(R(\omega), s|\omega) > \lambda$, the financial incentive to work $R(\omega) - s$ is larger than before tax income $\omega$: it is distorted upwards compared to laissez-faire and all the persons of productivity $\omega$ workers are better off.
Social weights larger than $\lambda$, corresponding to a group of employees whose average social weight is larger than the marginal cost of public funds, receive a financial incentive to work $R(\omega) - s$ larger than their productivity $\omega$, distorted upwards compared with laissez-faire.

The optimum may involve pooling, with regions where $R$ stays constant because of the monotonicity condition. In a pooling interval $[\omega_1, \omega_2]$, whenever $R - s$ is in the interior of the support of work opportunity costs, the first order conditions become
\[
\int_{\omega_1}^{\omega_2} \frac{\partial L}{\partial R}(R, s; \omega) \, dG(\omega) = 0 \quad (12)
\]
and
\[
\int_{\omega}^{\omega_2} \frac{\partial L}{\partial R}(R, s; \omega) \, dG(\omega) \leq 0 \quad (13)
\]
for all $\omega_1 \leq \omega \leq \omega_2$. Applied in a neighborhood of the extremities of the pooling interval, the last condition implies
\[
\frac{\partial L}{\partial R}(R, s; \omega_1) \geq 0 \quad \text{and} \quad \frac{\partial L}{\partial R}(R, s; \omega_2) \leq 0. \quad (14)
\]
Note that in a region of no-pooling, under regularity conditions, the derivative of after tax income $R(\omega)$ can be computed with the implicit function theorem
\[
\frac{\partial^2 L}{\partial R^2} \frac{\partial R}{\partial \omega} + \frac{\partial^2 L}{\partial R \partial \omega} = 0.
\]
At a local maximum, $\partial^2 L/\partial R^2$ is negative, so that for $R$ to be increasing, one must have $\partial^2 L/\partial R \partial \omega \geq 0$. Conversely, pooling only prevails if there is a region where $\partial^2 L/\partial R \partial \omega$ is negative (see equation [14]).

5 The shape of optimal tax schemes

The derivation of the optimal tax schedule is closely linked to the drawing describing the sign of $\partial L/\partial R$ in the plane $(\omega, \delta)$. We study this link, first by giving conditions for the absence of pooling in the general second best model, second by restricting the attention to utilities associated with a redistributive government.
5.1 Unrestricted second best tax schedules

How the distribution of work opportunity costs varies with productivity unsurprisingly is an important determinant of the shape of the optimal tax schedules. The two following assumptions help to describe the structure of the optimal tax problem, and to single out a class of economies where pooling does not occur at the optimum.

**Assumption 2.** The economy satisfies one of the three conditions stated below:

1. The distribution of work opportunity costs \( F(., \omega, c_U) \) does not depend on the productivity level \( \omega \).

2. The distribution of work opportunity costs \( F(., \omega, c_U) \) first order stochastically increases in \( \omega \). The marginal utility of income \( u'_1(c_E; \delta, c_U) \) is increasing with \( \delta \).

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\footnote{In the intensive model also, the sign of the *marginal* tax rate follows from the comparison of the cost of public funds with the average value of social weights of a specific group of workers. But the relevant groups differ in the intensive and extensive models. In the former case, what matters is the population of workers whose productivity is greater than or equal to a given level; in the latter, only workers with a given productivity are considered. This difference follows from the informational structures: the workers’ productivities are observed in the extensive model, whereas this information has to be extracted in the intensive case.}
3. The distribution of work opportunity costs \( F(.|\omega, c_U) \) first order stochastically decreases in \( \omega \). The marginal utility of income \( u_1'(c_E; \delta, c_U) \) is decreasing with \( \delta \).

Diamond (1980) worked under independence (case 1). To the best of our knowledge, there is no clear evidence as to whether work opportunity costs increase or decrease with productivity (the latter seems in line with the idea of specialization). As far as the marginal utility of income is concerned, case 2 seems most likely in circumstances where, as in the benchmark model, the work opportunity cost acts as a reduction of income, thus increasing its marginal utility.

The next assumption is best stated in terms of the joint distribution of productivities and work opportunity costs. Given a level of income out of work \( c_U \), if the government gives financial incentives \( r \) to the agents of productivity \( \omega \), a proportion \( F(r|\omega, c_U) \) of those agents will work, so that tax receipts equal \( (\omega - r)F(r|\omega, c_U) \). A marginal increase in \( r \) changes the receipts by \( (\omega - r)f(r|\omega, c_U) - F(r|\omega, c_U) \). We shall assume that this change is increasing with productivity \( \omega \), a property that always holds when the distribution of work opportunity costs is independent of productivity:

**Assumption 3.** The tax receipts of the government on the persons of productivity \( \omega \) when their incentives to work is \( r \), \( (\omega - r)F(r|\omega, c_U) \), have a positive second order cross partial derivative with respect to \( r \) and \( \omega \):

\[
\frac{\partial^2}{\partial r \partial \omega} [(\omega - r)F(r|\omega, c_U)] > 0.
\]

Assumptions 1 to 3 ensure that the cross-derivative of the Lagrangian is positive: \( \partial L/\partial r \partial \omega > 0 \). The analysis of Section 4 shows that there is no pooling: Pointwise maximization of the Lagrangian for each \( \omega \) yields the optimal tax scheme. Given a couple \((s, \lambda)\), for every \( R \) such that \( \delta < R - s < \delta \), there is a unique (possibly infinite) nonnegative number \( M(R; s, \lambda) \), such that

\[
\frac{\partial L}{\partial R} < 0 \text{ if and only if } \omega > M(R; s, \lambda).
\]

The geometrical construction of a schedule is shown on Figure 1, where productivity is on the horizontal axis while work opportunity costs or financial incentives to work are on the vertical axis. By construction, for \( \omega \) smaller than \( M \), on the left of the graph of \( M \), the derivative of the Lagrangian with respect to \( R \) is negative, while on the right of the graph it is positive.

To look for the pointwise maximum of the Lagrangian, draw the vertical line through \( \omega \). When it does not intersect the graph of \( M \) (i.e. \( \omega \) is outside the range of \( M \)), then the optimum is to have everybody unemployed \((R - s \leq \delta)\) when the line is on the left of the graph, or to have everyone employed \((R - s \geq \delta)\) when it is on the right of the graph.
Consider now an intersection point: if the slope of $M$ is negative at the point, this is a local minimum ($\partial L/\partial R$ is negative for smaller $R$’s, positive for larger $R$’s); the intersection points where the slope of $M$ is positive are local maxima. Furthermore, the optimal tax scheme is continuous whenever $M$ is nondecreasing everywhere. If the support of productivities is included in the range of $M$, the tax scheme exhibits some discontinuity whenever $M$ is decreasing on part of its domain. The pointwise maximum is the point that yields the higher $L$, comparing all the local maxima and the two corners (full employment, or no employment). All this procedure is summarized in the following proposition.

