Unions' Relative Concerns and Strikes in Wage Bargaining

Ana MAULEON
Vincent VANNETELBOSCH
Cecilia VERGARI
Abstract

We consider a model of wage determination with private information in a duopoly. We investigate the effects of unions having relative concerns on the negotiated wage and the strike activity. We show that an increase of unions' relative concerns has an ambiguous effect on the strike activity.

Keywords: relative position, wage bargaining, private information, strike activity.

JEL Classification: C70, J50, D60
1 Introduction

Clearly, it is likely that when unions bargain with firms over wages they are also influenced by relative wage considerations.\(^1\) Brown, Gardner, Oswald and Qian (2008), using data collected from 16,000 British workers, have found evidence that the welfare of a worker is not solely determined by his or her material circumstances but also depends on his or her relative wage and the rank-ordered position of his or her wage within a comparison set.\(^2\) The purpose of this note is to provide a theoretical study of how relative concerns will affect the outcome of wage negotiations in presence of private information in a duopoly. To describe the wage bargaining process, we adopt Rubinstein’s (1982) alternating-offer bargaining model with two-sided incomplete information, which allows the occurrence of strikes at equilibrium. An increase in unions’ relative concerns has a twofold effect on the strike activity. On one hand, it raises the potential payoffs for the union and the firm, and hence longer strikes or lockouts may be needed for screening the private information. On the other hand, each union is more inclined to concede and to accept rapidly a smaller wage increase than before since the smaller increase in wage is compensated by the increased utility due to more pride or less envy. Depending on which effect dominates, an increase in unions’ relative concerns will either raise or reduce the strike activity.

The note is organized as follows. In Section 2 the model under complete information is presented. Section 3 is devoted to the case with private information. Section 4 concludes.

2 Model

We consider a duopolistic industry for a single homogenous product, where the demand is linear and is given by \( p = a - q \), \( p \) is the market price, and \( q \) is the aggregate quantity demanded. Let \( q_i \) denote the quantity produced by firm \( i \), and let \( \Pi_i \) denote the profit of firm \( i \). There is no entry or threat of entry, and both firms are quantity setters (Cournot competition). Production technology exhibits constant returns to scale with labor as the sole input and is normalized in such a way that \( q_i = l_i \), where \( l_i \) is the labor input. The total labor cost to firm \( i \) of

\(^1\)Hopkins (2008) has provided a survey of different theoretical models of relative concerns and their relation to inequality. See also Sobel (2005).

\(^2\)Clark and Oswald (1996), using data on 5,000 British workers, have found evidence that workers’ reported satisfaction levels are inversely related to their comparison wage rates.
producing quantity $q_i$ is $q_i w_i$, where $w_i$ is the wage in firm $i$. Firm $i$’s profit is given by $\Pi_i = (p - w_i)q_i$. Firm $i$ is unionized, and enters into a closed-shop agreement with union $i$.

The objective of union $i$ is to maximize the following utility function:

$$U_i(w_i, w_j) = w_i - \gamma (w_j - w_i),$$

where $1 > \gamma \geq 0; i \neq j$. In this utility function, $\gamma$ captures the loss from disadvantageous inequality (envy) if $w_j > w_i$, or the win from advantageous inequality (pride) if $w_i > w_j$. This utility function is a special case of Fehr and Schmidt’s (1999) model of inequality aversion: $U_i(w_i, w_j) = w_i - \gamma (w_j - w_i)$ if $w_j > w_i$ and $U_i(w_i, w_j) = w_i - \beta (w_i - w_j)$ if $w_i > w_j$. The parameter $\gamma$ captures the envy or the dislike of others having higher wages. If the parameter $\beta$ is positive, then it captures the compassion, that is, low wages for others reduce one’s own utility. This is the assumption originally made by Fehr and Schmidt (1999). But if, contrary to Fehr and Schmidt’s assumptions, the parameter $\beta$ were negative, then it captures the pride as then lower wages for others raise an union’s utility. So, the utility function given in (1) reverts to assume that $\beta$ is negative and equal to $-\gamma$ in Fehr and Schmidt’s (1999) model. That is, we assume that pride is as strong as envy and there is no compassion. If $\beta = \gamma = 0$ the utility function given in (1) reduces to the standard utility function where the union maximizes the wage rate.

