Costly and Truthful Communication: Two Alternative Objectives

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Abstract

This paper is about costly and truthful communication. I show that in a situation where a sender tries to convince a receiver to accept a project, the receiver’s communication objective and effort depend on whether the receiver would reject or accept the project without any communication.

If without communication the receiver would reject the project, his communication objective is to identify and accept a high quality project. His effort depends positively on his gain of accepting a high quality project.

If without communication the receiver would accept the project, his communication objective is to identify and reject a low quality project. His effort depends positively on his loss from accepting a low quality project.

1 Introduction

Communication is crucial in economic interactions. Acquiring relevant information can help someone to make the best possible choices. Nevertheless, it often requires that the agents exert a costly effort. Otherwise, the agents would acquire all available information before each choice.

This paper studies the following situation: a sender (S) tries to convince a receiver (R) to accept a project through truthful communication. It may help to better understand many economic phenomena: a citizen/consumer that reacts positively or...
remains unconvinced by a public information campaign/informative advertisement; a worker that proposes to change a feature of the company product to his manager; a recruiting committee that decides whether or not to hire a candidate; a politician that communicates to win over potential voters... As a reader of this paper, you might even consider yourself as a receiver choosing whether to read further or to stop after this paragraph.

S will be rewarded if R accepts the project: for example, it increases the profits of the advertising firm; it also helps the worker to get a future promotion, the candidate to achieve his professional goals, and the politician to get elected.

Both agents do not know whether the project will increase or decrease R’s payoff. Based on some prior information, they believe with probability \( \alpha \) that the project is of high quality and will increase R’s payoff by \( r_H \), and otherwise that it is of low quality and will decrease R’s payoff by \( |r_L| \) (with \( r_L < 0 < r_H \)). The model studied is therefore not about signalling: no agents benefit from private information before communication. For example, S and R do not know whether the product has a high or low valuation for the consumer, whether changing a feature of the product will raise or decrease the economic performance of the company, and whether the candidate has the required skills for the job.

R cannot evaluate the project quality by himself before his project acceptance decision. Nevertheless, S has information that, if understood by R, tells R whether the project is of high or low quality. Therefore, S may need to communicate if he wants R to accept the project.

Their communication may either fail or succeed. The higher both agents’ efforts to communicate, the more likely communication will succeed. If communication fails, R does not learn anything: he still believes with probability \( \alpha \) that it will increase his payoff by \( r_H \) and otherwise that it will decrease his payoff by \( |r_L| \). If communication succeeds, there are two possible outcomes: R either finds out that the project is of high quality, or that it is of low quality.

The job candidate has to spend time to prepare his interview. Conversely, the recruiter has to pay attention, decode and challenge the strengths and weaknesses of the candidate to find out whether he should hire him or not. Similarly, the advertising firm/worker has to exert an effort to convey information to the consumer/manager.

In such a situation, I show that R’s communication objective and effort depend on whether his payoff from accepting the project without communication is negative (NEG case) or positive (POS case). Let me first explain how communication efforts differ in these two cases, and then why, contrary to the existing literature (see the end of the introduction), the agents may communicate in the POS case.

First of all, note that R is only interested in a communication outcome that
induces him to change his project acceptance decision.

In the NEG case, without any additional information R rejects the project. R is thus only interested in finding out that the project is of high quality; it is the only communication outcome that convinces him to accept the project. R thus exerts a communication effort to enhance his chance of accepting a high quality project. Therefore, an increase in \( r_H \) (R’s gain from accepting a high quality project) and/or in \( \alpha \) (the probability before communication that the project is of high quality) raises the agents’ efforts. Moreover, since R never accepts a low quality project, an increase in \( |r_L| \) (his loss from accepting a low quality project) does not affect the agents’ efforts.

In the POS case, without communication R accepts the project. R is only interested in learning that the project is of low quality, it is the only communication outcome that induces him to reject the project. R thus exerts an effort to enhance his chance of rejecting a low quality project. Therefore, the higher \( |r_L| \), the higher the agents’ efforts. Moreover, since R never rejects a high quality project, an increase in \( r_H \) does not affect the agents’ efforts.

In the NEG case, the recruiter hires the job candidate if and only if after the interview, he is convinced of his skills. The higher the recruiter’s ex ante belief that the candidate has good qualifications, the higher the time the recruiter spends on the job interview and the more likely the firm hires the candidate.

In the POS case, the recruiter hires the candidate unless he identifies an important shortcoming during the interview. The higher the recruiter’s ex ante belief that the candidate has good qualifications, the lower the attention the recruiter pays during the interview.

With respect to the advertising example, it can be considered that an increase in the price of the product lowers \( r_H \) and \( |r_L| \). In both the NEG and POS cases, an increase in the price decreases R’s likelihood of buying the product. Nevertheless, in the NEG case, an increase in the price lowers the agents’ efforts and R’s likelihood of identifying a high quality product. In the POS case, an increase in the price raises the agents’ efforts and R’s likelihood of not buying a low quality product.

Regarding the manager/worker example, in the NEG case, the use of a carrot (rewarding the worker if his project is both accepted and of high quality), contrary to that of a stick (punishing the worker if his project is both accepted and of low quality), will increase the agents’ efforts and likelihood of launching a high quality project. The reason is that the agents’ efforts depend positively on R’s gain of accepting a high quality project.

In the POS case, the use of a stick, contrary to that of a carrot, will increase the agents’ efforts and likelihood of rejecting a low quality project. The reason is that the agents’ efforts depend positively on R’s loss of accepting a low quality project.
In the existing literature (see infra), R cannot communicate to avoid a low quality project. I show that this result is not robust to very plausible extensions of the model. First, S may communicate if he has an interest in the project quality or if he has social preferences. The reason is that if S cares positively about the project quality, S might be ready to communicate to prevent R to some extent from accepting a low quality project. S does not want the project to be accepted at all costs. This might be the case if the worker’s compensation is tied to the company’s profits. Similarly, the advertising firm has an interest in the consumer valuation of the product because it affects the company’s reputation and future buying decisions. Finally, a job candidate can be intrinsically motivated because of the company social impact, his career concerns might also make him worry that his skills fit with the firm needs.

Another reason why S may communicate in the POS case is that he may believe with some probability that R is the NEG case. Putting it differently, S does not know whether R will accept the project or not if communication fails. The worker is for example uncertain about how large the potential benefits for the company might be. The advertising firm might be unable to discriminate the communication between consumers in the NEG and POS case.

This paper extends the framework of the Dewatripont and Tirole’s modes of communication model (2005).

It therefore differs from the economic literature on communication of soft information initiated by Crawford and Sobel (1982) since this paper studies verifiable information: the agents do not benefit from private information before communication.\footnote{For a review, see Sobel (2010).}

It also differs from the economic literature on communication of hard information pioneered by Grossman (1981) and Milgrom (1981) because in this paper, S does not choose which pieces of information to withhold.

Besides, contrary to both the literature on hard and soft information, it is considered that communication success is endogenous to S and R’s efforts, and that since communication requires time and devotion, the disclosure and absorption of information are both costly.

Moreover, it is built upon the elaboration likelihood model of persuasion in the psychology literature which considers two modes of communication: issue-relevant and cue communication. In this paper, only the former is studied: R carefully thinks about and examines information pertinent to the merits of a project to determine whether he should accept or reject it.\footnote{For a review, see Petty et al. (2005).}
The rest of the paper is organized as follows. In section 2, the setup of the model is presented. In section 3, I show that there are two communication objectives by considering separately the following two cases: I first discuss the case of social preferences/S’s project quality incentives; and then I study the case of a sender who is uncertain on the receiver’s revenue from accepting a high quality project. In section 4, I conclude.

2 Model setup

There are 2 agents: S, the sender, and R, the receiver. R is the decision maker and has 2 choices: rejecting the project, yielding zero revenue for both agents and accepting the project that, if implemented, yields revenue \( s > 0 \) for S, but might lead to a loss for R.