**Proposition 1.** Under Assumptions 1 to 3, there is no pooling at the optimum. Any optimal income scheme is increasing in the range of productivity for which the probability of being employed lies in $(0,1)$. Furthermore, such a scheme is continuous if and only if the function $M$ defined in (15) is globally increasing.

The no-pooling property holds in the interval of productivities for which there are non negligible sets of both employees and unemployed, say $[\omega_l, \omega_h]$. Then $R$ is increasing and no employee has interest in faking a smaller productivity than her own: she then would receive a smaller after tax income. People with lower productivities than $\omega_l$ are unemployed and receive the subsistence income $s$: they are treated as all the unemployed whose productivities are not observed by the government. Persons with a higher productivity than $\omega_h$ are all working. Typically, the optimal function $R$ is constant for $\omega$ larger than $\omega_h$, and the high skilled workers under consideration are indifferent between working at their full productivities $\omega$ or at any productivity in the interval $[\omega_h, \omega]$. We suppose that they do not shirk.

To clarify the logic of the argument, we spell out what happens at a discontinuity point of the tax schedule, for instance when one jumps from $R_- < \omega + s$ to $R_+ > \omega + s$ at some productivity $\omega$. At $R_-$, there is a (relatively) small number of employees, $F(R_- - s|s)$ and their average social weights $p_E(R_-,s)$ is smaller than $\lambda$, since we supposed that the point lies below the 45 degree line on the graph. At $R_+$, there are many more employees: all the persons with a work opportunity cost in the interval $[R_- - s, R_+ - s]$ who are unemployed when their productivities are slightly smaller than $\omega$ take a job when their productivity is larger than $\omega$. Since the $M$ curve then is above the 45 degree line, the average social weight of the employees $p_E(R_+,s)$ is larger than $\lambda$. For a discontinuity to occur, when the distributions are smooth (Assumption 1), the average social weight of the persons of work opportunity costs in $[R_- - s, R_+ - s]$ has to be larger than that of persons with work opportunity costs lower than $(R_- - s)$.

### 5.2 Redistribution

The previous paragraph stresses that, as in Stiglitz (1982), the social weights $p_E$ play a crucial role in shaping the income tax schedule. Intuitively, the type
of discontinuities in the schedule that we just described is only likely to occur empirically when the parameter \( \alpha \) is multidimensional, and some component kicks in when the work opportunity cost becomes larger than \( R_\omega - s \), making social weight locally an increasing function of income. We now lay down conditions under which the government objective involves standard redistribution, that is social weights are decreasing with incomes and/or productivities.

5.2.1 The social weights of the employees

Typically in the intensive model the more productive the agents, the richer they are at the optimum: with identical preferences social weights decrease with income. This behavior of the social weights, \( p_E \), decreases with \( R \), is not straightforward in the extensive model. Indeed brute differentiation yields

\[
\frac{\partial p_E}{\partial R} = \frac{1}{F(R - s|\omega, s)} \int_{\delta}^{R-s} u''_1(R; \delta, s) dF(\delta|\omega, s)
- \frac{f(R - s|\omega, s)}{F(R - s|\omega, s)} p_E(R, s|\omega)
+ u'_1(R; R - s, s) \frac{f(R - s|\omega, s)}{F(R - s|\omega, s)}.
\]

The first two terms are negative, following the intuition, but the last term is positive: an increase in \( R \) brings newcomers into employment, whose weights may be high. Nevertheless one can derive the desired property under two restrictive assumptions that bear on the two factors in the last term. The first one states that the new entrants in the labor force do not have too high a marginal utility of income:

**Assumption 4.** The marginal utility of income of the new entrants into the labor force is nonincreasing in \( R \) for all \( s \) and \( \omega \): \( u'_1(R; R - s, s) \) is nonincreasing in \( R \).

This assumption does not seem overly restrictive. For instance, in the benchmark case described at the end of Section 2, the marginal social utility of newcomers equals \( U'(s) \) and does not depend on \( R \). The second assumption, the log concavity of the cumulative distribution of work opportunity costs, is a stronger requirement, but is satisfied by a number of distributions.

**Assumption 5.** The distribution of work opportunity costs is log-concave.

---

We provide here structural foundations to Saez (2002)'s statement, page 1049:

*If the government values redistribution, then the lower the earnings level of the individual, the higher the social marginal value of an extra dollar for that individual.*
The latter two assumptions guarantee that the first aspect of redistribution mentioned earlier prevails, namely that the social weights of workers are nonincreasing in income.

**Proposition 2.** Under Assumptions 4 and 5, the average social weight of the employees of productivity \( \omega \), \( p_E(R, s|\omega) \), is a nonincreasing function of \( R \).

The second redistributive property concerns the monotonicity of social weights with productivity. The following assumption is relevant in that respect.

**Assumption 6.**

1. The distribution of work opportunity costs stochastically decreases with productivity (i.e. the function \( F(\delta|\omega, s) \) increases in \( \omega \)).
2. The second derivative \( u''_{R\delta} \) is nonnegative.

The first part of the assumption is satisfied whenever higher productivity on the market goes with smaller opportunity cost of going to work, or less productivity in tasks at home. The second part of the assumption, which holds in the benchmark model \( (u(R; \delta, s) = U(R - \delta)) \), is associated with social weights which increase with the work opportunity cost, all other things equal: individuals suffer from having a large opportunity cost, akin to a handicap and society wants to compensate them for this. The assumption fails when work opportunity costs reflect laziness and carry a social stigma. Note that there is a tension between Assumption 4 and Assumption 6.2. The former supposes that \( u''_{RR} + u''_{R\delta} \leq 0 \), so that for both to be satisfied, one needs \( -u''_{RR} \geq u''_{R\delta} \geq 0 \).

**Proposition 3.** Under Assumption 6, the average social weight of the employees \( p_E(R, s|\omega) \) is a nonincreasing function of \( \omega \).

**Proof:** It is a straightforward consequence of the assumptions. From [8],

\[
\frac{\partial p_E(R, s|\omega)}{\partial \omega} = -\frac{p_E(R, s|\omega)}{F(R - s|\omega, s)} \frac{\partial F(R - s|\omega, s)}{\partial \omega} + \frac{1}{F(R - s|\omega, s)} \frac{\partial}{\partial \omega} \int_{\delta}^{R-s} u'_1(R; \delta, s) \, dF(\delta|\omega, s).
\]

The first term is negative from the fact that \( F \) increases with \( \omega \), while the negativity of the second follows from Assumption 6 and the properties of stochastic dominance.

5.2.2 Utilitarian tax schedules

We proceed to describing the shapes of the optimal tax schedules under Assumptions 4 to 6 which ensure that the social weights of the employees decrease both with their incomes and their productivities. Note that these assumptions do not preclude pooling (Proposition 1 does not apply). The next proposition

16
states that, when the government has a redistributive objective, either taxes distort downwards the incentives to work everywhere, or they distort it upwards up to some threshold productivity level, and downwards for larger productivities. Furthermore taxes are a continuous function of income in the region where labor supply is distorted downwards. Discontinuities can occur for low incomes, when financial incentives to work are distorted upwards: indeed we shall see in Section 6.2 a number of circumstances where such discontinuities are likely to occur.

