



INTERNATIONAL ASSOCIATION FOR RESEARCH AND TEACHING
Economics, Finance, Operations Research, Econometrics and Statistics

++ research ++ teaching ++

ECORE DISCUSSION PAPER

2010/10

Market Coverage and the Nature of Product Differentiation: A Note

Xavier WAUTHY

CORE DISCUSSION PAPER
2009/81

**Market coverage and the nature
of product differentiation: a note**

Xavier WAUTHY¹

December 2009

Abstract

In this note, we analyze the equilibrium outcomes of pricing games with product differentiation in relation with the extent of market coverage. It is a received idea in the IO literature that the horizontal and vertical models of product differentiation are almost formally equivalent. We show that this idea turns out to be wrong when the full market coverage assumption is relaxed. We then argue that there exist two fundamentally different classes of address-models of differentiation, although their difference is not perfectly captured by the standard horizontal/vertical typology.

Keywords: price competition, product differentiation.

JEL Classification: L13

¹ Facultés universitaires Saint-Louis, CEREC, Bruxelles, Belgium and Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: xwauthy@fusl.ac.be

I am grateful to Jean Gabszewicz for his constructive comments. This research has been conducted with the financial support of BELSPO IAP research program under contract IAP 06/09. I retain responsibility for any remaining errors.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.

1 Introduction

In this note, we propose to adapt the traditional distinction between horizontal and vertical differentiation to build an alternative typology that better reflects the nature of strategic incentives that firms face when choosing their products' characteristics. It has been recognized for long that the introduction of product differentiation is instrumental in escaping from the Bertrand paradox. Firms are naturally inclined to differentiate their products in order to increase their market power, in particular under price competition. Address-models of product differentiation, as inspired by Hotelling (1929), are particularly suggestive. They make it explicit that, in order to relax price competition, firms actually need a two-dimensional heterogeneity: products have to differ in at least one characteristic, but the population of consumers must exhibit heterogeneous tastes as well. The relationship between these two dimensions of heterogeneity is crucial in characterizing the nature of product differentiation.

It is commonplace nowadays to distinguish between models of horizontal differentiation and models of vertical differentiation. However, the extent to which this distinction really matters for equilibrium outcomes is not clear. Several recent papers suggest that the horizontal and vertical approaches of differentiation are to a large extent equivalent. In particular, Cremer and Thisse (1991) show that "*every model belonging to a very large class of Hotelling-type models (including all the commonly used specifications) is actually a special case of a vertical differentiation model.*" This claim seems to be confirmed in Irmen and Thisse (1998): their analysis of products' characteristics choices in a multidimensional setting suggests that the difference between horizontal and vertical characteristics does not really matter.¹ As a matter of fact, IO textbooks also seem to have chosen their side. They focus indeed much more on similarities than on differences. For instance Tirole (1988) starts the analysis of vertical differentiation by claiming that "*The study of vertical differentiation so closely resembles that of horizontal differentiation...*". Shy (1996), in his chapter 12, builds the analysis of his vertical differentiation model as a particular case of the Hotelling model. Martin (2000) also emphasizes similarities between the two models. In a very recent contribution, Schmidt (2009) endorses a comparable point of view: building on a theoretical set-up which is supposed to represent either vertical or horizontal differentiation depending on the interpretation given to the model's variables, he establishes some policy recommendations which are invariant to the nature of differentiation.

Curiously enough then, vertical differentiation is also known to be a necessary condition for the finiteness property to hold. Generalizing the early arguments of Gabszewicz and Thisse (1979), Shaked and Sutton (1983) show that under vertical differentiation there exists an upper bound to the number of firms which may co-exist in the market in the long run, even when entry cost is arbitrarily small. This property is known as the finiteness property. Under this property,

¹The reader is referred to Cremer and Thisse (1991) and Irmen and Thisse (1998) for additional references on the distinction between horizontal and vertical differentiation.

we may expect natural oligopolies to prevail, though exclusively under vertical differentiation. By contrast, the number of firms which may co-exist under horizontal differentiation tends to infinity when entry cost tends to zero. In view of the preceding statements, this last result is surprising: if vertical and horizontal models of product differentiation are essentially two faces of the same coin, why is it that, in the long run, they possibly lead to radically different equilibrium market structures?