R can be of 2 types: he is of type 1, \( R_1 \), with probability \( \lambda \in [0, 1] \) and he is of type 2, \( R_2 \), with probability \( 1 - \lambda \).

\( R_i \)'s revenue from accepting the project is either \( r_{Hi} \) or \( r_{L} \), \( i \in \{1, 2\} \), with \( r_{H1} > r_{H2} > 0 > r_{L} \). Parameters \( r_{Hi} \) and \( r_{L} \) respectively represent \( R_i \)'s revenue from accepting a high and a low quality project. \( R_1 \) and \( R_2 \) only differ in their revenue from accepting a high quality project.

Let the parameter \( \alpha \) denote the ex ante (before communication) probability of \( r_{Hi} \), measuring the riskiness of the project. It also represents the alignment of the 2 agents’ interests regarding the project: \( \alpha \in (0, 1) \). If \( \alpha \) is close to 1 (0), \( R_i \) and \( S \)'s project revenues are highly and positively (negatively) correlated.

The sequential structure (presented below in Figure 1) is the following:

**Stage 0:** Nature decides \( R_i \)'s type\(^3\) and the project quality.

**Stage 1:** Both agents simultaneously choose a costly communication effort to resolve the uncertainty on the project quality: \( e_S \) and \( e_{R_i} \in [0, 1] \) respectively for S and \( R_i \). S has information that, if correctly understood by \( R_i \), tells both agents whether \( R_i \)'s revenue from accepting the project is \( r_{Hi} \) or \( r_{L} \).

Communication involves increasing and convex private costs \( C_S(e_S) \) for S and \( C_R(e_{R_i}) \) for \( R_i \), with \( \frac{\partial C_S}{\partial e_S}(1) = \frac{\partial C_R}{\partial e_{R_i}}(1) = \infty \). For simplicity, there are no communication setup costs and \( R_i \)'s (S's) marginal cost of effort is equal to zero when \( R_i \)

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\(^3\)Because of the S's uncertainty on \( R_i \)'s type, \( R_i \) has private information that he cannot communicate to S. The game would only change if there was an additional communication stage where R can signal this private information. Dewatripont and Tirole (2005) study the case where S can communicate private information about the value of the congruence parameter \( \alpha \).
(S) does not communicate. Cost functions $C_S(e_S)$ and $C_R(e_{Ri})$ are assumed to be continuous and differentiable on $[0, 1)$.

**Stage 2:** The agents’ efforts and pure chance then determine the communication outcome $j$:

1) with probability $\alpha e_S e_{Ri}$, communication succeeds and the agents find out that the project is of high quality, $j = H$;

2) with probability $(1 - \alpha) e_S e_{Ri}$, communication succeeds and the agents learn that the project is of low quality, $j = L$;

3) with probability $1 - e_S e_{Ri}$, communication fails and the agents do not grasp the quality of the project, $j = F$; as before communication, they believe with probability $\alpha$ that the project is of high quality and otherwise that it is of low quality.

Communication is more likely to succeed when S explains the project better and when R pays more attention to S’s message.

**Stage 3:** Finally, without needing to understand S’s message for a proper implementation of the project, $R_i$ chooses whether or not to accept the project. $R_i$ chooses his project acceptance decision $a_i^j$: the variable $a_i^j$ is equal to 1 if $R_i$ accepts the project after communication outcome $j$, and is equal to 0 otherwise.

![Figure 1: The sequential structure](image)

Therefore, S’s strategy consists of his level of effort. In contrast, $R_i$’s strategy combines his level of effort and his project acceptance decisions after each possible communication outcome.

Let me end this setup by stating and explaining S’s and $R_i$’s utility functions, respectively $U_S$ and $U_{Ri}$:

\[
U_S = \lambda [E(\Pi^1_S) + \beta_S E(\Pi_{Ri})] + (1 - \lambda) [E(\Pi^2_S) + \beta_S E(\Pi_{R2})] - C_S(e_S)
\]

\[
U_{Ri} = E(\Pi_{Ri}) + \beta_R E(\Pi^1_S) - C_R(e_{Ri})
\]

$E(\Pi_{Ri})$ and $E(\Pi^1_S)$ represent respectively $R_i$’s expected revenue and S’s expected revenue when he is matched with $R_i$.  

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$R_i$ may care about $S$’s revenue and vice versa: parameter $\beta_R$ ($\beta_S$) captures the extent to which $R_1$ and $R_2$ ($S$) take(s) $S$’s ($R_1$ and $R_2$’s) expected revenue(s) into consideration.

Parameters $\beta_S$ and $\beta_R$ measure the agents’ social/other-regarding preferences. If $\beta_S$ ($\beta_R$) is strictly positive, $S$ ($R_i$) has altruistic concerns. If it is strictly negative, $S$ ($R_i$) is envious. If it is zero, $S$ ($R_i$) is selfish. Alternatively, $\beta_S$ can represent $S$’s exogenous incentives in the project success; because of $\beta_S$, $S$’s payoff from the project depends on its quality.

Finally, it is assumed that $-\frac{r_{L2}}{s} < \beta_R < -\frac{r_{H2}}{s}$ so that $R_1$ and $R_2$ both strictly prefer to accept a high quality project than to reject it, and to reject a low quality project than to accept it.

3 Two communication objectives

To start the analysis, let me first look at $R_i$’s project acceptance decision in stage 3 after each possible communication outcome:

1) If $R_i$ finds out that the project is of high quality ($j = H$), $R_i$ strictly prefers to accept the project ($a_i^H = 1$; cf. assumption that $\beta_R > -\frac{r_{H2}}{s}$).

2) If $R_i$ learns that the project is of low quality ($j = L$), $R_i$ strictly prefers to reject the project ($a_i^L = 0$; cf. assumption that $\beta_R < -\frac{r_{L2}}{s}$).

The classes of strategies playing $a_i^H = 0$ and/or $a_i^L = 1$ are therefore strictly dominated or equivalent to a strategy playing action $a_i^H = 1$ and $a_i^L = 0$.

3) If communication fails ($j = F$) and if the project yields $R_i$ a negative (positive) expected payoff, $R_i$ prefers to reject (accept) the project.

As it will turn out, $R_i$’s effort choice depends on his project acceptance decision.

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Even if the primary objective of communication is to transmit information, communication may also generate emotions (empathy, attraction, envy...) and lead agents to judge one another. One’s opinion of the other person is either positive or negative and may therefore involve social preferences: an agent may care about the other agent’s payoff.

Moreover, there is evidence in the psychology literature about the possible effect of emotions (such as guilt, envy, compassion...) on persuasion: e.g. Dillar and Peck (2000) and Nabi (2002). According to Petty et al. (1988), agents’ feelings, moods and emotions can influence their evaluations of people and issues.

The goal is to provide insights into the impact of social preferences on communication and not to compare the effects of various models of social preferences. This is the same reasoning as in Besley and Ghatak (2005) who study the interdependence of incentives and productivity between the for-profit sector and the mission-oriented sector through occupational choice; as in Driscoll and Holden (2004) who explain inflation persistence through social preferences; as in Fehr et al. (2001) who examine the impact of fairness considerations on contractual choices; as in Itoh (2004) who analyzes the optimal contract between a principal and an agent in the presence of moral hazard and social preferences; or as in Rotemberg and Saloner (1993) who study the relationship between incentives, leadership styles and the firm environment.
when communication fails. Therefore, an important threshold can be defined: \( \alpha^*_{Ri} \), the level of \( \alpha \) for which \( R_i \) is indifferent between \( a^F_i = 1 \) and \( a^F_i = 0 \), is defined as 
\[
\alpha^*_{Ri} = \frac{-r_{L} - \beta_{R}s}{r_{Hi} - r_{L}}.
\]
This is formalized in Lemma 1.