The union’s utility function given in (1) implies that the union places no value on employment. Although this may seem implausible, the notion that, in negotiating wages, unions do not take into account the employment consequences of higher wages has a long tradition, and is often stated by union leaders (Mauleon and Vannetelbosch, 2005). This assumption is made to obtain closed-form solutions in order to carry out the analysis under incomplete information. Cramton and Tracy (2003) have concluded that disputes are largely motivated by the presence of private

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3De Bruyn and Bolton (2008) have investigated whether fairness considerations are stable across bargaining situations to be quantified and used to forecast bargaining behavior accurately. Using data from experiments on two-person bargaining, they have found that the positive reciprocity assumption of the Fehr-Schmidt model ($\beta > 0$) is violated.

4Brown, Gardner, Oswald and Qian (2008) have studied how British workers do make wage comparisons. They have regressed employee satisfaction against both the average wage in the firm and the employee’s rank in wages. Rank is found to be more important in predicting satisfaction. This is more supportive of rivalrous preferences than of inequity aversion. So, workers seem to feel envy or pride rather than envy or compassion when making wage comparisons.

5Mauleon and Vannetelbosch (2005) have shown that, if the union is not too powerful, it is optimal for the union that seeks to maximize the rents to send to the negotiating table delegates who seeks to maximize the wage.
information and the sharply conflicting interests of the union and the firm over the wage.

Interactions between market competition and wage bargaining are analyzed according to a two-stage game. In stage one, wages are negotiated at the firm-level in both firms. In stage two, each firm chooses its output (and hence employment) levels, taking as given both (i) the output decisions of the other firm and (ii) the negotiated wages. The model is solved backwards. In the last stage of the game, the wage levels have already been determined. Both firms compete by choosing their outputs simultaneously to maximize profits, with the price adjusting to clear the market. The unique Nash equilibrium of this stage game yields

\[
q_i(w_i, w_j) = \frac{a - 2w_i + w_j}{3}; \quad \Pi_i(w_i, w_j) = \left(\frac{a - 2w_i + w_j}{3}\right)^2;
\]

for \(i, j = 1, 2, i \neq j\). The Nash equilibrium outputs of a firm (and hence, the equilibrium level of employment) are decreasing with its own wage, but are increasing with the other firm’s wage and total industry demand.

The negotiations occur simultaneously in both firms and the agents are unaware of any proposals made (or settlement reached) in related negotiations. Production and market competition occur only when either both firms have come to an agreement with their workers, or when one firm has settled with its union and the other union has decided to leave the negotiation forever. Hence, each union-firm pair takes the decisions of the other pair as given while conducting its own negotiation.

Each negotiation proceeds as in Rubinstein’s (1982) alternating-offer bargaining model. The union and the firm make alternate wage offers, with the firm making offers in odd-numbered periods and the union making offers in even-numbered periods. The length of each period is \(\Delta\). The negotiation starts in period 0 and ends when one of the negotiators accepts an offer. No limit is placed on the time that may be expended in bargaining and perpetual disagreement is a possible outcome. The union is assumed to be on strike in every period until an agreement is reached. The union and the firm have time preferences with constant discount rates \(r_u > 0\) and \(r_f > 0\), respectively. To capture the notion that the time it takes to come to terms is small relative to the length of the contract, we assume that the time between periods is very small. As the interval between offers and counteroffers shortens and shrinks to zero, the alternating-offer model has a unique limiting subgame perfect equilibrium (SPE), which approximates the Nash bargaining solution to the bargaining problem (see Binmore, Rubinstein and Wolinsky, 1986). Thus the predicted wages are given by \(w_i^* = \text{arg max} \{\alpha \cdot \log U_i - (1 - \alpha) \cdot \log \Pi_i\}\), where \(\alpha \in (0, 1)\) is the union bargaining power which is equal to \(r_f/(r_u + r_f)\) and the status quo payoffs
are zero. Then, the equilibrium wages \((w^*_i)\), outputs \((q^*_i)\), and consumer surplus \((CS^*)\) are

