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Abstract

This paper asks whether revealing the identity of dropping bidders is in the interest of the auctioneer in an ascending price auction with asymmetric bidders and interdependent values. We show that revealing no information about bidders' identities may increase the expected revenue. In this setup, we identify the underlying mechanism for the failure of the often-heard recommendation that more transparency increases revenue. We also consider bidder ranking over auction formats.

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1 Introduction

The use of auctions, in particular open ascending auctions, has increased dramatically following the development of electronic commerce on the Internet. One question that arises is the way in which an auctioneer should design the auction in order to maximize her expected revenue. The relevant design features include auction format, participation rules and information feedback to bidders. All these aspects influence bidding behavior, including the bidders' decision to participate in the auction and the risk of collusion.

The aspect we focus on in this paper is whether in an English auction the auctioneer should reveal the identity of bidders who drop out. Information revelation about the identity of the dropping bidder is implicitly assumed in the standard way English auctions are modeled - the so-called *button-auction*. However, it is not a necessary feature of this auction format. In art auctions, for example, bidders often use a representative in order to hide their identities (Cassady (1967)). At eBay auctions, bidders usually know the identity and thus have access to the profile of the highest current bidder, but the seller can use the *private auction* option to hide bidders' identities.

Milgrom and Weber (1982) were among the first to study information revelation in auctions. They tackle the question of whether the auctioneer should reveal information about the object for sale in a symmetric common values model.[1] They find that committing to always reveal such information is beneficial to the auctioneer. Their analysis has been the building block of the so-called *linkage principle* according to which *always providing the bidders with as much information as possible about the value of the good increases expected revenue* (see Perry and Reny (1999) for an extension to multi-unit auctions; Benoît and Dubra (2006) for taking into account the seller's commitment and Board (2006) for an analysis that focuses on the number of bidders in the auction).

We depart from the usual information revelation setup and are concerned with information on the identity of bidders and not that of the object. This affects the expected value that bidders attach to the object only to the extent that values are interdependent and that each bidder values the signals of other bidders differently (i.e. bidders are asymmetric). In particular, revealing or not the identity of the dropping bidder has no impact in Milgrom and Weber's symmetric common values model. Moreover, the information revealed can definitely not be considered as an extra signal independent of bidders.

In an example in which one bidder has interdependent values and two other bidders have private values, Feinberg and Tennenholtz (2005) show that revealing the identity of dropping bidders can increase or decrease the expected revenue depending on the values of the parameters and the signal distribution. In this paper, we extend their model allowing for

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[1] Symmetry in this context means that all bidders have the same ex-ante distribution of signals, have valuations that depend symmetrically on their and others' signals and are affected by the revelation in the same way.

more than one interdependent values bidder; we derive a necessary condition for an ex-post equilibrium to exist and characterize the unique ex-post equilibrium. The characterization of a unique ex-post equilibrium allows us to thoroughly compare the two implementations of the auction we are interested in, i.e. when identities are revealed on the one hand and when they are concealed on the other. We find that revealing or concealing bidders' identity generically affects expected revenue and we identify a class of environments where choosing not to reveal the identity of dropping bidders yields higher expected revenue. Our proof allows us to identify and discuss the relation with the linkage principle. We also discuss implications for auction design.

The paper is organized as follows. Section 2 describes the model and the related potential applications. Equilibrium behavior is analyzed in Section 3. The revenue ranking between the ascending auction with full information and the restricted one as well as the analysis of the bidding behavior is examined in Section 4. Section 5 concludes.

2 The model

Payoff Structure. There is a single object for sale and $n = 3$ risk-neutral bidders, that is the minimum number of bidders to make the question of revelation of the identity of dropping bidders interesting. Bidders are organized into two groups: (a) *interdependent values bidders* who care about the signals received by other bidders and (b) *private values bidders* who care only about their own signal. Let n_d and n_p respectively denote the number of interdependent values and private values bidders. The sets of interdependent values and private values bidders are denoted N_d and N_p respectively. Let $N = N_d \cup N_p$. Prior to the auction, each bidder i privately observes a signal, that is a realization of the random variable $X_i \in \mathbb{R}_+$, denoted x_i. The value $v_i \in \mathbb{R}_+$ of the object to bidder i is given by

$$\begin{aligned} v_i(X_1, ..., X_n) &= \sum_{j=1}^{n} \alpha_{ij} X_j && \text{if bidder } i \in N_d \\ v_i(X_1, ..., X_n) &= X_i && \text{if bidder } i \in N_p \end{aligned} \tag{1}$$

where $\sum\limits_{j=1}^{n} \alpha_{ij} = 1$ with $\alpha_{ij} \geq 0$ and $\alpha_{ii} > \sum\limits_{j \neq i} \alpha_{ij}$.