**Proposition 4.** Suppose that Assumptions 1 (continuity of the distributions) and 4 to 6 (redistribution) hold.

Then there exists a productivity threshold \( \omega_m, \omega_m < \delta \), such that the financial incentive to work is undistorted or distorted downwards for \( \omega > \omega_m \), while it is undistorted or distorted upwards for \( \omega_m \geq \omega \geq \delta \). In the region where incentives are distorted downwards, after tax income is a continuous function of before tax income.

According to Proposition 4, if the financial incentive to work is distorted downwards (upwards) at a given productivity level, it must be distorted downwards (upwards) or undistorted for higher (lower) productivities. Figure 2 illustrates the Proposition. Laissez-faire is represented by the curve \( R - s = \max(\delta, \omega) \): there is no distortion along the 45 degree line. The dotted curve corresponds to

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\[ R - s \]

\( \delta \)

\( \omega_m \)

\( \delta \)

Figure 2: Two ‘well behaved’ optimal tax schemes
the case $\omega_m = \delta$. The graph of after tax income lies below the 45 degree line \[10\] all the workers are taxed; their financial incentive to work is distorted downwards compared to laissez-faire, where they would be better off than in the second best. This situation is typical of a highly redistributive government and holds in particular under the Rawlsian objective, where $p_E$ is zero (see e.g. Choné and Laroque (2005)). The bold solid curve illustrates the case $\omega_m > \delta$ \[11\] The figure shows a case where there is no pooling and where incentives to work $R(\omega) - s$ are a continuous increasing function of productivity. There is a low skilled region, $\delta \leq \omega \leq \omega_m$, where incentives are distorted upwards. \[12\]

**Remark 5.1.** Suppose that in addition to the assumptions of Proposition 4, the distribution of work opportunity costs is independent of productivity. Then the marginal tax rate is nonnegative in the region where incentives to work are distorted downwards.

### 6 The full program

#### 6.1 The marginal cost of public funds

The preceding section has studied the shape of the optimal tax schemes given the levels of subsistence income $s$ and of the marginal cost of public funds $\lambda$. Two equations are needed to determine these two quantities. One is the feasibility condition \[7\]. The other is a first order necessary condition, associated with a small translation of all incomes, i.e. an equal marginal change in both $s$ and $R(\omega)$ for all $\omega$. It is derived in the Appendix in general circumstances, allowing for pooling. The social weights of the employees, defined in \[8\], and of the unemployed

$$p_U(R, s|\omega) = E_\alpha \left[ \tilde{v}_1(s; \alpha) \mid \delta > R - s, \omega, s \right], \quad (16)$$

are the ingredients of the Lemma.

**Lemma 1.** Under Assumption \[7\] there exists a function $\rho(\omega)$, $\rho(\omega) < 1$, such that at an optimum

$$\int_{\Omega} \{(1 - \rho) F p_E(R(\omega), s|\omega) + (1 - F) p_U(R(\omega), s|\omega)\}dG(\omega) = \int_{\Omega} \lambda(1 - \rho F)dG(\omega), \quad (17)$$

\[10\] More precisely, we have: $R(\omega) \leq \max(\omega + s, \delta + s)$ for all $\omega$.

\[11\] It is similar to Figure IIa in Saez (2002), who discusses from a more applied perspective the occurrence of negative marginal tax rates.

\[12\] Other properties of the optimal tax schedules, in relationship with the Rawlsian criterion may be worth recalling. Theorem 6 of Choné and Laroque (2005) applies here: all the utilitarian optimal incentive schemes are located above the Rawlsian curve. Theorem 3 of Laroque (2005) also applies: any incentive scheme above the Laffer curve which does not overtax and such that $R(\omega) - s \leq \omega$ is associated with a second best optimal allocation.
where $F$ stands for $F(R(\omega) - s|\omega, s)$, the proportion of employed among agents with productivity $\omega$.

Absent income effects, $\rho$ is identically equal to 0. When leisure is a normal good, $\rho$ is a non positive function of $\omega$.

In the case where work opportunity costs do not depend on the level of the subsistence income (no income effects), the lemma indicates that the familiar equality of the marginal cost of public funds to the average of the agents marginal utilities of income holds here: $\rho$ is equal to zero, and (17) simplifies into

$$
\int_{\delta}^{R-s} u'_1(R; \delta, s) \, dF(\delta|\omega, s) + \int_{R-s}^{\delta} v'_1(s; \delta) \, dF(\delta|\omega, s) = \lambda.
$$

In the presence of income effects, the coefficients of $p_E$ and of $p_U$ are weighted. The weights are nonnegative and sum up to the coefficient of $\lambda$: $F(1 - \rho) + (1 - F) = 1 - \rho F$. When leisure is a normal good, $\rho$ is negative and the employees are given more importance than in the absence of income effects.

### 6.2 When are negative tax rates optimal?

When does a redistributive government implement negative taxes? Theory helps to answer the question. The intuition is the following. Marginal employees are indifferent between working and not working. Suppose that the utilities of the unemployed are a nondecreasing function of their work opportunity costs, as in the benchmark model. Then a redistributive government would impute them a lower social weight than that of the marginal employees. From (17), it follows that the social weight of the lower income employees is larger than the marginal cost of public funds if there is any redistribution at all. Then, by continuity, there is a range of productivities at the bottom of the skill distribution where the social weights of the workers is larger than the marginal cost of public funds. It follows that there are upward labor distortions of financial incentives to work at low productivities at the optimum.

The following proposition illustrates this fact that holds for the benchmark economy, for any redistributive objective that satisfies the assumptions of Section 5.2:

**Proposition 5.** Consider a benchmark economy of Section 2 satisfying Assumption 1, with $\omega \leq \delta$. Then, at the optimum, the financial incentives to work are distorted upwards for the lowest skilled workers.

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13Otherwise all the social weights on the left hand side of (17) are at most equal to $\lambda$, so that for the equality to hold, all weights must be equal to $\lambda$, which corresponds to the laissez-faire equilibrium.
The proof of Proposition 5 (see Appendix F) shows that there exists a productivity \( \omega_0, \omega_0 > \delta \), such that \( R(\omega) > \omega + s \) for all \( \omega \) in \( (\delta, \omega_0) \) and that one of the two following properties holds:

1. None of the agents of productivity smaller than \( \delta \) work at the optimum, \( R(\delta) - s = \delta \), and the slope of the income schedule at \( \delta \) is larger than 1. Such a configuration is shown on Figure 2.

2. There are (a non negligible set of) agents with productivity \( \delta \) who work: \( R(\delta) > s + \delta \). Furthermore, if \( \omega < \delta \), a nonnegligible set of agents of productivity smaller than \( \delta \) work at the optimum. Such a configuration is shown on Figure 3.