\[
w^*_i = \frac{\alpha (1 + \gamma) a}{4 - \alpha (3 - \gamma)} ; \quad q^*_i = \frac{4(1 - \alpha) a}{3 (4 - \alpha (3 - \gamma))} ; \quad CS^* = \frac{32(1 - \alpha)^2 a^2}{9 (4 - \alpha (3 - \gamma))^2}.
\]

Of course, we have that \(\partial w^*_i / \partial \alpha > 0\) (\(\partial U^*_i / \partial \alpha > 0\)) and \(\partial \Pi^*_i / \partial \alpha < 0\). More interestingly, we find that \(\partial w^*_i / \partial \gamma > 0\) (\(\partial U^*_i / \partial \gamma > 0\)), \(\partial q^*_i / \partial \gamma < 0\), \(\partial \Pi^*_i / \partial \gamma < 0\), and \(\partial CS^* / \partial \gamma < 0\).

**Proposition 1.** An increase of unions’ relative concerns increases wages but decreases outputs, profits and consumer surplus.

Take as given the wage negotiated in the other firm. As \(\gamma\) increases, each union becomes less inclined to accept lower wages because of having to suffer from more envy. In addition, each union is now more persistent to obtain higher wages because of getting more pride. Hence, the more the union cares about relative concerns the higher the negotiated wages and the lower the profits of the firm.

3 Maximum delay in reaching an agreement

Both the asymmetric Nash bargaining solution and Rubinstein’s model predict efficient outcomes of the bargaining process. In particular, agreement is reached immediately. This is not true if we introduce incomplete information into the bargaining. In this case, the early rounds of negotiation are used for information transmission between the two negotiators. We now suppose that negotiators have private information. Neither negotiator knows the impatience (or discount rate) of the other party. It is common knowledge that the firm’s discount rate is included in the set \([r^P_f, r^P_f]\), where \(0 < r^P_f \leq r^F_f\), and that the union’s discount rate is included in the set \([r^P_u, r^P_u]\), where \(0 < r^P_u \leq r^U_u\). The superscripts ”\(\Gamma\)” and ”\(P\)” identify the most impatient and most patient types, respectively. The types are independently drawn from the set \([r^P_i, r^U_i]\) according to the probability distribution \(p_i\), for \(i = u, f\). This uncertainty implies bounds on the union’s bargaining power which are denoted by \(\bar{\alpha} = r^P_f / (r^U_f + r^P_f)\) and \(\bar{\alpha} = r^P_u / (r^U_u + r^P_u)\). The wage bargaining game may involve delay (strikes or lockouts), but not perpetual disagreement, in equilibrium.\(^{7}\) In fact, delay is positively related to the distance between the discount rates of the most and

\(^{6}\)These relationships hold under an alternative specification where unions maximize the surplus and have relative concerns: \(U_i(w_i, l_i, w_j, l_j) = w_i l_i - \gamma(w_j l_j - w_i l_i)\).

\(^{7}\)Watson (1998) has characterized the set of perfect Bayesian equilibrium (PBE) payoffs which may arise in Rubinstein’s alternating-offer bargaining game and constructed bounds (which are

4
least patient types of the players. If the range of types is reduced, then this leads to a smaller range of possible payoffs and less delay. Delay can occur even when the game is close to one of complete information (as the type distributions converge to point mass distributions).