In this note, we argue that the similarities between the vertical and the horizontal approach, as emphasized for instance by Cremer and Thisse (1991), Schmidt (2009) and in most IO textbooks, are misleading. Those similarities are indeed formally established by relying on a two-stage model where firms choose first products' characteristics and then compete in prices. The model is solved under the key assumption that the market is fully covered, i.e. consumers never refrain from buying, whatever the choice of products' characteristics. As generally argued in the literature, this assumption greatly simplifies the exposition. Unfortunately it also hides a key difference between horizontal and vertical differentiation. Actually, the extent of market coverage, i.e. whether full coverage prevails or not in equilibrium, should be endogenous to any two-stage model of differentiation. Once partial market coverage is considered, a key difference emerges between the prototype models of differentiation. In one class of models, akin to the concept of variety differentiation, firms may actually get rid of price competition through product differentiation whereas this is never possible under the other class, akin to the concept of quality differentiation.

2 An Example

Schmidt (2009) considers a population of consumers indexed by their type x . Types are uniformly distributed in the $[0, 1]$ interval with a density equal to 1. Each type x is characterized by an indirect utility function $W(x, q, p) = u(x, q) - p$. He consumes at most one unit of product with attribute $q \geq 0$.

Definition 1 *A market is said to satisfy full coverage if and only if, at prevailing prices all types x buy one product.*

There are $n \geq 2$ products with attribute $q_i \geq 0$, $i = 1 \dots n$. We assume that $q_i \geq q_j$ if and only if $i \geq j$.

Let us then characterize the type of differentiation relying on the definition proposed in Schmidt (2009):

Definition 2 *A vertically (horizontally) differentiated market is a market where consumers have an identical (differing) preference ordering over the feasible product attributes.*

There exist different versions of the definition in the literature but the present one is quite representative. In particular, it is perfectly in line with the more restrictive definition put forward

in Gabszewicz and Thisse (1986) or Tirole (1988) according to which vertical differentiation prevails when consumers agree on the ordering of products when they are sold at the same price. This definition also captures the intuitive parallel which is made between vertical and quality differentiation on the one hand and horizontal and variety differentiation on the other hand.

As recalled in Schmidt (2009), it is often convenient to rely on the following complementary definition

Definition 3 *The preferred product version of type x , $\tilde{q}(x)$, is the version that yields the highest surplus to type x , when all feasible versions are offered at their marginal cost $c(q)$.*

Relying on this second definition, it is clear that in a model where differentiation relies on quality levels, the preferred product version should be the same for all consumers if quality is not costly, i.e. if $c(q) = c \geq 0$. By contrast, we expect this preferred product to be specific to each x under variety differentiation. Notice that this second definition is also instrumental in checking whether the finiteness property holds or not. To put it simply,² the finiteness condition holds whenever the preferred product is the same for all consumers in the market. This condition is more demanding than vertical differentiation as defined above. As originally argued in Cremer and Thisse (1991) and recalled in Schmidt (2009), if the finiteness property does not hold, a formal equivalence is easily established between the vertical differentiation model with quadratic quality cost and the Hotelling model with quadratic transportation costs. Equivalence means here that in equilibrium, the choice of the products' characteristics and the prices are identical.

Let us consider the following example which describes the two prototype models of product differentiation in the literature:³

$$\text{Model V: } U(\cdot) = V + q + xq - p \text{ with } n = 2 \text{ and } c(q) = \frac{q^2}{2}.$$

Assuming that V is large enough to ensure full market coverage, the unique subgame perfect equilibrium is easily established. A little more work is required to establish the equivalence with a Hotelling model under quadratic transportation costs, but not much!⁴ The corresponding specification of the model for this last case is:

$$\text{Model H : } U'(\cdot) = V + t + \frac{(x-t)^2}{4} - p \text{ with } n = 2 \text{ and } t = 2q - 1.$$

The basic intuition for understanding why this equivalence prevails is the following. By assuming that V is arbitrarily large, we ensure that all consumers buy one unit either of good 1 or 2. Firms therefore compete for market shares and products attributes matter only for deciding which firm gets which side of the market. The level of the prices which are to be considered

²The interested reader is referred to Shaked and Sutton (1983) for a detailed exposition.