**Lemma 1** If \( \alpha < \alpha^*_{Ri} \), \( R_i \) plays \( a^F_i = 0 \); if \( \alpha > \alpha^*_{Ri} \), \( R_i \) plays \( a^F_i = 1 \); and if \( \alpha = \alpha^*_{Ri} \), \( R_i \) is indifferent between playing \( a^F_i = 0 \) and \( a^F_i = 1 \).

Proof: see appendix A.

If \( R_i \) prefers to reject the project when communication fails (\( \alpha \leq \alpha^*_{Ri} \)), \( R_i \) only accepts the project if he is convinced that it is of high quality.

If \( R_i \) prefers to accept the project when communication fails (\( \alpha \geq \alpha^*_{Ri} \)), \( R_i \) accepts the project unless he is convinced that it is of low quality.

The variable \( a^F_i \) influences the agents’ expected revenue in the following way:

\[
E(\Pi_{Ri}) = \begin{cases} 
  e_{SE_R} \alpha r_{Hi} & \text{if } a^F_i = 0 \\
  \alpha r_{Hi} + (1 - e_{SE_R})(1 - \alpha)r_{L} & \text{if } a^F_i = 1.
\end{cases}
\]

\[
E(\Pi_S) = \begin{cases} 
  e_{SE_R} \alpha s & \text{if } a^F_i = 0 \\
  (1 - e_{SE_R}(1 - \alpha))s & \text{if } a^F_i = 1.
\end{cases}
\]

When \( a^F_i = 0 \), \( R_i \) [\( S \)] earns revenue \( r_{Hi} \) [\( s \)] if communication is successful (with probability \( e_{SE_R} \)) and if the project is of high quality (with probability \( \alpha \)).

When \( a^F_i = 1 \), \( R_i \) always accepts the project yielding \( R_i \) [\( S \)] an expected revenue \( \alpha r_{Hi} + (1 - \alpha)r_{L} \) [\( s \)] unless \( R_i \) learns that the project is of low quality (with probability \( e_{SE_R}(1 - \alpha) \)).

The purpose of this paper is to study the characteristics of equilibria involving communication. Therefore, in the next two subsections, I show that there are two communication objectives by studying separately social preferences/\( S \)'s project quality incentives and \( S \)'s uncertainty on the receiver’s type.

### 3.1 Social preferences/\( S \)'s project quality incentives

In this subsection, I consider that \( \lambda = 1 \), i.e. \( S \) is certain that he is matched with \( R_1 \), to investigate the implications of social preferences/project quality incentives.

I will show that there are four possible types of equilibrium:

- **I)** the *communicating to accept a high quality project* (\( CH \)) equilibrium;
- **II)** the *communicating to reject a low quality project* (\( CL \)) equilibrium;
- **III)** the *no communication* (\( H \)) equilibrium in which \( R_1 \) always rejects the project;

\[5\]The possible equilibria when considering social preferences and \( S \)'s uncertainty on the receiver’s type together is presented in appendix B.3.
IV) the no communication (L) equilibrium in which \( R_1 \) always accepts the project.

The second equilibrium is the novelty of this paper because it does not exist without considering social preferences/project quality incentives.

I) In the CH equilibrium, the agents communicate and \( R_1 \) accepts the project if and only if he finds out that the project is of high quality (\( a_1^{F*} = 0 \)).

In this equilibrium, \( R_1 \) and S’s utilities are:

\[
U_{R1} = e^*_S e^*_R (r_{H1} + \beta_R s) - C_R(e^*_R) \\
U_S = e^*_S e^*_R (s + \beta_S r_{H1}) - C_S(e^*_S)
\]

Therefore, \( R_1 \) and S’s optimal efforts are implicitly given by:

\[
\frac{\partial C_R}{\partial e_{R1}} (e^*_R) = e^*_S \alpha (r_{H1} + \beta_R s) \\
\frac{\partial C_S}{\partial e_S} (e^*_S) = e^*_R \alpha (s + \beta_S r_{H1})
\]

Note that the agents’ optimal efforts depend on \( r_{H1} \) and not on \( r_L \). This means that the agents communicate to increase the probability of identifying a high quality project, and therefore to avoid an error of type II (a false negative). They are only interested in communication outcome \( H \) (they find out that the project is of high quality). \( R_1 \)’s project acceptance decision when he learns that the project is of low quality is the same as when communication fails: he rejects the project.

Therefore, since the probability of communication outcome \( H \) is equal to \( e^*_S e^*_R \alpha \), the higher the value of \( \alpha \), the higher both agents’ efforts will be.

II) In the CL equilibrium, S and \( R_1 \) communicate and \( R_1 \) accepts the project unless he finds out that the project is of low quality (\( a_1^{F*} = 1 \)).

In this equilibrium, \( R_1 \) and S’s utilities are:

\[
U_{R1} = \alpha r_{H1} + (1 - \alpha)(1 - e^*_S e^*_R) r_L - C_R(e^*_R) + \beta_R E(\Pi^1_S) \\
U_S = [1 - e^*_S e^*_R (1 - \alpha)] s - C_S(e^*_S) + \beta_S E(\Pi_{R1})
\]

Therefore, \( R_1 \) and S’s optimal efforts are implicitly given by:

\[
\frac{\partial C_R}{\partial e_{R1}} (e^*_R) = e^*_S (1 - \alpha)(-r_L - \beta_R s) \\
\frac{\partial C_S}{\partial e_S} (e^*_S) = e^*_R (1 - \alpha)(-s - \beta_S r_L)
\]
Note that the agents’ optimal efforts depend on \( r_L \) and not on \( r_{H1} \). This means that the agents communicate to increase the probability of correctly identifying a low quality project, and therefore to avoid an error of type I (a false positive). They are only interested in communication outcome \( L \) (they find out that the project is of low quality). \( R_1 \)’s project acceptance decision when he learns that the project is of high quality is the same as when communication fails: he accepts the project.

Therefore, since the probability of communication outcome \( L \) is equal to \( e^*_S e^*_R (1 - \alpha) \), the higher the value of \( \alpha \), the lower both agents’ efforts will be.

This equilibrium is in certain respects the opposite of the \( CH \) equilibrium: \( R_1 \) communicates to avoid getting \( r_L \) and not to earn \( r_{H1} \). In this equilibrium, \( S \) prevents \( R_1 \) to some extent from accepting a low quality project. \( S \) does not want the project to be accepted by \( R_1 \) at all costs.

III) In the \( H \) equilibrium, \( S \) and \( R_1 \) do not communicate \( (e^*_S = e^*_R = 0) \) and \( R_1 \) always rejects the project \( (a^*_F = 0) \).

In this equilibrium, \( R_1 \) and \( S \)’s utilities are zero.

IV) In the \( L \) equilibrium, \( S \) and \( R_1 \) do not communicate \( (e^*_S = e^*_R = 0) \) and \( R_1 \) always accepts the project \( (a^*_F = 1) \).

In this equilibrium, \( R_1 \) and \( S \)’s utilities are:

\[
U_{R1} = \alpha r_{H1} + (1 - \alpha) r_L + \beta_R s \\
U_S = s + \beta_S (\alpha r_{H1} + (1 - \alpha) r_L)
\]

Let me now state the conditions of existence of the equilibria.

**Proposition 1**

I) The communicating to accept a high quality project (\( CH \)) equilibrium exists provided that \( \alpha \leq \alpha^*_{R1} \) and \( \beta_S > -\frac{s}{r_{H1}} \).

II) The communicating to reject a low quality project (\( CL \)) equilibrium exists provided that \( \alpha \geq \alpha^*_{R1} \) and \( \beta_S > -\frac{s}{r_L} \).

III) The no communication (\( H \)) equilibrium in which \( R_1 \) always rejects the project exists provided that \( \alpha \leq \alpha^*_{R1} \).

IV) The no communication (\( L \)) equilibrium in which \( R_1 \) always accepts the project exists provided that \( \alpha \geq \alpha^*_{R1} \).

Proof: see appendix B.1.