First, note that v_i is a continuous function and is at least strictly increasing in its argument X_i. Second, we assume here that the weight a bidder i puts on her own signal in her valuation (namely α_{ii}) is always strictly larger than the sum of the weights that other bidders' signals have in her valuation (that is α_{ij} for all $i \neq j$). This assumption is used among others by Dasgupta and Maskin (2000) and Izmalkov (2004) to ensure that the equilibrium in the English auction is efficient. In our case, it also helps to rank revenues in the two auction formats.

Starting from this general model, we focus on two special cases that help articulate the key factors behind the result:

1. A common values auction with a fringe of consumption-driven one-time bidders: N_d is here the set of regular bidders and N_p contains one-time bidders, who do not have any particular expertise and are driven by very different motives. These bidders have private values. Given the idiosyncrasy of the private values bidders' signal, regular bidders do not consider their signals when estimating the valuation of the object. Thus, in a three bidders' auction, if we suppose that bidder 3 is the one-time fringe bidder, we have

$$\begin{aligned} v_1(X_1, X_2, X_3) &= \alpha_{11}X_1 + \alpha_{12}X_2 \\ v_2(X_1, X_2, X_3) &= \alpha_{22}X_2 + \alpha_{21}X_1 \\ v_3(X_1, X_2, X_3) &= X_3 \end{aligned}$$

2. An auction with experts and non-experts: in this case, N_p can be interpreted as the set of expert bidders, who trust their information and do not update the value they assign to the object upon learning others' information. N_d is then the set of non-experts. An example could be an auction at Sotheby's where experts know the willingness to pay for an object in their respective market. This interpretation could be applied to a collector/non-collector application. Feinberg and Tennenholtz's (7) model is a special case of this environment, that is the two experts have the same importance in bidder 1's valuation . With three bidders, we have

$$\begin{aligned} v_1(X_1, X_2, X_3) &= \alpha_{11}X_1 + \alpha_{12}X_2 + \alpha_{13}X_3 \\ v_2(X_1, X_2, X_3) &= X_2 \\ v_3(X_1, X_2, X_3) &= X_3 \end{aligned}$$

   Another practical and popular example is an auction on eBay : suppose an item is for sale and bidder 1 has submitted a bid. Then, another potential bidder, bidder 2, can acquire information by clicking on his name and reviewing items bought or sold by bidder 1 and thus infer that the former is a expert bidder or a non-expert bidder.[2]

Note that we do not cover all the space of environments allowed by (1). In particular, those involving three interdependent values bidders and two interdependent values where $\alpha_{ij} > 0$ *for all* $j \in N_p$. However, the two cases described in the paper are sufficient to introduce the main question and discuss the relation with the linkage principle.

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[2] Of course, an eBay auction presents an extra characteristic from which we abstract here: the number of (potential) bidders is unknown at start of and during the auction.

Information. N_d, N_p and preferences are common knowledge. Signals are privately observed. They are uniformly distributed over $[0, 1]$. Furthermore, this distribution is common knowledge.

Auction rules. We consider two distinct implementations of the *English button-auction.* In both cases, the auctioneer raises the price continuously. All bidders are initially active and they remain so until they drop out irrevocably. Ties are broken by assigning the object randomly and with equal probability to one of the highest bidders. The two auctions differ in the information revealed when a bidder drops out. In the anonymous auction, bidders only observe the dropping price of the bidder who drops out, not his identity. In the transparent auction, bidders observe both the dropping price and the identity of the dropping bidder. The transparent auction is what is usually referred to as the *English button-auction* (Milgrom and Weber (1982)).