When the distribution of work opportunity costs is uniform and independent of productivity, the analytic computations can be carried out in some detail (see Appendix F.2). The bold line represents the optimal tax scheme. None of the agents with very low productivities, \( \omega < \omega_0 \), work. But for all \( \omega \) larger than or equal to \( \omega_0 \), a fraction of the agents does. In fact the upward distortion to labor supply here is particularly strong: some agents with productivity smaller than the minimal cost of going to work participate in the labor force. Note that here, with a uniform distribution of work opportunity costs, after tax income is a concave function of income, implying a progressive tax system for productivities above \( \omega_0 \).

\[ \text{Figure 3: Strong upward distortion (benchmark economy)} \]

\[ \text{Figure 3 is drawn in this particular situation.} \]
Moving away from the standard labor supply model, there are situations where the social weight attached to the unemployed agents is larger than that attached to the employees: for instance this may be the case in the presence of ‘involuntary’ unemployment, or when a large opportunity cost to work is associated with a handicap (the marginal social weight \( \psi'(s; \delta, \omega) \) is increasing with \( \delta \)). It is then easy to think of economies where at the optimum the average social weight of the unemployed is larger than the marginal cost of public funds and the social weight of the lowest paid workers is smaller. In these economies, after tax income is everywhere smaller than productivity.

Similarly, the analysis has proceeded under the assumption that the social welfare function is smooth, so that the distribution of the agents’ weights has no mass point. The case of a Rawlsian planner who puts all the weight on the least favored agent in the economy corresponds here to a situation where \( p_E \) is equal to zero everywhere. Then the tax rate is always positive. This is in line with the results of Choné and Laroque (2005). On the other hand, other social choice criteria, such as the one advocated in Fleurbaey and Maniquet (2006), may put the weight on the deserving ‘working poors’. As a consequence, in an intensive model with multiple dimensions of heterogeneity, Fleurbaey and Maniquet (2006) show that optimal taxation implies subsidizing as much as possible the poorest workers.

7 An example without heterogeneity

The previous analysis crucially hinges on the existence of heterogeneity in work opportunity costs. This section shows how the results break down where everyone has the same work opportunity cost, as in Homburg (2002), a situation where Assumption I does not hold. Let \( \delta_0 \) be the common value of the work opportunity cost. In the benchmark model (see end of Section 2), the objective takes the form

\[
L(R, s; \omega) = \begin{cases} 
U(s) - \lambda s & \text{if } R - s < \delta_0, \\
U(R - \delta_0) + \lambda[\omega - R] & \text{if } R - s > \delta_0,
\end{cases}
\]

The Lagrangian is equal to any of the two above quantities when \( R - s = \delta_0 \): the agent is indifferent between working or not, and the planner can choose the preferred outcome. The problem is to maximize the integral of the Lagrangian subject to the feasibility constraint over \( R, R \) nondecreasing.

Absent pooling, the first order condition (10) with respect to \( R(\omega) \) for an unconstrained worker of productivity \( \omega \) would be:

\[
U'(R(\omega) - \delta_0) = \lambda.
\]

Since the solution \( R(\omega) \) does not depend on productivity, there is full pooling.
Furthermore the feasibility constraint (7) here becomes
\[ \int_{\Omega} [\omega - R + s] \mathbb{1}_{\text{workers}} dG(\omega) = s. \]

An optimum is characterized by two quantities \((R, s)\) linked by the feasibility constraint. Furthermore it has to specify the set of workers when \(R - s\) is equal to \(\delta_0\). Consequently, the first order conditions lead to three possibilities:

1. If \(R - \delta_0 < s\), nobody works. By feasibility \(s\) is equal to zero. The value of social welfare, \(U(0)\), is at a global minimum when the economy is productive enough.

2. If \(R - \delta_0 > s\), everybody works. The constant income \(R\) is equal to the average productivity in the economy \(\int_{\Omega} \omega dG(\omega)\). The social welfare is equal to
\[ U \left( \int_{\Omega} \omega dG(\omega) - \delta_0 \right), \]
which is only defined when \(\int_{\Omega} \omega dG(\omega) \geq \delta_0\).

3. Finally, when there is indifference between working or not, \(R - \delta_0 = s\), and looking at the expression of \(L(R, s; \omega)\), the planner decides to put to work the agents with productivity \(\omega\) at least as large as \(R - s\). The feasibility condition gives the value of \(s\)
\[ \int_{\Omega} \max[\omega - \delta_0, 0] dG(\omega) = s \geq \int_{\Omega} \omega dG(\omega) - \delta_0. \]

This last case is the optimum whenever there is a non negligible set of agents with \(\omega > \delta_0\). Then there is no upward distortion of labor supply. The utilitarian optimum does not leave any surplus to the workers and everyone is treated equally. Heterogeneity in the form of some dispersion of work opportunity costs gives more scope for redistribution, associated with the unknown value of \(\delta\).

8 Conclusion

We have studied optimal taxation in the extensive framework: agents decide to stay inactive or to work -possibly in an occupation requiring a skill below their type. Starting from a general participation model, we have derived a two-dimensional reduced form involving productivity and work opportunity cost. Allowing for general distributions of these parameters and for income effects, we have characterized optimal, incentive-compatible tax schedules. We have given sufficient conditions for the social weight of the employed workers to decrease.
with both income and productivity. When these conditions hold, upwards distortions of the financial incentives to work can occur for low-skilled workers only. Under a simple specification of utility, we have shown that upwards distortions indeed arise for a positive mass of agents.

Overall, negative average tax rates and upward distortions of the financial incentives to work of the less skilled workers seem to be, in the extensive model, the rule rather than the exception. This property occurs in particular in the standard benchmark model, provided social weights vary continuously with the utility levels. This is in sharp contrast with the results that follow from the intensive model a la Mirrlees.

References


Appendix

A Relationship between the structural and the reduced forms

The following proposition provides a characterization of reduced-form specification which holds only in the one-dimensional setting.

**Proposition A.1.** When there is one dimension of heterogeneity \((\alpha \text{ is scalar})\), the reduced-form utility satisfies

\[
{u_\delta}(c_E; \delta, c_U)/u_{c_U}(c_E; \delta, c_U) \text{ does not depend on } c_E. \tag{18}
\]

Conversely, any reduced-form model \((u, v)\) satisfying conditions (4) and (18) derives from a structural model \((\tilde{u}, \tilde{v})\) with unidimensional heterogeneity.

When there is more than one dimension of heterogeneity, the restriction (18) is not necessary.

To prove Proposition A.1, we consider successively the case where the dimension of \(\alpha\) is one and the case where it is higher than 1.