Let me comment the conditions of existence not explained by Lemma 1.

I) The \( CH \) equilibrium can only exist if \( \beta_S > -\frac{s}{r_{H1}} \). \( S \) should not care too negatively about \( R_1 \)’s revenue: when \( R_1 \) accepts a project of high quality, \( S \) benefits from a revenue \( s \) but may suffer from negative concerns \( \beta_S r_{H1} \) if \( \beta_S \) is negative. \( S \)’s
expected payoff from the project should be positive. Otherwise, if \( \alpha \leq \alpha_{R1}^{*} \), S does not communicate implying that \( R_1 \) never accepts the project.

II) The CL equilibrium cannot exist if \( \beta_S \leq \frac{\alpha}{-r_L} \). In the CL equilibrium, S sacrifices part of his expected revenue \((e_S^{*} e_{R1}^{*}(1 - \alpha) s)\) to prevent \( R_1 \) to some extent from accepting a low quality project \((-e_S^{*} e_{R1}^{*}(1 - \alpha) r_L)\). Interpreting the parameter \( \beta_S \) as S’s social preferences means that S’s altruistic concerns must dominate his interest about his own expected revenue \((\beta_S > \frac{\alpha}{-r_L})\). Alternatively, it means that S’s project quality incentives should be sufficiently important to cause him to fear a low quality project.\(^6\)

To illustrate the effect of S’s social preferences/project quality incentives on his efforts, Figure 2 depicts S’s equilibrium effort for every possible value of \( \alpha \) in two situations: \( \beta_S = 0.6 \) (dashed curve) and \( \beta_S = 0 \) (solid curve).\(^7\) In both situations, I consider that \( C_R(e_{R1}) \) is equal to \( \frac{(e_{R1})^3}{3} \) if \( e_{R1} < 1 \) and to \( \infty \) otherwise, \( C_S(e_S) \) is equal to \( \frac{(e_S)^2}{2} \) if \( e_{R1} < 1 \) and to \( \infty \) otherwise, \( r_{H1} = 2, r_L = -2, \beta_R = 0 \) and \( s = 0.5 \).

![Figure 2: S’s social preferences/project quality incentives and effort](image)

Figure 2 shows that in the NEG case (accepting the project before communication would yield \( R_1 \) a negative payoff), it is the CH equilibrium that prevails in

\(^6\)One could argue that communication is *ceteris paribus* more likely to succeed if the project is of low quality than if it is of high quality. This would decrease the agents’ efforts in the CH equilibrium and increase the agents’ efforts in the CL equilibrium. There would not necessarily be a downward discontinuity in both agents’ efforts between the CH and the CL equilibria when \( \alpha \) crosses \( \alpha_{R1}^{*} \) (if \( \beta_R, \beta_S < 1 \) and if S and/or R’s efforts are not maximal in the CL equilibrium when \( \alpha = \alpha_{R1}^{*} \)). Moreover, this would not affect any equilibria conditions of existence since R’s project acceptance decision for each communication outcome \( a_1^{c} \) does not depend on the agents’ efforts.

\(^7\)In this graph, I disregard the no communication equilibrium when a Pareto-dominant equilibrium exists. The corresponding graph for \( R_1 \)’s effort is presented in appendix D.1.
both situations \((e^*_S = \alpha^3r_{H1}(s+\beta_Sr_{H1})^2)\). Therefore, the higher the value of \(\alpha\), \(r_{H1}\) and/or \(\beta_S\), the higher S’s effort will be. \(R_1\) exerts an effort to get \(r_{H1}\) (the \textit{ex ante} probability that the project is of high quality is equal to \(\alpha\)) and S communicates therefore to convince \(R_1\) that the project is of high quality.

In the POS case, the situation with social preferences/S’s project quality incentives \((\beta_S = 0.6)\) is completely different from the one without social preferences \((\beta_S = 0)\).

When S does not care about \(R_1\)’s project revenue \((\beta_S = 0)\), there cannot be any communication in equilibrium. S has the real authority, he does not communicate and \(R_1\) always accepts the project.

When S cares about \(R_1\)’s project revenue \((\beta_S = 0.6)\), it is the CL equilibrium that prevails \((e^*_{SL} = (1-\alpha)^3(-r_L)(-s - \beta_Sr_L)^2)\). Therefore, the higher the value of \(\alpha\) and/or \(r_L\), the lower S’s effort will be. \(R_1\) exerts an effort to avoid \(r_L\) (the \textit{ex ante} probability that the project is of low quality is equal to \(1-\alpha\)) and S therefore communicates to prevent \(R_1\) to some extent from accepting a low quality project.

Moreover, there is a downward discontinuity in both agents’ efforts between the \(CH\) and the \(CL\) equilibria when \(\alpha\) crosses \(\alpha^*_R\). This is the case if \(\beta_R \beta_S < 1\) and if S and/or R’s efforts are not maximal in the \(CL\) equilibrium when \(\alpha = \alpha^*_R\) (proof: see appendix C). This can be explained through S’s marginal revenue of communication. First, remember that \(\beta_S\) should be strictly higher than \(\frac{s}{r_L}\) for S to be willing to communicate. Second, notice that the higher the value of \(\beta_R\), the higher (lower) S’s effort will be in the \(CH\) (CL) equilibrium when \(\alpha = \alpha^*_R\).

### 3.2 Uncertainty

In this subsection, I consider that \(\beta_R = \beta_S = 0\) to investigate the implications of S’s uncertainty on R’s type.

I will show that there are five possible types of equilibrium.

\textbf{I)} In the CHH equilibrium, S, \(R_1\) and \(R_2\) communicate to accept a high quality project \((a^*_1 = a^*_2 = 0)\).

In this equilibrium, \(R_1\), \(R_2\) and S’s utilities are:

\[
U_{R_1} = e^*_S e^*_{R1} \alpha r_{H1} - CR(e^*_{R1}) \\
U_{R_2} = e^*_S e^*_{R2} \alpha r_{H2} - CR(e^*_{R2}) \\
U_S = e^*_S \left[\lambda e^*_{R1} + (1-\lambda)e^*_{R2}\right] \alpha s - CS(e^*_S)
\]
Therefore, $R_1$, $R_2$ and S’s optimal efforts are implicitly given by:

\[
\frac{\partial C_R}{\partial e_{R_1}}(e^{*H}_{R_1}) = e^{*H}_S \alpha r_{H1} \\
\frac{\partial C_R}{\partial e_{R_2}}(e^{*H}_{R_2}) = e^{*H}_S \alpha r_{H2} \\
\frac{\partial C_S}{\partial e_S}(e^{*H}_S) = [\lambda e^{*H}_{R_1} + (1 - \lambda)e^{*H}_{R_2}] \alpha s
\]

The agents’ optimal efforts depend on $r_{H1}$ and $r_{H2}$ and not on $r_L$. Note also that $R_1$ and $R_2$’s efforts are interdependent because of S’s effort choice.

II) In the CLH equilibrium, $R_1$ communicates to reject a low quality project and $R_2$ communicates to accept a high quality project ($a_{F_1} = 1$ and $a_{F_2} = 0$).

In this equilibrium, $R_1$, $R_2$ and S’s utilities are:

\[
U_{R_1} = \alpha r_{H1} + (1 - \alpha)(1 - e^{\lambda L} e^{*L}_{R_1})r_L - C_R(e^{*L}_{R_1}) \\
U_{R_2} = e^{\lambda L} e^{*H}_{R_2} \alpha r_{H2} - C_R(e^{*H}_{R_2}) \\
U_S = s \left[ \alpha \left( \lambda + (1 - \lambda)e^{\lambda L} e^{*H}_{R_2} \right) + (1 - \alpha)\lambda \left( 1 - e^{*L} e^{*L}_{R_1} \right) \right] - C_S(e^{*L}_S)
\]

Therefore, the agents’ optimal efforts are implicitly given by:

\[
\frac{\partial C_R}{\partial e_{R_1}}(e^{*L}_{R_1}) = e^{\lambda L}_S (1 - \alpha)(-r_L) \\
\frac{\partial C_R}{\partial e_{R_2}}(e^{*H}_{R_2}) = e^{\lambda L}_S \alpha r_{H2} \\
\frac{\partial C_S}{\partial e_S}(e^{*L}_S) = (1 - \lambda)e^{*L}_{R_2} \alpha s - \lambda e^{*L}_{R_1}(1 - \alpha) s
\]

Note that $R_1$’s optimal effort depends on $r_L$ and not on $r_{H1}$.