3 Equilibrium Behavior

A strategy p_i^m for bidder i in auction format $m \in \{T, A\}$ is a mapping from the realization of her signal x_i and the current state of the world s to a dropping price. The set of the relevant states of the world S^T in the transparent auction consists of the set of active bidders together with the dropping price e of the first bidder who drops out (if no bidder has dropped out so far, $e = +\infty$). Consider distinct $i, j, k \in N$. Thus, for bidder i, $s \in S_i^T = [\{\{i, j, k\}, \{i, j\}, \{i, k\}\} \times (\mathbb{R}_+ \cup \{+\infty\})]$ and $p_i^T(x_i, s) : [0, 1] \times S_i^T \rightarrow \mathbb{R}_+$.

The set of the relevant states of the world S^A in the anonymous auction consists of the number of active bidders and e. Thus, for bidder i, $s \in S_i^A = [\{2, 3\} \times (\mathbb{R}_+ \cup \{+\infty\})]$ and $p_i^A(x_i, s) : [0, 1] \times S_i^A \rightarrow \mathbb{R}_+$.

The bidders' vector of strategies in an auction is denoted $p^m = (p_1^m, p_2^m, p_3^m)$ and p_{-i}^m denotes the vector of strategies but for i's.

In this section, we derive the unique subgame perfect ex-post equilibrium[3] in the two auction formats. Ex-post equilibrium (Crémer and McLean (1985) and Maskin (1992) is a refinement of a Bayesian Nash Equilibrium: it involves *no regret* in the sense that even if, after the fact, bidders were told the signal realizations of other bidders, they would not change their strategy. Note that the nature of the information that will cause *no regret* differs across the two auction formats. In the transparent auction, bidders will know the signal realization of the other bidders and how they connect to the bidder identities. In the anonymous auction, bidders only know the signal realizations.

Because the English auction is a dynamic game, we require strategies to form an ex-post equilibrium for all possible subgames. Formally, define $\pi_i^m(p, x, s)$ as bidder i's ex-post profit conditional on being in state of the world s, signal realizations $x \in [0, 1]^3$ and strategies p being played. A strategy p^{*m} constitutes a subgame perfect ex-post equilibrium if

$$\pi_i^m(p^{*m}, x, s) \geq \pi_i^m(p_i, p_{-i}^{*m}, x, s)$$

where $m \in \{T, A\}$ $\forall i, \forall x, \forall s$ and $\forall$ alternative strategy $p_i \in \mathbb{R}_+$. It is important to underline that the notion of (ex-post) equilibrium is here strongly related to the implementation (T or A) the auctioneer has chosen. Indeed, bidders derive their best responses (and their equilibrium strategies) on the basis of the information they can acquire in the implementation they are facing.

Lemma 1 shows that a necessary condition for an ex-post equilibrium to exist is that strategies are strictly increasing in their own signal.

Lemma 1. *Any ex-post (subgame perfect) equilibrium strategy candidate, p_i^{*m}, is strictly increasing in x_i $\forall i$, $\forall s$ and $\forall m \in \{T, A\}$.*

Proof: (i) We start with the transparent auction. Consider state of the world s. Define the set of remaining bidders $\mathcal{R}$ and the corresponding indicator function $\mathrm{I}_{j \in \mathcal{R}}$.

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[3] See Bikchandani et al. (2002) for a characterization of the set of perfect Bayesian equilibria in symmetric separating strategies.

Ad interim - i.e. when bidder i knows his own signal - bidder i's expected valuation conditional on winning at price p_{final} is given by:

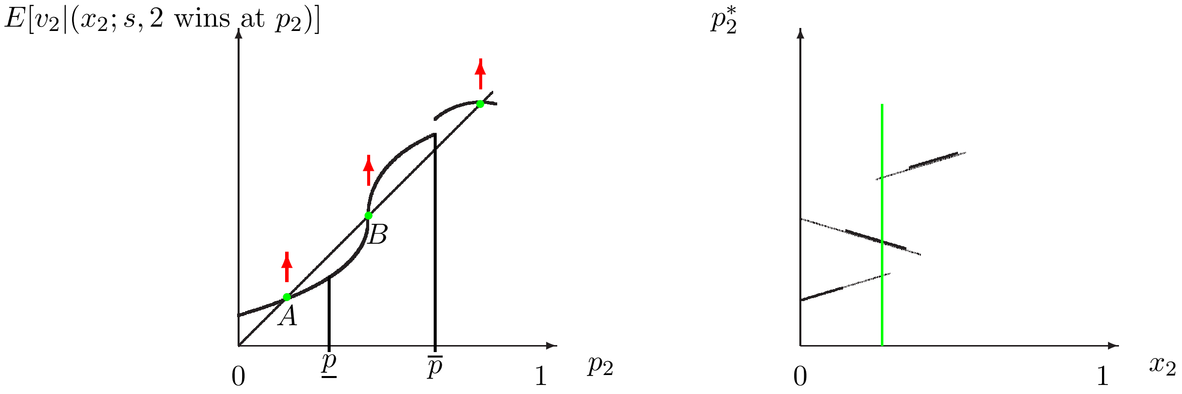
$$\begin{aligned} E[v_i|x_i; s, i \text{ wins at } p_{final}] = \alpha_{ii}x_i \quad &+ \quad \sum_{j \neq i} \alpha_{ij} \left(\mathrm{I}_{(j \in \mathcal{R})} E[X_j|s, (\text{bidder } j \text{ drops at } p_{final})] \right. \\ &+ \quad \left. (\mathrm{I}_{(j \notin \mathcal{R})}) E[X_j|s, (\text{bidder } j \text{ drops at } e_j)] \right) \end{aligned} \tag{2}$$

where e_j is the dropping price for bidder j. Let $F(.)$ be the c.d.f. of the final price p_{final} and $\pi_i^T(p_i, p_{-i}^*, x_i, s)$ bidder i's expected payoff. Similarly to Milgrom and Shannon (10), the envelope integral formula implies at equilibrium that the expected payoff of bidder i satisfies

$$\pi_i^T(p_i^*, p_{-i}^*, x_i, s) \quad = \quad \int^{p_i*} (E[v_i|x_i; s, i \text{ wins at } p_{final}] - p_{final}) dF(p_{final})$$

$p_i^*(x_i, s)$ is a selection from $argmax_{p_i}\ \pi_i^T(p_i, p_{-i}^*, x_i, s)$ and such that $F(.)$ is everywhere strictly increasing. $p_i^*(x_i, s)$ thus satisfies the following first order condition that yields $\forall x_i, \forall s$

$$E[v_i|x_i; s, i \text{ wins at } p_i^*(x_i, s)] - p_i^*(x_i, s) = 0. \tag{3}$$



1.(a) 1.(b)

Figure 1: Panel (a) shows candidates for ex-post equilibrium. When x_2 increases, the function $E[v_2|x_2; s, 2 \text{ wins at price } p_2]$ is shifted upwards for all states of the world s. Panel (b) represents both the intersection points (dotted lines) and the selected piecewise strictly monotonic strategy against own signal (thick lines).

Claim 1. : *Any equilibrium candidate must be piecewise strictly monotonic.*

Proof. Consider any s and suppose without loss of generality that bidder 2 is still in. Let $p_2(.)$, the equilibrium strategy by bidder 2. It must satisfy the necessary condition (3) for

an equilibrium. Panel (a) of Figure 1 represents an example of an expected valuation function as a function of winning prices. $E[v_2|x_2; s, 2 \text{ wins at } p_2]$ might have discontinuities, might be increasing or decreasing. The only constraint we place on it is that it crosses at least once the 45 degree line for existence. Fixed points correspond to equilibrium candidates because they satisfy necessary condition (3). Note that there might be multiple fixed points.
Arrows indicate that as x_2 increases, the expected valuation of bidder 2 shifts upwards continuously [cf eq.(2)] so that fixed points shift continuously too (some might disappear and others might appear). Note that fixed points are shifted upwards and downwards continuously when the expected valuation function crosses the 45 degree line from above and below respectively (See fixed points A and B respectively in Panel (a) on Figure 1). Consider a function of x_2 made up of a selection of such fixed points. By construction, this selection satisfies the necessary condition for an equilibrium and thus constitutes an equilibrium candidate. Because of the properties of fixed points as x_2 increases, this function is piecewise strictly increasing or decreasing. Discontinuity points correspond to jumps (up or down) from one fixed point to another as depicted in Panel (b) on Figure 1.