A.1 The unidimensional case

Consider the structural model (1) and assume that \(\alpha\) is scalar and that, for any given \(c_U\), the map \(\alpha \rightarrow \delta(\alpha; c_U)\) is one-to-one. We note \(\delta^{-1}(\cdot; c_U)\) the inverse function:

\[
\delta^{-1}(\delta(c_U); c_U).
\]

\[u(c_E; \delta(c_U); c_U) = \tilde{u}(c_E; \delta^{-1}(\delta(c_U); c_U))\] \[\text{and } u_{c_U}(c_E; \delta, c_U) = \tilde{u}_{c_U}(c_E; \delta^{-1}(\delta(c_U); c_U)). \tag{19}\]

Condition (2) is equivalent to condition (4). It follows from (19) that

\[
{u_\delta}(c_E; \delta, c_U) = \tilde{u}_\alpha(c_E; \delta^{-1}(\delta(c_U); c_U)) \quad \text{and} \quad u_{c_U}(c_E; \delta, c_U) = \tilde{u}_{c_U}(c_E; \delta^{-1}(\delta(c_U); c_U)).
\]

which implies (18). Now, note that

\[u(c_E; \delta(\alpha; c_U), c_U) = \tilde{u}(c_E; \alpha).\]

It follows that, for any given \((c_E, \alpha)\), the function \(s \rightarrow \delta(\alpha; c_U)\) satisfies the following ordinary differential equation:

\[
{u_\delta}(c_E; \delta(\alpha; c_U), c_U)\delta_{c_U}(\alpha; c_U) + u_{c_U}(c_E; \delta(\alpha; c_U), c_U) = 0. \tag{20}
\]

The income effect, that is the dependence of \(\delta(\alpha; c_U)\) on \(c_U\), is given by (20).
Conversely, consider two functions \( u(c_E; \delta, c_U) \) and \( v(c_U; \delta) \) satisfying conditions (4) and (18). Thanks to (18), the ordinary differential equation (20) does not depend on \( c_E \). For a given initial condition at some point \( x_0 \), the solution of this equation does not depend on \( c_E \). Using \( \alpha \) to parameterize the initial condition, say \( \delta(\alpha; x_0) = \alpha \), we get a family of functions \( c_U \rightarrow \delta(\alpha; c_U) \) that characterize the behavior of the agent and the income effect. We define

\[
\begin{align*}
\tilde{u}(c_E; \alpha) &= u(c_E; \delta(\alpha; c_U), c_U) \\
\tilde{v}(c_U; \alpha) &= v(c_U; \delta(\alpha; c_U)).
\end{align*}
\]

Thanks to (20), the definition of \( \tilde{u} \) is consistent, i.e. \( \tilde{u} \) does not depend on \( c_U \).

A.2 Counter-example with multiple heterogeneity

When the dimension of \( \alpha \) is greater than 1, the condition (18) is not necessary any more, as the following example shows. We fix \( M > 1 \) and suppose that \((\alpha_1, \alpha_2)\) is uniformly distributed on the square \([0, 1]^2\). Let \( \tilde{u} \) and \( \tilde{v} \) be defined by

\[
\begin{align*}
\tilde{u}(c_E; \alpha) &= -\frac{1}{M + c_E - \alpha_1} \\
\tilde{v}(c_U; \alpha) &= -\frac{1}{M + (1 + \alpha_2)c_U}.
\end{align*}
\]

The functions \( \tilde{u} \) and \( \tilde{v} \) are strictly increasing and strictly concave in income for all \( \alpha = (\alpha_1, \alpha_2) \). It is easy to check that \( \tilde{u} \) holds with \( \delta(\alpha_1, \alpha_2; c_U) = \alpha_1 + c_U \alpha_2 \) and that \( \alpha_1 \), conditionally on \( \delta \), is uniformly distributed\(^{15}\). If we note \( [\alpha_1(\delta, c_U), \alpha_1(\delta, c_U)] \) the support of the conditional distribution, we have

\[
u(c_E; \delta, c_U) = \frac{-1}{\alpha_1(\delta, c_U) - \alpha_1(\delta, c_U)} \int_{\alpha_1(\delta, c_U)}^{\alpha_1(\delta, c_U)} \frac{1}{M + c_E - \alpha_1} d\alpha_1.
\]

In the region where \( c_U < \delta < 1 \), we have \( \alpha_1(\delta, c_U) = \delta - c_U; \alpha_1(\delta, c_U) = \delta \) and

\[
u(c_E; \delta, c_U) = \frac{1}{c_U} \ln \frac{M + c_E - \delta}{M + c_E - \delta + c_U}
\]

which does not satisfy (18). Note also that the distribution of the opportunity cost is given, form small \( \delta \), by \( F(\delta|\omega, c_U) = \delta^2/c_U \). This distribution satisfies Assumption 1 as \( F/f \) tends to 0 as \( \delta \) goes to \( \delta = 0 \).

B Proof of Proposition 1

The existence and properties of \( M(R; s, \lambda) \) follow from the fact that, under Assumptions 3 and 2, the cross derivative \( \partial^2 L/\partial R \partial \omega \) is positive. Indeed:

\[
\frac{1}{\lambda} \frac{\partial L}{\partial R} = \frac{\partial (\omega - \delta) F(\delta|\omega)}{\partial R} + \frac{1}{\lambda} \int_{\delta}^{R-s} u'(R; \delta, s) dF(\delta|\omega, s).
\]

\(^{15}\)Leisure is a normal good as \( \delta \) increases with \( c_U \).
The first term is increasing in $\omega$ from Assumption 3, the second is nondecreasing from Assumption 2.

Pointwise maximization leads to the global maximum ignoring the monotonicity constraint, so that the only thing to prove is the monotonicity of the pointwise maximizer. It is a straightforward consequence of single crossing:

$$L(R(\omega'), \omega') - L(R(\omega'), \omega) - L(R(\omega), \omega') + L(R(\omega), \omega) = \int_{\omega}^{R(\omega')} \int_{\omega}^{\omega'} \frac{\partial^2 L}{\partial R \partial \omega} dR \, d\omega.$$ 

The left hand side is non negative from pointwise maximization. The right hand side is of the same sign as $[R(\omega') - R(\omega)](\omega' - \omega)$.

\[\text{C Proof of Proposition 2}\]

In this section, we show that

$$p_E(R, s|\omega) = \frac{1}{F(R - s|\omega, s)} \int_{\delta(\omega)}^{R-s} \frac{\partial u}{\partial R}(R; \delta) \, dF(\delta|\omega, s) = E_\delta \left[ \frac{\partial u}{\partial R} | \delta \leq R - s, \omega, s \right]$$

(22)
decreases with $R$. To simplify notations, we drop the $s$ and $\omega$ arguments which are kept constant in the proof. An integration by parts yields

$$p_E(R) = \frac{\partial u}{\partial R}(R; R - s) - \frac{1}{F(R - s)} \int_{\delta(\omega)}^{R-s} \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) F(\delta) \, d\delta,$$

or

$$p_E(R) = \frac{\partial u}{\partial R}(R; R - s) - E_\delta \left[ \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) \frac{\bar{F}(\delta)}{f(\delta)} | \delta \leq R - s \right].$$