The uncertainty on the receiver’s communication objective explains why S communicates in this equilibrium. S is uncertain that he is matched with a receiver that would accept the project without communication. If S knew before communication that he is matched with $R_1$, S would not communicate.

No equilibrium in which $R_1$ and $R_2$ both communicate to reject a low quality project ($a_{F_1} = a_{F_2} = 1$) can exist. The reason is that if S knows with certainty that he is matched with a receiver who accepts the project when communication fails, S strictly prefers not to communicate.

III) In the HH equilibrium, S, $R_1$ and $R_2$ do not communicate ($e^{*}_S = e^{*}_{R_1} = e^{*}_{R_2} = 0$), and $R_1$ and $R_2$ always reject the project ($a_{F_1} = a_{F_2} = 0$).

In this equilibrium, $R_1$, $R_2$ and S’s utilities are zero.
IV) In the LH equilibrium, $S$, $R_1$ and $R_2$ do not communicate ($e^*_{S} = e^*_{R1} = e^*_{R2} = 0$), $R_1$ always accepts the project and $R_2$ always rejects the project ($a^F_1 = 1$ and $a^F_2 = 0$).

In this equilibrium, $R_1$, $R_2$ and $S$’s utilities are:

$$U_{R1} = \alpha r_{H1} + (1 - \alpha) r_L$$
$$U_{R2} = 0$$
$$U_S = \lambda s$$

V) In the LL equilibrium, $S$, $R_1$ and $R_2$ do not communicate ($e^*_{S} = e^*_{R1} = e^*_{R2} = 0$), and $R_1$ and $R_2$ always accept the project ($a^F_1 = a^F_2 = 1$).

In this equilibrium, $R_1$, $R_2$ and $S$’s utilities are:

$$U_{R1} = \alpha r_{H1} + (1 - \alpha) r_L$$
$$U_{R2} = \alpha r_{H2} + (1 - \alpha) r_L$$
$$U_S = s$$

A crucial threshold must be defined before developing the conditions of existence of the equilibria. Let $\alpha^*_S = \frac{\lambda e^*_L}{(1-\lambda)e^*_{H1} + \lambda e^*_{R2}}$. If $R_1$ exerts a strictly positive effort $e^*_{R1}$ with $a^F_1 = 1$, if $R_2$ exerts a strictly positive effort $e^*_{R2}$ with $a^F_2 = 0$, the threshold $\alpha^*_S$ represents the minimum parameter $\alpha$ above which $S$ strictly prefers to communicate.

**Proposition 2**

I) The CHH equilibrium exists provided that $\alpha \leq \alpha^*_R$.

II) The CLH equilibrium exists provided that $\max\{\alpha^*_R; \alpha^*_S\} \leq \alpha \leq \alpha^*_S$.

III) The HH equilibrium exists provided that $\alpha \leq \alpha^*_R$.

IV) The LH equilibrium exists provided that $\alpha^*_R \leq \alpha \leq \alpha^*_S$.

V) The LL equilibrium exists provided that $\alpha \geq \alpha^*_R$.

**Proof:** see appendix B.2.

I) The CHH equilibrium exists if and only if $R_1$ does not deviate from communicating to accept a high quality project ($a^F_1 = 0$) to communicating to reject a low quality project ($a^F_1 = 1$): $\alpha \leq \alpha^*_R$ (see Lemma 1). Since $\alpha^*_R$ is lower than $\alpha^*_S$, the maximum value of $\alpha$ under which $R_1$ does not deviate to $a^F_1 = 1$ is lower than the maximum value of $\alpha$ under which $R_2$ does not deviate to $a^F_2 = 1$. This explains why a CHL equilibrium, in which $R_1$ communicates to accept a high quality project and $R_2$ communicates to reject a low quality project, does not exist.

There are no other conditions of existence since $S$ ($R_1$ and $R_2$) always has (have) some interest in communicating if $R_1$ and $R_2$ ($S$) communicate(s). Remember that:

- $R_1/R_2/S$’s marginal cost of effort is zero if he does not communicate;
- there are no setup costs of communication; and that
- S’s (R1’s/R2’s) marginal revenue of communication is strictly positive if R1 and
R2 (S) communicate(s).

II) I will comment the only condition of existence of the CLH equilibrium that
is not explained by Lemma 1.

The condition \( \alpha^*_S \leq \alpha \) is necessary for S not to deviate to a zero effort strategy.

Remember first that S communicates in this equilibrium because he wants to
convince R2 that the project is of high quality \( (a^F_2 = 0) \) and not because he wants
to enable R1 to reject a low quality project \( (a^F_1 = 1) \).

In this equilibrium,
- if S is matched with R1 (with probability \( \lambda \)), if communication succeeds with R1
(with probability \( e^{LH}_S e^{LH}_R \)) and if R1’s revenue from accepting the project is negative
(with probability \( 1 - \alpha \)), then S does not earn revenue \( s \);
- if S is matched with R2 (with probability \( 1 - \lambda \)), if communication succeeds
with R2 (with probability \( e^{LH}_S e^{LH}_R \)) and if R2’s revenue from accepting the project
is positive (with probability \( \alpha \)), then S earns revenue \( s \).

Note that the higher \( \alpha \), the lower the probability that S does not earn revenue
\( s \) if S is matched with R1 and the higher the probability that S earns revenue \( s \) if S
is matched with R2.

S is thus prepared to communicate if in expectation, S’s gain from commun-
icating and being matched with R2 is higher than S’s loss from communicating and
being matched with R1.\(^8\)

To illustrate the effect of S’s uncertainty on his effort, Figure 3 compares S’s
equilibrium effort for every possible value of \( \alpha \) in a situation with uncertainty (\( \lambda =
0.25 \) and \( r_{H1} = 3 \), solid curve), to one without uncertainty (\( \lambda = 1 \) and \( r_{H1} = 1.5 \),
dashed curve).\(^9\) In both situations, I consider that \( C_R(e_R) \) is equal to \( \frac{(e_R)^2}{2} \) if \( e_R < 1 \)
and to \( \infty \) otherwise, \( C_S(e_S) \) is equal to \( \frac{(e_S)^3}{3} \) and to \( \infty \) otherwise, \( r_{H2} = 1, r_L = -2 \)
and \( s = 1.5 \). Therefore, the receiver’s revenue from accepting a high quality project
is in expectation the same in both situations.

\(^8\)If communication is *ceteris paribus* more likely to succeed when the project is of low quality
than when it is of high quality, it would decrease the agents’ efforts in both the CHH and CLH
equilibria and it would raise \( \alpha^*_S \).

\(^9\)In this graph, I disregard the no communication equilibrium when a Pareto-dominant equilib-
rium exists. The corresponding graph for R1 and R2’s efforts is presented in appendix D.2.
Figure 3: S's uncertainty and effort

Figure 3 shows that if S is certain that he is matched with $R_1$ ($\lambda = 1$), for the same reasons as explained for Figure 2, the CH equilibrium ($e^H_S = \alpha^2 s r_{H1}$) prevails in the NEG case ($\alpha \leq \alpha^*_{R1}(r_{H1} = 1.5)$) and there cannot be any communication in equilibrium in the POS case ($\alpha > \alpha^*_{R1}(r_{H1} = 1.5)$).

Let me explain S’s effort if S is uncertain that he is matched with $R_1$ ($\lambda = 0.25$).