We propose to identify strike activity with the maximum delay time in reaching an agreement. Only on average is this measure a good proxy for actual strike duration.\textsuperscript{8} In the appendix we compute the maximum delay in equilibrium which shows that an agreement is reached in finite time and that delay time equals zero as incomplete information vanishes (in that \( r^p_t \) and \( r^l_t \) converge). The maximum real delay time in reaching an agreement is given by \( D(\gamma) = \min \{ D^u(\gamma), D^f(\gamma) \} \) where

\[
D^u(\gamma) = -\frac{1}{r^u_f} \cdot \log \left[ \frac{r^p_f}{r^l_f} \cdot \frac{(1 + \gamma)r^l_f + 4r^p_u}{(1 + \gamma)r^p_f + 4r^l_u} \right] \quad (2)
\]
is the maximum real time the union would spend negotiating, and

\[
D^f(\gamma) = -\frac{1}{r^f_u} \cdot \log \left[ \left( \frac{r^p_u}{r^l_u} \right)^2 \cdot \left( \frac{(1 + \gamma)r^l_u + 4r^p_f}{(1 + \gamma)r^p_u + 4r^l_f} \right)^2 \right] \quad (3)
\]
is the maximum real time the firm would spend negotiating. In fact, \( D^u(\gamma) \) is the maximum real time the union would spend negotiating if it were of the most patient type. Similarly, \( D^f(\gamma) \) is the maximum real time the firm would spend negotiating if it were of the most patient type. So, \( D^u(T) \) and \( D^f(T) \) are the upper bounds on the maximum time the union of type \( r_u \) and the firm of type \( r_f \) would spend negotiating. Since \( D^u(T) \) and \( D^f(T) \) are positive, finite numbers, the maximum real delay in reaching an agreement is finite and converges to zero as \( r^l_t \) and \( r^p_t \) become close. We have that \( \partial D^u(\gamma)/\partial \gamma < 0 \) and \( \partial D^f(\gamma)/\partial \gamma > 0 \).

met) on the agreements that may be made. The bounds and the PBE payoffs set are determined by the range of incomplete information and are easy to compute because they correspond to the SPE payoffs of two bargaining games with complete information. These two games are defined by matching one player’s most impatient type with the opponent’s most patient type. In addition, Watson (1998) has constructed equilibria with delay in which the types of each player behave identically (no information is revealed in equilibrium), players use pure strategies, and players make non-serious offers until some appointed date.

\textsuperscript{8}It is not uncommon in the literature on bargaining to analyze the maximum delay before reaching an agreement. See, for instance, Cramton (1992) and Cai (2003). In the literature on strikes, three different measures of strike activity are usually proposed: the strike incidence, the strike duration, and the number of work days lost due to work stoppages. Since we allow for general distributions over types and we may have a multiplicity of PBE, we are unable to compute these measures of strike activity.
Proposition 2. An increase of unions’ relative concerns decreases the maximum real time the union would spend negotiating but increases the maximum real time the firm would spend negotiating.

The intuition behind this proposition is as follows. An increase of unions’ relative concerns (γ increases) raises the potential payoffs for the union and the firm, and in expanding the payoff set (or range of possible payoffs), also increases the scope for delay (longer strikes or lockouts may be needed for screening the private information). Hence, ∂D(u)(γ)/∂γ > 0. However, for the union, there is a second effect at play. When γ increases, taking as given the wage agreement in the other negotiation, each union is more inclined to concede and to accept rapidly a smaller wage increase than before since the smaller increase in wage is compensated by the increased utility due to more pride or less envy. This second effect dominates the first one. Hence, ∂D(u)(γ)/∂γ < 0.

We now provide an example of the maximum delay. In this example, let r^p_i = r^p_i (1), r^l_j = r^l_j (1), r^l = 0.36 - r^p with r^p ∈ [0.04, 0.18]. Table 1 gives the integer part of the maximum delay for the different values of the parameter γ. We observe that (i) the real delay time in reaching an agreement is not negligible: many bargaining rounds may be needed in equilibrium before an agreement is reached; (ii) D^u and D^l are increasing with the amount of private information |r^p_i - r^l_j|; (iii) the maximum delay D(γ) is increasing more with γ when γ is small, and may decrease when γ and the amount of private information |r^p_i - r^l_j| are large. For instance, take r^p = 0.05. Then, we observe that D(0) = 35; D(1/4) = 41; D(1/2) = 45; D(3/4) = 48; and D(1) = 46. Results (i) and (ii) hold in general.