Now differentiating (22) gives

$$\frac{\partial p_E}{\partial R}(R) = E_\delta \left[ \frac{\partial^2 u}{\partial R^2}(R; \delta) | \delta \leq R - s \right] - \frac{f(R - s)}{F(R - s)} p_E(R) + \frac{f(R - s)}{F(R - s)} \frac{\partial u}{\partial R}(R; R - s).$$

Substituting the expression obtained for $p_E(R)$:

$$\frac{\partial p_E}{\partial R}(R) = E_\delta \left[ \frac{\partial^2 u}{\partial R^2}(R; \delta) + \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta) \frac{f(R - s)}{F(R - s)} \frac{F(\delta)}{f(\delta)} | \delta \leq R - s \right].$$

By the log-concavity of $F$, the factor

$$\frac{f(R - s)}{F(R - s)} \frac{F(\delta)}{f(\delta)}$$

is smaller than 1. By concavity of the utility function, the first term is negative. By the assumption on the marginal utility of income of the newcomer into the labor force

$$\frac{\partial^2 u}{\partial R^2}(R; \delta) + \frac{\partial^2 u}{\partial R \partial \delta}(R; \delta)$$

is non positive. The result follows. \[\blacksquare\]
D Proof of Proposition 4

Under the assumptions of the proposition, pooling cannot be ruled out. Accordingly, the following proof accounts for the possibility of pooling.

D.1 Position of the after tax income curve with respect to the 45 degree line

By Propositions 2 and 3

\[
\frac{\partial p_E(R, s|\omega)}{\partial R} + \frac{\partial p_E(R, s|\omega)}{\partial \omega} \leq 0.
\]

It follows that \( p_E(R, s|\omega) \) is nondecreasing as one goes up the 45 degree line. Consider in turn the three possibilities: \( p_E(\omega + s, s|\omega) \) is everywhere smaller than \( \lambda \), \( p_E(\omega + s, s|\omega) \) is everywhere equal to or larger than \( \lambda \), or finally there is some \( \omega \) in \( (\delta + s, \bar{\delta} + s) \) where \( p_E(\omega + s, s|\omega) \) is equal to \( \lambda \) (and some where it is smaller).

1. For all \( \omega \), \( p_E(\omega + s, s|\omega) \leq \lambda \). It follows from the monotonicity properties of \( p_E \) that \( p_E \leq \lambda \) above the 45 degree line \( \omega = R - s \). Now, from \( \partial L/\partial R \) is negative for all \( \omega \) such that \( R > \max(\omega + s, \delta + s) \). Then any nondecreasing function \( R(\omega) \) above the 45 degree line is dominated by the nondecreasing function \( \tilde{R}(\omega) \), defined by \( \tilde{R}(\omega) = \min(\max(\omega + s, \delta + s), R(\omega)) \). It follows that \( R \) must coincide with \( \tilde{R} \), that is to say: \( R \leq \max(\omega + s, \delta + s) \) for all \( \omega \), which expresses that the financial incentives to work are anywhere distorted downwards or undistorted. Proposition 4 therefore holds with \( \omega = \omega \) (as well as with \( \omega = \delta \)).

2. For all \( \omega \), \( p_E(\omega + s, s|\omega) \geq \lambda \). It follows from the monotonicity properties of \( p_E \) that \( p_E \geq \lambda \) below the 45 degree line \( \omega = R - s \). From \( \partial L/\partial R \) is positive for all \( \omega \) such that \( s + \delta < R < s + \omega \). Then any nondecreasing function \( R(\omega) \) below the 45 degree line is dominated by the nondecreasing function \( \tilde{R}(\omega) \), defined by \( \tilde{R}(\omega) = \max(\omega + s, R(\omega)) \). It follows that \( R \geq \omega + s \) for all \( \omega \). But then, by feasibility \( \gamma \), the subsistence income \( s \) is equal to zero and \( \omega = R(\omega) \). This is the laissez-faire allocation.

3. There is an \( \omega \), say \( \omega_0 \), such that \( p_E(\omega_0 + s, s|\omega_0) = \lambda \). It follows from the monotonicity properties of \( p_E \) that \( p_E \leq \lambda \) for \( R \geq \omega + s \) and \( \omega \geq \omega_0 \) and that \( p_E \geq \lambda \) for \( R \leq \omega + s \) and \( \omega \leq \omega_0 \). From \( \partial L/\partial R \) is positive for all \( (R, \omega) \) such that \( \omega_0 > \omega > R - s \), and negative for all \( (R, \omega) \) such that \( \omega_0 < \omega < R - s \). Take any function \( R(\omega) \). Define \( \tilde{R} \), associated with \( R \), through

\[
\tilde{R}(\omega) = \begin{cases} 
\min[R(\omega_0), \max(\omega + s, R(\omega))] & \text{if } \omega \leq \omega_0, \\
\max[R(\omega_0), \min(\omega + s, R(\omega))] & \text{if } \omega \geq \omega_0.
\end{cases}
\]
Then, whenever $R$ is nondecreasing, $\tilde{R}$ is also nondecreasing and yields a social utility level at least as high as $R$. As a consequence, any optimal $R$ must be such that the associated $\tilde{R}$ coincides with $R$. Letting $\omega_m = R(\omega_0) - s$, the optimal tax scheme is above or on the 45 degree line for $\omega \leq \omega_m$ and is below or on the 45 degree line for $\omega \geq \omega_m$.

D.2 Continuity of taxes when incentives are distorted downwards

From (9), $\partial L/\partial R$ has the same sign as the function $H$ defined by

$$H(R, \omega) = (\omega - R + s) - \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \left[ 1 - \frac{p_E(R, s|\omega)}{\lambda} \right].$$

A straightforward computation gives:

$$\frac{\partial H}{\partial R} = -1 - \left( 1 - \frac{p_E(R, s|\omega)}{\lambda} \right) \frac{\partial (F/f)}{\partial R} + \frac{F(R - s|\omega, s)}{f(R - s|\omega, s)} \frac{\partial p_E}{\partial R}.$$

In the half plane $\omega \geq R - s$, any point such that $H(R, \omega)$ is equal to zero satisfies $p_E(R, s|\omega) \leq \lambda$. From the log-concavity of $F$ and the monotonicity of $p_E$ (which follows from the assumptions of the Proposition), it follows that at any such point

$$\frac{\partial H}{\partial R} \leq -1.$$

Therefore, from the implicit function theorem, the equality $H(R, \omega) = 0$ can be solved in the half plane into a continuously differentiable function $R = \rho(\omega)$.

We now are in a position to proceed with the proof. Consider an optimal tax scheme that enters the region $\omega > R - s$ where labor supply is distorted downwards, at the point of abscissa $\omega_m$ defined in the first part of the Proposition. From the first order conditions, there are two possibilities, depending on whether there is pooling or not, at $\omega_m$:

1. There is no pooling at $\omega_m$ and $p_E(\omega_m + s, s|\omega_m)$ is equal to $\lambda$. Then the solution for $\omega \geq \omega_m$ is obtained by applying an ironing technique to the graph of $H(R, \omega)$. From the properties of $H$ and the function $\rho$, the solution is continuous.