If both $R_1$ and $R_2$ are in the NEG case ($\alpha \leq \alpha^*_{R1}(r_{H1} = 3)$), the CHH equilibrium prevails ($e^*_{H} = \alpha^2 s (\lambda r_{H1} + (1 - \lambda) r_{H2})$). $R_1$ and $R_2$ exert an effort to respectively get $r_{H1}$ and $r_{H2}$ and S therefore communicates to convince $R_1$ and $R_2$ that the project is of high quality.

If $R_1$ is the POS case and $R_2$ is in the NEG case ($\alpha^*_{R1}(r_{H1} = 3) \leq \alpha \leq \alpha^*_{R2}$), contrary to $R_2$, $R_1$ is ready to accept the project when communication fails. Therefore, if S could distinguish $R_1$ from $R_2$, S would be willing to communicate with $R_2$ but not with $R_1$. S wants to convince $R_2$ that the project is of high quality (with probability $\alpha$), but he does not want to prevent $R_2$ from accepting a low quality project (with probability $1 - \alpha$). The higher $\alpha$, the higher S’s benefit from communicating with $R_2$ ($(1 - \lambda) e_s e_{r_2} \alpha s$) and the lower S’s loss from communicating with $R_1(-\lambda e_s e_{r_2}(1 - \alpha) s)$.

Therefore, if $\alpha^*_{R1} \leq \alpha \leq \alpha^*_{S}$, S does not communicate because S’s benefit from communicating with $R_2$ is lower than his loss from communicating with $R_1$. If $\alpha^*_{S} \leq \alpha \leq \alpha^*_{R2}$, the CLH equilibrium prevails: S is willing to communicate and the higher the value of $\alpha$, the higher S’s effort will be ($e^*_{S} \leq \alpha^2 (1 - \lambda) r_{H2} s + (1 - \alpha)^2 \lambda r_{LS}$). If both $R_1$ and $R_2$ are in the POS case ($\alpha \geq \alpha^*_{R2}$), S is not willing to communicate because both $R_1$ and $R_2$ are ready to accept the project without any communication.
4 Conclusion

In a situation where a sender tries to convince a receiver to accept a project through truthful communication, I have shown that there are two communication objectives. If without any additional information the receiver would reject the project, his communication effort depends positively on his gain of accepting a high quality project and on the *ex ante* probability that the project is of high quality. He communicates to enhance his chance of accepting a high quality project and to avoid a type II error, a false negative.

If without any additional information the receiver would accept the project, his effort depends positively on the loss from accepting a low quality project and on the *ex ante* probability that the project is of low quality. He communicates to enhance his chance of rejecting a low quality project and to avoid a type I error, a false positive.

The next research step is to carry out an economic experiment to test these predictions, namely whether there are two communication objectives. A possible direction for future research is to extend the model to a buyers-sellers situation to give insights on the price and communication strategies in markets.\(^{10}\)

\(^{10}\)This extension could be built on the models of Johnson and Myatt (2006), Lewis and Sappington (1994), and Moscarini and Ottaviani (2001). For a review, see Bagwell (2007) or DellaVigna and Gentzkow (2010).
Appendixes

A Lemma

Whatever the communication efforts \((e_S \text{ and } e_{Ri})\) chosen in the stage 1, \(R_i\) strictly prefers playing \(a_i^F = 0\) \([a_i^F = 1]\) provided that:

\[
U_{Ri}(a_i^F = 0) > [<] U_{Ri}(a_i^F = 1) \Leftrightarrow e_S e_{Ri} \alpha (r_{Hi} + \beta_{Rs}) - C_R(e_{Ri})
\]

This also explains why \(R_i\) is indifferent between playing \(a_i^F = 0\) and \(a_i^F = 1\) when \(\alpha = \alpha_{Ri}^*\).

B Equilibria

B.1 Social preferences/S’s project quality incentives

I) The CH equilibrium exists provided that the following conditions hold:

1) \(R_1\) does not deviate to \(a_1^F = 1\) provided that \(\alpha \leq \alpha_{R1}^*\): see Lemma 1.

2) \(S\) does not deviate to \(e_S = 0\) provided that \(S\)’s marginal revenue of effort is strictly positive:

\[
e_{R1}^* \alpha (s + \beta_S r_{H1}) > 0 \Leftrightarrow \beta_S > -\frac{s}{r_{H1}}
\]

Recall that there are no communication setup costs, and that if \(e_S = 0\), \(S\)’s marginal cost of effort is null.

3) \(R_1\) does not deviate to \(e_{R1} = 0\) provided that \(R_1\)’s marginal revenue of effort is strictly positive:

\[
e_{R1}^* \alpha (r_{H1} + \beta_{Rs}) > 0 \Leftrightarrow \beta_R > -\frac{r_{H1}}{s}
\]

Recall that there are no communication setup costs, and that if \(e_{R1} = 0\), \(R_1\)’s marginal cost of effort is null.

II) The CL equilibrium exists provided that the following conditions hold:

1) \(R_1\) does not deviate to \(a_1^F = 0\) provided that \(\alpha \geq \alpha_{R1}^*\): see Lemma 1.

2) \(S\) does not deviate to \(e_S = 0\) provided that \(S\)’s marginal revenue of effort is
strictly positive:
\[ e^*_R (1 - \alpha)(-s - \beta s r_L) > 0 \iff \beta_S > \frac{s}{-r_L} \]

3) \( R_1 \) does not deviate to \( e^{R_1} = 0 \) provided that \( R \)’s marginal revenue of effort is strictly positive:
\[ e^*_S (1 - \alpha)(-r_L - \beta s) > 0 \iff \beta_R < \frac{-r_L}{s} \]

III) The H equilibrium exists provided that the following conditions hold:
1) \( R_1 \) does not deviate to \( a^{F_1} = 1 \) if \( \alpha \leq \alpha^*_R \): see Lemma 1;
2) \( R_1 \) (S) does not deviate to a strictly positive effort because communicating when S (\( R_1 \)) is not exerting any effort is useless. It decreases \( R_1 \)’s (S’s) payoff and it does not affect the other agent’s revenue.

IV) The L equilibrium exists provided that the following conditions hold:
1) \( R_1 \) does not deviate to \( a^{F_1} = 0 \) if \( \alpha \geq \alpha^*_R \): see Lemma 1;
2) S and \( R_1 \) do not deviate to a strictly positive effort (same reason as for the previous equilibrium).

B.2 Uncertainty

First of all, note that \( \alpha^*_R \leq \alpha^*_L \) since \( r_H > r_L \).

I) The CHH equilibrium exists provided that the following conditions hold:
1) \( R_1 \) (\( R_2 \)) does not deviate to \( a^{F_1} = 1 \) (\( a^{F_2} = 1 \)) provided that \( \alpha \leq \alpha^*_R \) (\( \alpha \leq \alpha^*_L \)):
   see Lemma 1.
2) \( R_1 \) (\( R_2 \) [S’s] never deviates to \( e^{R_1} = 0 \) (\( e^{R_2} = 0 \)) since \( R_1 \)’s (\( R_2 \)’s) [S’s] marginal revenue of communication is strictly positive: \( e^*_S(1 - \alpha)s r_H > 0 \) (\( e^*_L(1 - \alpha)s r_H > 0 \))
   \[ (\lambda e^*_R + (1 - \lambda)e^*_R) \alpha s > 0 \].