³The example is directly taken from Schmidt (2009)

⁴The interested reader is referred to Schmidt (2009) for a more general and detailed argument

are strictly below the lowest reservation prices for the two products. As a consequence, only the price differential matters for the consumers and it is sufficient to identify the position of the indifferent consumer to define firms' demands.

Let us denote this consumer by $\tilde{x}(p_1, p_2)$ and assume wlog that $q_1 < q_2$. It is immediate to see that $D_1(p_1, p_2) = \tilde{x}(\cdot)$ and $D_2(p_1, p_2) = 1 - \tilde{x}(\cdot)$ in either model V or H . In such a case, the vertical or horizontal nature of the differentiation is formally irrelevant. In the vertical interpretation, the level of the marginal cost being quadratic in quality, the low quality firm can always secure a positive market share on the left of the interval whereas in the Hotelling interpretation, the firm located on the left side always secures a positive market share at the left extreme of the interval. In other words, the behaviour of firms' demands at the price competition stage are the same under vertical and horizontal differentiation. Moreover, the quadratic cost assumption, be it on transportation cost or quality cost, ensures that this is the case for all possible relevant price subgames. At this step, it seems fair to conclude that model V and model H are somewhat similar, if not strictly equivalent.

We question now the robustness of this result to the market full coverage assumption. To which extent should we expect to obtain a comparable equivalence should V not be arbitrarily large? The answer is almost immediate: we should not expect the previous equivalence to hold anymore!

By definition, if the market is not fully covered, prices are such that for at least one consumers' type, the best available option is to refrain from consuming, i.e. $\exists x \neq 0$ such that $W(x, q_i, p_i) < 0$, $i = 1, 2$. The relevant question is then : where do these refraining consumers locate in the $[0, 1]$ interval?

Let us start with model V . According to the specification of $U(\cdot)$, all consumers agree on a ranking according to which the preferred characteristics is the largest q . Therefore, vertical differentiation prevails. But more importantly, the surplus function associated with this specification is strictly increasing in x . We may define by \bar{x}_i the type x which satisfies $U(x, q_i, p_i) = 0$. Since $q_1 < q_2$, $\forall x < \bar{x}_1$, not buying is preferred to buy i . All types $x \geq \bar{x}_1$ buy one of the two products. The following property immediately follows:

Result 1 *In the case utility is defined by $U(\cdot) = V + q + xq - p$, for any feasible products' characteristics, if the market is not fully covered, non buying consumers are located in a single sub-interval of $[0, 1]$. Moreover, firms' market shares are necessarily connected by an indifferent consumer.*

Firms' demands are defined by $D_2(p_1, p_2) = 1 - \tilde{x}$, $D_1(p_1, p_2) = \tilde{x} - \bar{x}_1$. A key feature of this specification is that, because firms' market are connected, firms compete with each other even though the market is not covered. This result must be contrasted with the specification of demand in a non-covered market in model H . In model H , it is clear that the surplus function is not monotonic in type x . More precisely, the sign of $\frac{\partial U'(\cdot)}{\partial x}$ depends on $x - t$. An immediate

consequence is the following result:

Result 2 *In the case utility is defined by $U'(\cdot) = V + t + \frac{(x-t)^2}{4} - p$, if the market is not fully covered, the position of non buying consumers in the $[0, 1]$ interval depends on the specification of the products' characteristics t_i . Non-buying consumers may be located in disconnected intervals and firms' market shares need not be connected by an indifferent consumer.*

Suppose in particular that $t_1 = -\frac{1}{4}$ and $t_2 = \frac{5}{4}$, which define the equilibrium values in the unique subgame perfect equilibrium under full coverage. Clearly enough, if V is such that the market is not covered for some relevant price levels, the non-buying consumers will be located in the middle of the $[0, 1]$ interval. Market shares are not connected anymore, which implies that firms do not directly compete with each other but rather behave like local monopolists.

Summing up, we observe that, in sharp contrast with the fully covered market case, the specification of demand functions at the price competition stage will most often differ fundamentally depending on whether model V or model H applies. As a consequence, payoffs function differ and the formal equivalence of equilibrium outcomes disappears. To sum up, we may claim:

Proposition 1 *The equivalence result established in Cremer and Thisse (1991) is not robust to the introduction of non-covered market configurations.*

This proposition is certainly not surprising. It nevertheless recalls that the similarities between vertical and horizontal models of product differentiation are over-emphasized.