□

Claim 2. *: Any equilibrium candidate must be strictly increasing.*

Proof. Consider a candidate $p_2^*(x_2, s)$. Suppose $E[v_2|x_2; s, 2 \text{ wins at price } p] - p < 0$ for some $p < p_2^*(x_2, s)$. Suppose point B corresponds to $p_2^*(x_2, s)$ and consider $p = \underline{p}$ in Panel (a). If bidder 2 wins at that price $p = \underline{p}$, he will experience regret because he is better off losing at that price. A contradiction with ex-post equilibrium.
Likewise, suppose $E[v_2|x_2; s, 2 \text{ wins at price } p] - p > 0$ for some $p > p_2^*(x_2, s)$. Suppose point B corresponds to $p_2^*(x_2, s)$ and consider $p = \overline{p}$ in Panel (a). Thus even if bidder 2 wins at that price, he will experience regret because it could have been possible for him to win the object at a higher price leading to a larger payoff. Again, a contradiction with ex-post equilibrium. Putting both arguments together, we conclude that at any ex-post equilibrium candidate, $E[v_2|x_2, s, 2 \text{ wins at price } p] - p$ has a unique fixed point, with $E[v_2|x_2; s, 2 \text{ wins at price } p] > p \ \forall p < p_2^*(x_2, s)$ and $E[v_2|x_2; s, 2 \text{ wins at price } p] < p$ $\forall p > p_2^*(x_2, s)$. This means that generically $E[v_2|x_2; s, 2 \text{ wins at price } p]$ crosses p once and from above. The expected valuation corresponding to the fixed point generated by an increase in bidder's own signal can obviously never be linked to a price at the left of p_2^* (precisely in the interval $[0, p_2^*]$) resulting in an upwards shift of the fixed point. We conclude that as x_2 increases, $p_2^*(x_2, s)$ increases. □

(ii) We now turn to the anonymous auction. Consider any s and consider the three distinct bidders $\{i, j, k\}$ with bidder i is still in the auction. When $s = (3, +\infty)$, the same reasoning applies to establish the Lemma for the anonymous auction since no bidder has dropped out: bidders in that state of the world have identical information as in the transparent auction. Thus, we focus on the case $s = (2, e)$ in the two auction formats. Suppose one bidder has already dropped and let γ the probability that k is the other remaining bidder in the auction. Ad interim, bidder i's expected valuation in the Anonymous auction if p_{final} is the final price :

$$\begin{aligned} E[v_i|x_i; s, i \text{ wins at price } p_{final}] &= \gamma\Big[\alpha_{ii}x_i + \alpha_{ik}E[X_k|(\text{bidder k drops at } p_{final})] \\ &+ \alpha_{ij}E[X_j|(\text{bidder j drops at e})]\Big] \\ &+ (1-\gamma)\Big[\alpha_{ii}x_i + \alpha_{ik}E[X_k|(\text{bidder k drops at e})] \\ &+ \alpha_{ij}E[X_j|(\text{bidder j drops at } p_{final})]\Big] \qquad (4) \end{aligned}$$

We can then apply the same reasoning as in part (i) to argue that $E[v_i|x_i; s, i \text{ wins at price } p_{final}]$ is continuous in x_i, that any equilibrium candidate satisfies (3) and must be a strictly increasing function of x_i. This completes the proof. □

We are now ready to characterize the equilibrium both in the transparent and the anonymous auction. Note that in Table 1 and Table 2, column titles $\{1, 2, 3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$ refer to the identity of the remaining bidders. The state of the world is made of the identity of the remaining bidders and e, the dropping price of the first bidder (if any).

Proposition 1 (Equilibrium in the transparent auction)**.**

(i) Suppose bidders 1 and 2 have interdependent values, bidder 3 has private values and $\alpha_{13} = \alpha_{23} = 0$ *(model with a fringe of one-time bidders). The strategies (i.e. dropping prices) given in Table 1 constitute the unique subgame perfect ex-post equilibrium :*

Table 1: Dropping prices in the transparent auction with a fringe of one-time bidders*

	$\{1,2,3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$
$p_1^T(x_1,s)$	x_1	x_1	$\alpha_{11}x_1 + (1-\alpha_{11})e$	$-$
$p_2^T(x_2,s)$	x_2	x_2	$-$	$\alpha_{22}x_2 + (1-\alpha_{22})e$
$p_3^T(x_3,s)$	x_3	$-$	x_3	

*where e denotes the observed dropping price.