II) The CLH equilibrium exists provided that the following conditions hold:
1) \( R_1 \) does not deviate to \( a^{F_1} = 0 \) (\( a^{F_2} = 1 \)) provided that \( \alpha \geq \alpha^*_R \) (\( \alpha \leq \alpha^*_L \)):
   see Lemma 1.
2) \( R_1 \) (\( R_2 \)) never deviates to \( e^{R_1} = 0 \) (\( e^{R_2} = 0 \)) since \( R_1 \)’s (\( R_2 \)’s) marginal revenue of communication is strictly positive: \( -r_L e^*_L(1 - \alpha) > 0 \) (\( e^*_L \alpha r_H > 0 \)).
   Recall that there are no communication setup costs, and that if \( e^{R_1} = 0 \) (\( e^{R_2} = 0 \), \( R_1 \)’s (\( R_2 \)’s) marginal cost of effort is zero.
3) $S$ does not deviate to $e_S = 0$ provided that $S$’s marginal revenue is positive:

$$(1 - \lambda)e^{H\prime}_{R2} \alpha s - \lambda e^{L}_{R1}(1 - \alpha)s > 0 \Leftrightarrow \alpha \geq \alpha^*_S$$

Recall that there are no communication setup costs, and that if $e_S = 0$, $S$’s marginal cost of effort is null.

III) The HH equilibrium exists provided that the following conditions hold:

1) $R_1$ ($R_2$) does not deviate to $a^F_1 = 1$ ($a^F_2 = 1$) if $\alpha \leq \alpha^*_R$: see Lemma 1;

2) $R_1$ and/or $R_2$ ($S$) do(es) not deviate to a strictly positive effort because communicating when $S$ ($R_1$ and $R_2$) is (are) not exerting any effort is useless.

IV) The LH equilibrium exists provided that the following conditions hold:

1) $R_1$ ($R_2$) does not deviate to $a^F_1 = 0$ ($a^F_2 = 1$) if $\alpha \geq \alpha^*_R$: see Lemma 1;

2) $S$, $R_1$ and $R_2$ do not deviate to a strictly positive effort (same reason as for the previous equilibrium).

V) The LL equilibrium exists provided that the following conditions hold:

1) $R_1$ ($R_2$) does not deviate to $a^F_1 = 0$ ($a^F_2 = 0$) if $\alpha \geq \alpha^*_R$: see Lemma 1;

2) $S$, $R_1$ and $R_2$ do not deviate to a strictly positive effort (same reason as for the previous equilibrium).

B.3 Social preferences and uncertainty

I will show that there are six possible types of equilibrium.

I) In the CHH equilibrium, $S$, $R_1$ and $R_2$ communicate to accept a high quality project ($a^F_1 = a^F_2 = 0$).

In this equilibrium, $R_1$, $R_2$ and $S$’s utilities are:

$$U_{R_1} = e^{H}_{S} e^{H}_{R_1} \alpha (r_{H1} + \beta_{RS}) - C_{R}(e^{H}_{R1})$$
$$U_{R_2} = e^{H}_{S} e^{H}_{R_2} \alpha (r_{H2} + \beta_{RS}) - C_{R}(e^{H}_{R2})$$
$$U_{S} = e^{H}_{S} \alpha \left[ \lambda e^{H}_{R1}(s + \beta_{SR}r_{H1}) + (1 - \lambda)e^{H}_{R2}(s + \beta_{SR}r_{H2}) \right] - C_{S}(e^{H}_{S})$$
Therefore, \( R_1, R_2 \) and S’s optimal efforts are implicitly given by:

\[
\begin{align*}
\frac{\partial C_R}{\partial e_{R1}}(e_{R1}^*) &= e_{R1}^* \alpha(r_{H1} + \beta R S) \\
\frac{\partial C_R}{\partial e_{R2}}(e_{R2}^*) &= e_{R2}^* \alpha(r_{H2} + \beta R S) \\
\frac{\partial C_S}{\partial e_S}(e_{S}^*) &= \alpha \left[ \lambda e_{R1}^*(s + \beta S r_{H1}) + (1 - \lambda) e_{R2}^*(s + \beta S r_{H2}) \right]
\end{align*}
\]

II) In the CLH equilibrium, \( R_1 \) communicates to reject a low quality project and \( R_2 \) communicates to accept a high quality project (\( a_1^{F*} = 1 \) and \( a_2^{F*} = 0 \)).

In this equilibrium, \( R_1, R_2 \) and S’s utilities are:

\[
\begin{align*}
U_{R_1} &= \alpha(r_{H1} + \beta R S) + (1 - \alpha)(1 - e_{S}^{LL} e_{R1}^{sL})(r_{L} + \beta R S) - C_R(e_{R1}^*) \\
U_{R_2} &= e_{S}^{LL} e_{R2}^{sL} \alpha(r_{H2} + \beta R S) - C_R(e_{R2}^*) \\
U_S &= \alpha \left[ \lambda(s + \beta S r_{H1}) + (1 - \lambda)e_{S}^{LL} e_{R2}^{sL}(s + \beta S r_{H2}) \right] + \\
&\quad (1 - \alpha) \lambda \left( 1 - e_{S}^{LL} e_{R1}^{sL} \right) (s + \beta S r_{L}) - C_S(e_{S}^{LL})
\end{align*}
\]

Therefore, the agents’ optimal efforts are implicitly given by:

\[
\begin{align*}
\frac{\partial C_R}{\partial e_{R1}}(e_{R1}^*) &= e_{R1}^{sL}(1 - \alpha)(-r_{L} - \beta R S) \\
\frac{\partial C_R}{\partial e_{R2}}(e_{R2}^*) &= e_{R2}^{sL} \alpha(r_{H2} + \beta R S) \\
\frac{\partial C_S}{\partial e_S}(e_{S}^{sL}) &= (1 - \lambda) e_{R2}^{sH} \alpha(s + \beta S r_{H2}) - \lambda e_{R1}^*(1 - \alpha)(s + \beta S r_{L})
\end{align*}
\]

III) In the CLL equilibrium, S, \( R_1 \) and \( R_2 \) communicate to reject a low quality project (\( a_1^{F*} = a_2^{F*} = 1 \)).

In this equilibrium, \( R_1, R_2 \) and S’s utilities are:

\[
\begin{align*}
U_{R_1} &= \alpha(r_{H1} + \beta R S) + (1 - \alpha)(1 - e_{S}^{LL} e_{R1}^{sL})(r_{L} + \beta R S) - C_R(e_{R1}^*) \\
U_{R_2} &= \alpha(r_{H2} + \beta R S) + (1 - \alpha)(1 - e_{S}^{LL} e_{R2}^{sL})(r_{L} + \beta R S) - C_R(e_{R2}^*) \\
U_S &= \alpha \left[ s + \lambda \beta S r_{H1} + (1 - \lambda) \beta S r_{H2} \right] \\
&\quad + (1 - \alpha) \left[ 1 - \lambda e_{S}^{LL} e_{R1}^{sL} + (1 - \lambda) e_{S}^{LL} e_{R2}^{sL} \right] (s + \beta S r_{L}) - C_S(e_{S}^{LL})
\end{align*}
\]

Therefore, \( R_1, R_2 \) and S’s optimal efforts are implicitly given by:

\[
\begin{align*}
\frac{\partial C_R}{\partial e_{R1}}(e_{R1}^*) &= \frac{\partial C_R}{\partial e_{R2}}(e_{R2}^*) = e_{S}^{sL}(1 - \alpha)(-r_{L} - \beta R S) \\
\frac{\partial C_S}{\partial e_S}(e_{S}^{sL}) &= -e_{R1}^*(1 - \alpha)(s + \beta S r_{L})
\end{align*}
\]
IV) In the HH equilibrium, \( S, R_1 \) and \( R_2 \) do not communicate \( (e^*_S = e^*_R_1 = e^*_R_2 = 0) \), and \( R_1 \) and \( R_2 \) always reject the project \( (a^F_1 = a^F_2 = 0) \).

In this equilibrium, \( R_1, R_2 \) and \( S \)'s utilities are zero.

V) In the LH equilibrium, \( S, R_1 \) and \( R_2 \) do not communicate \( (e^*_S = e^*_R_1 = e^*_R_2 = 0) \). \( R_1 \) always accepts the project and \( R_2 \) always rejects the project \( (a^F_1 = 1 \text{ and } a^F_2 = 0) \).