3 A Complementary Typology of Differentiation Models

The vast majority of contributions that rely on address-models as vehicles to capture product differentiation retain the market full coverage assumption. No doubt this assumption simplifies the exposition! It turns out however that it is not made without loss of generality. In the previous section, we have shown that it was critical in establishing the formal equivalence between some classes of vertical and horizontal models. We investigate now in more depth what is further revealed by the analysis of non-covered market configuration.

Assuming that the market is covered amounts to assume that the elasticity of aggregate demand is zero. We now relax this assumption. Suppose then that all firms decrease their price equally. The critical question is: who benefits from the increase in market size? Under utility function $U'(\cdot)$ there is no clear answer to this question. It always depends on the products' characteristics and since the ranking of products' characteristics is not unanimous in the population, the identity of those consumers who start buying and the firm to which they turn is essentially indeterminate. Under utility function $U(\cdot)$ instead, it is always the case that the first consumer who starts buying as a consequence of a price decrease must be located in the vicinity of \bar{x}_1 .

As a consequence, additional consumers are always captured exclusively by the firm initially serving the marginal consumer \bar{x}_1 .

This last observation has far-reaching implications if one considers stage games with differentiation precommitments. In two-stage games of differentiation-then-price competition, firms aim at segmenting the population of consumers through their choice of products' attributes. The scope for relaxing price competition depends on the way heterogeneity in products' attributes is combined with population' heterogeneity. By differentiating their products, firms actually decide on a particular sharing of consumers' types. In this perspective, a key concern is the possible existence of a hierarchy among consumers established by the firms. We argue indeed that in order to better assess the nature of product differentiation, it is useful to put the standard approach on its head: instead of asking whether consumers are unanimous or not in their ranking of products' characteristics (as in Definition 1), one may ask whether *firms* are unanimous or not in *their* ranking of consumers' types. Consider the following definition of a "preferred consumer", which is a slightly adapted version of Definition 2:

Definition 4 *The preferred consumer of a firm which sells characteristic q is the consumer that benefits from the highest gross surplus in the population when consuming product q .*

Extending this definition, one may wish to consider the preference ordering that firms would establish over the set of consumers, depending on their own characteristics. It is then obvious that in model V , the ranking established by firms does not depend on their precise characteristics whereas it always does in model H . This mere fact induces a very different structure of competition between the firms. In model V , indeed, firms will inevitably end up competing for the same set of consumers, whatever the degree of product differentiation they retain in the first stage. If, for some choice of characteristics, the market ends up being non-covered in a price equilibrium, firms will nevertheless remain direct competitors. If $q_1 < q_2$, then firm 2 will always end up selling to its preferred consumers whereas this will never be the case for firm 1. By contrast, in model H , firms define their own ranking of consumers by deciding on their characteristics. It may then happen that by differentiating their products, the firms induce a non-covered market configuration where they end up not competing with each other. To put it differently, product differentiation induces heterogeneous rankings of consumers in model H whereas it preserves the homogeneous ranking in model V . We propose the following typology to summarize the above intuition:

Definition 5 *A market is Absolutely Ordered (AO) whenever all firms have the same preferred consumer, irrespective of the product characteristics chosen by these firms. A market is Relatively Ordered (RO) whenever the preferred consumer of a firm depends on its chosen characteristic.*

Notice that our typology is in line with the usual classification based on the quality vs variety distinction. Quality differentiation entails AO market with it, precisely because the basic

assumption is that all consumers value quality. The heterogeneity in consumers' type therefore reflects differences in levels of willingness to pay for quality upgrades; but whatever the quality level, the preferred consumer is the one with the highest willingness to pay. By contrast, the mere idea of variety differentiation entails *RO* markets: when choosing a particular product variety, a firm inevitably induces a specific ranking of consumers based on the preferences of these consumers towards that particular variety. Definition 4 is also very much in line with Definition 1. In the standard formulations retained in the literature, we expect that a vertically differentiated market is *AO* whereas a horizontal one is *RO*. Notice however that we may easily think of vertically differentiated markets which satisfy *RO*. Consider for instance the following location model: the population of consumers is uniformly distributed in an interval $[a^-, a^+] \in [0, 1]$. There are two feasible locations $\{0, 1\}$. Depending on the position of interval $[a^-, a^+]$, we may either have horizontal or vertical differentiation in the sense of Definition 1, but it will always be the case that the market is *RO*: a firm locating at 0 has type a^- as its preferred consumer whereas a^+ is the preferred consumer of a firm located at 1.