2. There is pooling at $\omega_m$. From (14), at the end of the pooling interval, say at $\omega_1 > \omega_m$, $\partial L/\partial R$ and $H$ are less than or equal to zero. We claim that these quantities are zero: indeed by the first order conditions, we must have $\partial L/\partial R \geq 0$ and $R \geq R(\omega_1)$ in the right neighborhood of $\omega_1$, while $\partial H/\partial R$ is negative. It follows that as in 1. the remainder of the solution is obtained by ironing the graph of $H(R, \omega)$, for $\omega \geq \omega_1$.  

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D.3 Proof of Remark 5.1

When the distribution of work opportunity costs is independent of $\omega$, the equation $\partial L/\partial R = 0$ is equivalent to:

$$\omega = R - s + \frac{F(R - s|s)}{f(R - s|s)} \left[ 1 - \frac{p_E(R, s)}{\lambda} \right].$$

From the logconcavity of $F$ and the fact that $p_E$ decreases with $R$, the right hand side is an increasing function of $R$ with a slope at least as large as 1 in the region where $p_E$ is smaller than $\lambda$. The result follows directly at points where $\partial L/\partial R = 0$; it holds trivially in a pooling region, where the marginal tax rate is 1.

E Proof of Lemma 1

Writing the effect of a translation of the overall income (increasing $R(\omega)$ for all $\omega$ and $s$ by the same small amount) shows that the expectation in $w$ of

$$F(R(\omega) - s|\omega, s)p_E(R(\omega), s|\omega) + [1 - F(R(\omega) - s|\omega, s)]p_U(R(\omega), s|\omega)$$

$$+ \lambda \left( [\omega - R + s] \frac{\partial F}{\partial s}(R(\omega) - s|\omega, s) - 1 \right)$$

is zero.

Consider first points $\omega$ where $R(\omega)$ is strictly increasing and the first order condition (10) holds. For these points, we set $\rho = \frac{1}{\int f(\omega)}$. Thanks to (5), we get: $\rho \leq 1$. It follows from the first order condition (10) that

$$\lambda [\omega - R + s] \frac{\partial F}{\partial s}(R(\omega) - s|\omega, s) = \lambda [\omega - R + s] \rho f = \rho F(\lambda - p_E).$$

Now consider a pooling interval $[\omega_1, \omega_2]$ where $R$ is constant and equal to $\bar{R}$. Suppose first that $\bar{R} - s \leq \omega_1$. Then using (5) and the first pooling condition (12), we get

$$\lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} \text{d}G(\omega) \leq \lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) f \text{d}G(\omega) = \int_{\omega_1}^{\omega_2} (\lambda - p_E) F \text{d}G(\omega).$$

For $\omega \in [\omega_1, \omega_2]$, we set

$$\rho = \frac{\lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} \text{d}G(\omega)}{\int_{\omega_1}^{\omega_2} (\lambda - p_E) F \text{d}G(\omega)}.$$

Since $\int_{\omega_1}^{\omega_2} (\lambda - p_E) F \text{d}G(\omega) > 0$, we get $\rho \leq 1$. 

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Suppose now that $\bar{R} - s \geq \omega_2$. The same computation yields
\[
\int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega) \geq \lambda \int_{\omega_1}^{\omega_2} (\omega - \bar{R} + s) f dG(\omega) = \int_{\omega_1}^{\omega_2} (\lambda - pE) F dG(\omega).
\]
We define $\rho$ as above, and again $\rho \leq 1$. (Notice that, in this case, $\int_{\omega_1}^{\omega_2} (\lambda - pE) F dG(\omega) < 0$.)

Finally suppose that $\omega_1 < \bar{R} - s < \omega_2$. Then, using (5) and the second pooling condition (13), we get
\[
\int_{\bar{R} - s}^{\omega_2} \lambda (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG \leq \int_{\bar{R} - s}^{\omega_2} \lambda (\omega - \bar{R} + s) f dG \leq \int_{\bar{R} - s}^{\omega_2} (\lambda - pE) F dG.
\]
For $\omega \in [\bar{R} - s, \omega_2]$, we define $\rho$ as
\[
\rho = \frac{\int_{\bar{R} - s}^{\omega_2} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega)}{\int_{\bar{R} - s}^{\omega_2} (\lambda - pE) F dG(\omega)}
\]
and get $\rho \leq 1$. Similarly we have
\[
\lambda \int_{\omega_1}^{\bar{R} - s} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG \geq \lambda \int_{\omega_1}^{\bar{R} - s} (\omega - \bar{R} + s) f dG \geq \int_{\omega_1}^{\bar{R} - s} (\lambda - pE) F dG
\]
For $\omega \in [\omega_1, \bar{R} - s]$, we define $\rho$ as
\[
\rho = \frac{\int_{\omega_1}^{\bar{R} - s} (\omega - \bar{R} + s) \frac{\partial F}{\partial s} dG(\omega)}{\int_{\omega_1}^{\bar{R} - s} (\lambda - pE) F dG(\omega)}
\]
and get, again, $\rho \leq 1$. (Notice that $\int_{\omega_1}^{R - s} (\lambda - pE) F dG(\omega) < 0$.)

Replacing in (23) the term $\lambda (\omega - R + s) \frac{\partial F}{\partial s}$ (or its integral on pooling intervals) by the expressions derived above yields the desired result.

Finally recall that absent income effects: $\partial F/\partial s = \rho = 0$, while when leisure is a normal good, $\partial F/\partial s$ is everywhere less than or equal to 0.

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**F  The benchmark economy**

**F.1  Proof of Proposition 5**

Recall that in the benchmark model (see the end of Section 2) workers get utility $U(R - \delta)$ and unemployed agents get utility $U(s)$. We note the (unconditional) social weight of the whole population of unemployed agents
\[
p_U(s) = \mathbb{E}_{\alpha, w} \left[ \bar{v}_i(s; \alpha) \mathbbm{1}_{R(\omega) - s < \delta} \right].
\]
Here $p_U(s) = U'(s)$. There are no income effects ($\partial F/\partial s = 0$) and equation (17) simplifies into

$$\lambda = \int \int U'[\max(s, R(\omega) - \delta)] \, dF(\delta|\omega) \, dG(\omega).$$

It follows that $\lambda \leq U'(s)$. The inequality has to be strict: otherwise $R(\omega) - s \leq \delta$ almost everywhere, and essentially nobody works. Then feasibility would imply $s = 0$, and everybody is unemployed with zero income. But this allocation is dominated by laissez-faire, a contradiction. Consequently $p_U(s) = U'(s) > \lambda$.