In this equilibrium, \( R_1, R_2 \) and \( S \)'s utilities are:

\[
U_{R_1} = \alpha r_{H_1} + (1 - \alpha) r_L + \beta r_S
\]

\[
U_{R_2} = 0
\]

\[
U_S = \lambda [s + \beta_S (\alpha r_{H_1} + (1 - \alpha) r_L)]
\]

VI) In the LL equilibrium, \( S, R_1 \) and \( R_2 \) do not communicate \( (e^*_S = e^*_R_1 = e^*_R_2 = 0) \), and \( R_1 \) and \( R_2 \) always accept the project \( (a^F_1 = a^F_2 = 1) \).

In this equilibrium, \( R_1, R_2 \) and \( S \)'s utilities are:

\[
U_{R_1} = \alpha r_{H_1} + (1 - \alpha) r_L + \beta r_S
\]

\[
U_{R_2} = \alpha r_{H_2} + (1 - \alpha) r_L + \beta r_S
\]

\[
U_S = s + \beta_S [\alpha (\alpha r_{H_1} + (1 - \alpha) r_{H_2}) + (1 - \alpha) r_L]
\]

Let me at present prove the conditions of existence of these six possible types of equilibrium.

I) The CHH equilibrium exists provided that:

1) \( R_1 \) (\( R_2 \)) does not deviate to \( a^F_1 = 1 \) (\( a^F_2 = 1 \)) provided that \( \alpha \leq \alpha^*_R_1 \ (\alpha \leq \alpha^*_R_2) \); see Lemma 1.

2) \( R_1 \) (\( R_2 \)) \( [S \)'s] never deviates to \( e^*_R_1 = 0 \) (\( e^*_R_2 = 0 \)) \( [S \]'s] since \( R_1 \)'s (\( R_2 \)'s) \( [S \]'s] marginal revenue of communication is strictly positive: \( e^*_S H \alpha(r_{H_2} + \beta r_S) > 0 \) \( (e^*_S H \alpha(r_{H_2} + \beta r_S) > 0) \)

\( [\alpha (\lambda e^*_R_1 (s + \beta_S r_{H_1}) + (1 - \lambda) e^*_R_2 (s + \beta_S r_{H_2})) > 0] \).

A crucial threshold must be defined before developing the conditions of existence of the CLH equilibrium. Let \( \alpha^*_S = \frac{\lambda e^*_R_1 (s + \beta_S r_{H_1})}{(1 - \lambda) e^*_R_2 (s + \beta_S r_{H_2}) + \lambda e^*_R_1 (s + \beta_S r_{H_1})} \). If \( R_1 \) exerts a strictly positive effort \( e^F_{R_1} \) with \( a^F_1 = 1 \), if \( R_2 \) exerts a strictly positive effort \( e^F_{R_2} \) with \( a^F_2 = 0 \) and if \( beta_S \leq \frac{s}{\alpha^*_R} \), the threshold \( \alpha^*_S \) represents the minimum parameter \( \alpha \) above which \( S \) strictly prefers to communicate.

II) The CLH equilibrium exists provided that:

1) \( R_1 \) does not deviate to \( a^F_1 = 0 \) (\( a^F_2 = 1 \)) provided that \( \alpha \geq \alpha^*_R_1 \ (\alpha \leq \alpha^*_R_2) \); see Lemma 1.
2) $R_1$ ($R_2$) never deviates to $e_{R_1} = 0$ ($e_{R_2} = 0$) since $R_1$’s ($R_2$’s) marginal revenue of communication is strictly positive: $-r_L e_{RL}^{LH}(1 - \alpha) > 0$ ($e_{RL}^{LH} \alpha r_{R_2} > 0$).

3) $S$ does not deviate to $e_S = 0$ provided that $S$’s marginal revenue is positive:

$$e^{LH}_S (1 - \alpha) > 0$$

III) The CLL equilibrium exists provided that:

1) $R_1$ ($R_2$) does not deviate to $a_1^F = 0$ ($a_2^F = 0$) if $\alpha \geq \alpha_{R_2}^*$: see Lemma 1;

2) $R_1$ ($R_2$) never deviates to $e_{R_1} = 0$ ($e_{R_2} = 0$) since $R_1$’s ($R_2$’s) marginal revenue of communication is strictly positive: $e_{RL}^{LH}(1 - \alpha)(s + \beta_s r_{R_2}) > 0$

3) $S$ does not deviate to $e_S = 0$ provided that $S$’s marginal revenue of effort is strictly positive:

$$-e_{RL}^{LH}(1 - \alpha)(s + \beta_s r_L) > 0 \Leftrightarrow \beta_S > \frac{s}{r_L}$$

IV) The HH equilibrium exists provided that:

1) $R_1$ ($R_2$) does not deviate to $a_1^F = 1$ ($a_2^F = 1$) if $\alpha \leq \alpha_{R_2}^*$: see Lemma 1;

2) $R_1$ and/or $R_2$ ($S$) do(es) not deviate to a strictly positive effort because communicating when $S$ ($R_1$ and $R_2$) is (are) not exerting any effort is useless.

V) The LH equilibrium exists provided that:

1) $R_1$ ($R_2$) does not deviate to $a_1^F = 0$ ($a_2^F = 1$) if $\alpha \geq \alpha_{R_1}^*$ ($\alpha \leq \alpha_{R_2}^*$): see Lemma 1;

2) $S$, $R_1$ and $R_2$ do not deviate to a strictly positive effort (same reason as for the previous equilibrium).

VI) The LL equilibrium exists provided that:

1) $R_1$ ($R_2$) does not deviate to $a_1^F = 0$ ($a_2^F = 0$) if $\alpha \geq \alpha_{R_2}^*$: see Lemma 1;

2) $S$, $R_1$ and $R_2$ do not deviate to a strictly positive effort (same reason as for the previous equilibrium).
C Downward discontinuity in the agents’ efforts

When \( \alpha = \alpha^*_{R1} \), \( R_1 \)'s effort/marginal revenue of communication is strictly lower in the CL equilibrium than in the CH equilibrium provided that:

\[
\begin{align*}
    e^*_S (1 - \alpha^*_{R1})(-r_L - \beta_R s) &< e^*_S \alpha^*_{R1} (r_H + \beta_R s) \\
\Leftrightarrow e^*_S \left(\frac{r_H + \beta_R s}{r_H - r_L}\right)(-r_L - \beta_R s) &< e^*_S \left(\frac{-r_L - \beta_R s}{r_H - r_L}\right)(r_H + \beta_R s) \\
\Leftrightarrow e^*_S &< e^*_S
\end{align*}
\]

Therefore, when \( \alpha = \alpha^*_{R1} \), \( R_1 \)'s effort is also the same (strictly higher) in the CL equilibrium as (than) in the CH equilibrium provided that \( e^*_S = e^*_S \) \( (e^*_S > e^*_S) \).

When \( \alpha = \alpha^*_{R1} \) and when \( e^*_L = e^*_H \), S’s effort/marginal revenue of communication is strictly lower in the CL equilibrium than in the CH equilibrium provided that:

\[
\begin{align*}
    e^*_L (1 - \alpha^*_{R1})(-s - \beta_S r_L) &< e^*_H \alpha^*_{R1} (s + \beta_S r_H) \\
\Leftrightarrow -r_H - \beta_S r_L \beta_R &< -r_L - \beta_R \beta_S r_H \Leftrightarrow \beta_S \beta_R < 1
\end{align*}
\]

Therefore, when \( \alpha = \alpha^*_{R1} \), both agents’ efforts are strictly lower in the CL equilibrium than in the CH equilibrium if \( \beta_S \beta_R < 1 \).

D R’s effort graphs

D.1 Social preferences/project quality incentives

![Diagram showing R's effort graphs with case NEG and case POS](image)

S’s social preferences/project quality incentives and \( R_1 \)'s effort
D.2 Uncertainty

S's uncertainty, and R1 and R2's efforts

References