Since the connections between Definitions 1 and 4 are so close, it is legitimate to question the usefulness of the *AO* – *RO* typology, as compared to the *VD* – *HD* one. The chief interest of the *AO* – *RO* typology is to put emphasis on a neglected key driving force at work at the level of product selection by the firms. What mainly governs the nature of strategic interaction and the scope for product differentiation is the extent to which product differentiation allows for segmenting the population of consumers. Under *RO*, product differentiation is apt to create localized competition (in the limit, it even destroys competition) while preserving the ability for each firm to sell to consumers with the highest surplus for their products. By contrast, under *AO*, only one firm will end up selling to the highest surplus consumers, all the other ones will be confined to selling to "second-rate" consumers. In other words, product differentiation induces a hierarchy between firms under *AO*, but not under *RO*. Central to the nature of product differentiation is thus whether firms compete for the same consumers (*AO*) or segment otherwise identical consumers (*RO*) through their product selection. Boccard and Wauthy (2009) illustrates the importance of our typology. In that paper it is shown that within the standard vertical differentiation model popularized by Tirole (1988), firms actually choose not to differentiate their products by quality when they are allowed to commit to limited capacity levels. The intuition underlying this result is simple: since capacity constraints already limit drastically price competition, firms are induced to select quality levels so as to maximize the consumer welfare (that they will capture at the price competition stage), because the market is of the *AO* type, they focus on the same set of consumers, for whom the surplus is maximized by selecting the best available quality. Hence firms do not differentiate their product in equilibrium. A comparable no-differentiation result would never obtain in a Hotelling model with capacity commitment. The Hotelling model belongs to the *RO* types of models with the particular feature that each consumer has his ideal type. In such a case, maximizing industry surplus

requires product differentiation for sure.

4 Final Remarks

In this note we have revisited the traditional distinction between horizontal and vertical differentiation. We started from a setup in which the two classes of model appear to be almost equivalent. Then we showed that the similarities are not robust to the introduction of a partial market coverage. Analyzing the behaviour of the model under partial market coverage led us to propose an complementary typology of product differentiation models, based on the preferences firms displays relative to consumers' type, rather than the usual approach which relies on the preferences of consumers relative to firms' products.

References

- [1] **Boccard N. and X.Y. Wauthy (2009)** ,Equilibrium vertical differentiation in a Bertrand model with capacity precommitment. *International Journal of Industrial Organization* forthcoming.
- [2] **Cremer H. and Thisse J.F. (1991)** , Location models of horizontal differentiation: a special case of vertical differentiation models. *The Journal of Industrial Economics*, 39,4, p.383-390.
- [3] **Gabszewicz J. and Thisse J-F. (1979)** Price competition, quality and income disparities. *Journal of Economic Theory*, 20 p 310- 359.
- [4] **Gabszewicz J. and Thisse J-F. (1986)** On the nature of competition with differentiated products *The Economic Journal*, 96 p 160-172.
- [5] **Hotelling H. (1929)** Stability in competition *The Economic Journal* 39 p41- 57.
- [6] **Irmen A. and Thisse J.F. (1998)** : Competition in multicharacteristics spaces: Hotelling was almost right *Journal of Economic Theory*, 78,1, p.76-102.
- [7] **Martin S. (2000)**, Advanced Industrial Organization, Blackwell
- [8] **Shaked A. and Sutton J.(1982)**: Natural Oligopolies *Econometrica*, 51, p.1469-1484
- [9] **Schmidt R. (2009)** Welfare in differentiated oligopolies with more than two firms, *International Journal of Industrial Organization*, 27, 501-507
- [10] **Shy O. (1996)**, Industrial Organization, MIT Press
- [11] **Tirole J. (1988)** *The theory of industrial organization* MIT Press.