4 Conclusion

We have considered a model of wage determination with private information in a duopoly. We have investigated the effects of unions having relative concerns on the negotiated wage and the strike activity. We have shown that an increase of unions’ relative concerns has an ambiguous effect on the strike activity. We have assumed that firms were competing à la Cournot and were producing homogeneous goods. Product differentiation does not affect qualitatively our results about the effect of unions’ relative concerns on wage negotiations. Mauleon and Vannetelbosch

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9We can interpret r_i as the annual discount rate and the numbers in Table 1 as the maximum number of days needed to reach an agreement. Indeed, the integer part of the maximum delays for Δ = 1/365 are exactly the numbers in Table 1. The data in Table 1 seem consistent with U.S. strike durations as reported in Cramton and Tracy (1994).
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Table 1: Maximum delay in reaching an agreement

(2003) have shown that, when unions maximize rents, wages and strikes are increasing with the degree of product differentiation, and the strike activity is smaller under Bertrand than under Cournot competition. However, an increase in market competition does not always reduce the strike activity. For instance, Mauleon and Vannetelbosch (2010) have shown that, from an initial situation of two-way intra-industry trade, an increase in product market integration decreases the strike activity. But, opening up markets to trade has an ambiguous effect on the wage and the strike activity.

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Appendix

A Maximum delay

The negotiation goes as in Rubinstein’s (1982) alternating-offer bargaining model. The firm and the union have time preferences with constant discount factors $\delta_f \in (0, 1)$ and $\delta_u \in (0, 1)$, respectively. It is assumed that each union-firm pair takes the other wage settlement as given during the negotiation. For any wage bargaining which leads to an agreement $w_i$ at period $n$, $\delta_f^n \Pi_i(w_i, l_i(w_i, w_j))$ and $\delta_u^n U_i(w_i, w_j)$ are, respectively, firm $i$’s payoff and union $i$’s payoff. For any wage bargaining which leads to perpetual disagreement, disagreement payoffs are set to zero. As in Binmore, Rubinstein and Wolinsky (1986), the SPE wage outcome is such that $\Pi_i(w_{iu}, l_i(w_{iu}, w_j)) = \delta_f \Pi_i(w_{if}, l_i(w_{if}, w_j))$ and $U_i(w_{if}, w_j) = \delta_u U_i(w_{iu}, w_j)$, where $w_{iu}$ is the SPE wage outcome if the union makes the first wage offer, and $w_{if}$ is the SPE wage outcome if the firm makes the first offer. Since the union makes the first offer, the unique symmetric SPE wages are given by

$$w_i^*(\delta_u, \delta_f) = \frac{(1 + \gamma)(1 - \sqrt{\delta_f})a}{2(1 - \sqrt{\delta_f \delta_u}) - (1 - \gamma)(1 - \sqrt{\delta_f})}, \ i = 1, 2,$$

which is also the SPE unions payoffs, $U_i^*(\delta_u, \delta_f)$, and from which we get the SPE profits,

$$\Pi_i^*(\delta_u, \delta_f) = \frac{4\delta_f (1 - \delta_u)^2 a^2}{9 [2(1 - \sqrt{\delta_f \delta_u}) - (1 - \gamma)(1 - \sqrt{\delta_f})]^2}, \ i = 1, 2.$$