Let $R_0$ be such that

$$U'(R_0 - \delta) = \lambda.$$ 

From the concavity of the utility function, $R_0 - \delta > s$, and the weight of the employees $p_E$ is larger than $\lambda$ whenever the income $R(\omega)$ is smaller than $R_0$. Define $\omega_0$ as $R_0 - s$: $\omega_0$ is larger than $\delta$. From the expression (9) of $\partial L/\partial R$, we have

$$\frac{\partial L}{\partial R}(R, s; \omega) > 0 \quad \text{for all } \delta + s < R \leq R_0 \text{ and } R - s \leq \omega.$$

(24)

For $\omega$ in $(\delta, \omega_0)$, the above inequality directly yields $R(\omega) > \omega + s$ when there is no pooling at $\omega$. Suppose now there is pooling at $\omega$ and consider the upper end of the pooling interval. The first order condition (14) imposes: $\partial L/\partial R \leq 0$ at this point. The inequality then shows that this point is above the 45 degree line $R = \omega + s$. It follows that the pooling interval entirely lies above the 45 degree line. As a consequence

$$R(\omega) > s + \omega \quad \text{for all } \omega \text{ such that } \delta < \omega \leq \omega_0.$$ 

All the low skilled workers (if $\omega = \delta$, there are no workers with productivity $\delta$) of productivity smaller than $\omega_0$ have financial incentives to work distorted upwards, which completes the proof.

We now investigate what happens at productivity $\omega = \delta$, in order to substantiate points 1. and 2. of the comment of the Proposition: what is the optimal $R(\delta)$? The derivative $\partial L/\partial R$ is zero for $R = s + \delta$, so that $s + \delta$ is an extremum of the function $L(\cdot, s; \delta)$. A straightforward calculation gives

$$\frac{\partial^2 L}{\partial R^2}(s + \delta, s; \delta) = (p_U - 2\lambda)f(\delta|\delta),$$

where we use the fact that the marginal worker is indifferent between working or being unemployed, so that $p_E$ is equal to $p_U$ at this point. For $p_U > 2\lambda$,

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16 This point contrasts with the proof of Proposition 4. The condition (24) holds for all $R - s < \omega$, while in point 3 of Section D.1 the additional restriction $\omega \leq \omega_0$ was necessary to ensure $\partial L/\partial R(R, s; \omega) > 0$.

17 For simplicity, the proof is presented under the assumption that $f(\delta|\delta) > 0$. It is easy to extend it to other cases, e.g. $f(\delta|\delta) = 0$ and $f'(\delta|\delta) > 0$. 

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the second derivative is positive, so that \( s + \delta \) is a local minimum in \( R \) of \( L \). This corresponds to point 2. The value of \( L(R(\tilde{\delta}), s; \tilde{\delta}) \) then is larger than \( L(s + \delta, s; \delta) = U(s) - \lambda s \), the value of the Lagrangian when everybody is unemployed. By continuity, the optimum tax scheme puts to work some agents of productivity \( \omega \) smaller than \( \delta \), whenever \( \omega < \delta \).

F.2 Example with a uniform distribution of work opportunity costs

When the distribution of work opportunity costs is uniform, the social weight of the employees is

\[ p_E(R) = \frac{1}{F(R-s)} \int_{\tilde{\delta}}^{R-s} U'(R - \delta) \frac{d\delta}{\delta - \tilde{\delta}} = \frac{U(R - \tilde{\delta}) - U(s)}{R - s - \tilde{\delta}}. \]

Integrating and substituting yields

\[ M(R; s, \lambda) = 2(R - s) - \delta - \frac{U(R - \tilde{\delta}) - U(s)}{\lambda}. \]

The function \( M(R; s, \lambda) \) is strictly convex in \( R \), \( M(s + \delta; s, \lambda) = \delta \) and \( M'_R(s + \delta; s, \lambda) = 2 - p_U(s)/\lambda < 1 \).

1) Case \( \lambda < p_U(s) \leq 2\lambda \). \( M(R; s, \lambda) \) is strictly increasing in \( R \) and Proposition 4 applies. The convexity of \( M(\cdot; s, \lambda) \) implies the concavity of \( R(\omega) \). The slope of \( R \) at \( \omega = \delta \) is the inverse of that of \( M \), so it is strictly greater than 1, as represented by the bold curve on Figure 2.

2) Case \( p_U(s) > 2\lambda \). Here the slope of \( M \) at \( s + \delta \) is negative (see Figure 3). We study the pointwise maximum of \( L(R, s; \omega) \) for \( R - s \geq \delta \). As it turns out to be increasing in \( \omega \), it satisfies the monotonicity condition and is the optimum.

Recall that \( L(s + \delta, s; \omega) = U(s) - \lambda s \), which we shall note \( L_{\min} \). Now,

\[ \frac{\partial L}{\partial R}(R, s; \omega) = \lambda(\omega - M(R; s, \lambda))f(R - s) = \frac{\lambda}{\delta - \tilde{\delta}} (\omega - M(R; s, \lambda)) \]

for \( \tilde{\delta} \leq R - s \leq 3 \) is a concave function of \( R \) which becomes negative for large enough \( R \). We consider three cases:

a. For \( \omega > \delta \), \( \partial L/\partial R(s + \delta, s; \omega) \) is positive. There is a single income \( R(\omega) \) in \( [s + \delta, s + \tilde{\delta}] \), solution to \( \omega = M(R; s, \lambda) \), which maximizes \( L(R, s; \omega) \).

b. For \( \omega = \delta \), \( \partial L/\partial R(s + \delta, s; \omega) \) is equal to zero. \( \partial^2 L/\partial R^2(s + \delta, s; \omega) = (p_U(s) - 1 - \lambda)/{(\delta - \tilde{\delta})} \) is positive, so that there is another root \( R(\delta) \), larger than \( s + \delta \) (\( R = s + \delta \) is a local minimum of \( L \)).
c. Finally consider $\omega < \delta$. The function $\partial L / \partial R(\cdot, s; \omega)$ is linear increasing in $\omega$: when $\omega$ decreases from $\delta$, its smallest root increases, its largest root (a local maximum of $L$), say $\Delta(\omega)$, decreases, until eventually they both disappear, say at $\omega_1$, $\omega_1 < \delta$. Note that $L(\Delta(\omega), s; \omega)$ is an increasing function of $\omega$. Since $L(s + \delta, s; \delta) = L_{\min}$, $L(\Delta(\omega_1), s; \omega_1)$ is smaller than $L_{\min}$. Let $\omega_2, \omega_2 > \omega_1$, be such that $L(\Delta(\omega_2), s; \omega_2)$ is equal to $L_{\min}$. Define $\omega_0 = \max(\omega_1, \omega_2)$, $R(\omega) - s = \Delta(\omega)$ for $\omega_0 \leq \omega \leq \delta$, and $R(\omega) - s = \delta$ for $\omega$ smaller than $\omega_0$.

It is easy to check that the $R(\omega)$ function thus defined indeed is the solution of the problem.  

\[ \Box \]