Suppose now that the players have private information. They are uncertain about each others’ discount factors. Player $i$’s discount factor is included in the set $[\delta^l_i, \delta^p_i]$, where $0 < \delta^l_i \leq \delta^p_i < 1$. Since we allow for general probability distributions over discount factors, multiplicity of PBE is not an exception. From Watson (1998), we have that for any PBE, the payoff of the union belongs to $[U^*_i(\delta^l_u, \delta^p_f), U^*_i(\delta^p_u, \delta^l_f)]$ and the payoff of the firm belongs to $[\Pi^*_i(\delta^l_u, \delta^p_f), \Pi^*_i(\delta^p_u, \delta^l_f)]$. The maximum number of bargaining periods the union would spend negotiating, $I(m^\alpha(\gamma))$, is given by $U^*_i(\delta^l_u, \delta^p_f) = (\delta^l_u)^{m^\alpha(\gamma)} U^*_i(\delta^p_u, \delta^l_f)$, from which we obtain $m^\alpha(\gamma) = (\log(\delta^l_u))^{-1} \log [U^*_i(\delta^l_u, \delta^p_f)/U^*_i(\delta^p_u, \delta^l_f)]$. Notice that $I(m^\alpha(\gamma))$ is simply the integer part of $m^\alpha(\gamma)$. It is customary to express the players’ discount factors in terms of discount rates, $r_u$ and $r_f$, and the length of the bargaining period, $\Delta$, according to the formula $\delta_i = \exp(-r_i\Delta)$. With this interpretation, player $i$’s type is identified with the discount rate $r_i$, where $r_i \in [r^p_i, r^l_i]$. We thus have that
\(\delta_t^i = \exp(-r_i^1 \Delta)\) and \(\delta_t^p = \exp(-r_i^p \Delta)\). Note that \(r_i^1 \geq r_i^p\) since greater patience implies a lower discount rate. As \(\Delta\) approaches zero, using l’Hopital’s rule we obtain that

\[
D^u(\gamma) = \lim_{\Delta \to 0} (m^u(\gamma) \cdot \Delta) = -\frac{1}{r_i^p} \cdot \log \left[ \frac{r_i^p}{r_i^f} \cdot \frac{(1 + \gamma)r_i^1 + 4r_i^p}{(1 + \gamma)r_i^p + 4r_i^u} \right],
\]

which is a positive, finite number. Notice that \(D^u(\gamma)\) converges to zero as \(r_i^p\) and \(r_i^1\) become close. We have

\[
\frac{\partial D^u(\gamma)}{\partial \gamma} = \frac{-4(r_i^1 r_i^p - r_i^f r_i^p)}{(1 + \gamma)r_i^p + 4r_i^u} \left( \frac{1 + \gamma}{1 + \gamma} \right) \frac{r_i^p}{r_i^f} > 0.
\]

The maximum number of bargaining periods the firm would spend negotiating, \(I(m^f(\gamma))\), is given by \(\Pi_i^\ast(\delta_i^u, \delta_i^f) = (\delta_i^p)^{m^f(\gamma)} \cdot \Pi_i^\ast(\delta_i^u, \delta_i^f)\), from which we obtain

\[
I(m^f(\gamma)) = (\log(\delta_i^p))^{-1} \log \left[ \frac{\Pi_i^\ast(\delta_i^u, \delta_i^f)}{\Pi_i^\ast(\delta_i^u, \delta_i^f)} \right],
\]

and as \(\Delta\) approaches zero,

\[
I_f(\gamma) = \lim_{\Delta \to 0} (m^f(\gamma) \cdot \Delta) = -\frac{1}{r_i^p} \cdot \log \left[ \frac{r_i^p}{r_i^f} \cdot \frac{1 + \gamma}{1 + \gamma} \left( \frac{1 + \gamma}{1 + \gamma} \right) \right],
\]

which is a positive, finite number. We have

\[
\frac{\partial I_f(\gamma)}{\partial \gamma} = \frac{8(r_i^1 r_i^p - r_i^f r_i^p)}{(1 + \gamma)r_i^p + 4r_i^u} \left( \frac{1 + \gamma}{1 + \gamma} \right) \frac{r_i^p}{r_i^f} > 0.
\]

The maximum real delay time before reaching an agreement is given by

\[
D(\gamma) = \min \{ D^u(\gamma), D^f(\gamma) \}.
\]

References