(ii) Suppose bidder 1 has interdependent values and bidders 2 and 3 have private values. The strategies given in Table 2 constitute the unique subgame perfect ex-post equilibrium :

Table 2: Dropping prices in the transparent auction with experts and non-expert bidders*

	$\{1,2,3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$
$p_1^T(x_1,s)$	x_1	$\frac{\alpha_{11}}{1-\alpha_{12}}x_1 + \frac{\alpha_{13}}{1-\alpha_{12}}e$	$\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + \frac{\alpha_{12}}{1-\alpha_{13}}e$	$-$
$p_2^T(x_2,s)$	x_2	x_2	$-$	x_2
$p_3^T(x_3,s)$	x_3	$-$	x_3	x_3

*where e denotes the observed dropping price.

Proof: See Appendix A ☐

We now turn to the anonymous auction. The column titles named 3 and 2 in Table 3 and Table 4 refer to the number of remaining bidders. The state of the world is made up of the remaining number of bidders and e, the dropping price of the first dropping bidder if applicable.

Proposition 2 (Equilibrium in the anonymous auction)**.**

(i) Suppose bidders 1 and 2 have interdependent values, bidder 3 has private values and that $\alpha_{13} = \alpha_{23} = 0$ *(model with a fringe of one-time bidders). The strategies (i.e. dropping prices) given in Table 3 constitute the unique subgame perfect ex-post equilibrium:*

Table 3: Dropping prices in the anonymous auction with a fringe of one-time bidders*

	3	2
$p_1^A(x_1, s)$	x_1	$a_1 x_1 + (1 - a_1)e$
$p_2^A(x_2, s)$	x_2	$a_2 x_2 + (1 - a_2)e$
$p_3^A(x_3, s)$	x_3	x_3

* where $a_1 = \frac{4\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}{4\alpha_{22} - 2\alpha_{12}}$, $a_2 = \frac{4\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}{4\alpha_{11} - 2\alpha_{21}}$
and e denotes the observed dropping price.

(ii) Suppose bidder 1 has interdependent values and bidders 2 and 3 have private values. The strategies given in Table 4 constitute the unique subgame perfect ex-post equilibrium:

Table 4: Dropping prices in the anonymous auction with experts and non-expert bidders*

	3	2
$p_1^A(x_1, s)$	x_1	$bx_1 + (1 - b)e$
$p_2^A(x_2, s)$	x_2	x_2
$p_3^A(x_3, s)$	x_3	x_3

* where $b = \frac{2\alpha_{11}}{2 - (\alpha_{12} + \alpha_{13})} = \frac{2\alpha_{11}}{(1 + \alpha_{11})}$
and e denotes the observed dropping price.

Proof: See Appendix B ☐

Comparing Propositions 1 and 2 leads to two direct implications. First, bidders follow an optimal strategy such that they put a different weight on their signals depending on the auction format. This is to be expected given that they receive different information in these two auction formats. As a result, they update their expectations of the good's value differently.
Second, in the case of the fringe of one-time bidders, from Table (3) follows:

Corollary 1. *In the fringe of one-time bidders case,* $a_i > \alpha_{ii}$

Proof: From Proposition 2, $a_i = \frac{4\alpha_{ii}\alpha_{jj} - \alpha_{ij}\alpha_{ji}}{4\alpha_{jj} - 2\alpha_{ij}}$.
Rearranging the inequality $a_i > \alpha_{ii}$ using $\sum\limits_{j=1}^{n} \alpha_{ij} = 1$, we obtain that it holds if

$$2\alpha_{ii} > \alpha_{ji}$$

This is always verified given our assumption that $\alpha_{ii} > \sum\limits_{j \neq i} \alpha_{ij}$. $\square$

Corollary 1 implies that bidders with interdependent values make higher bids in the anonymous auction than in the transparent auction. Indeed, they place a higher weight on their signal and a lower weight on the exit price in that case and, from the construction of the equilibrium, their own signal is larger than the exit price.
This does not directly imply that the revenue in the anonymous auction is higher than in the transparent auction because a thorough comparison must include the two states of the world in the transparent auction that correspond to $(2, e)$ in the Anonymous implementation. Thus the comparison is between $\alpha_{ii}x_i + (1 - \alpha_{ii})e$ and x_i, on the one hand, and $a_i x_i + (1 - a_i)e$ on the other hand for $i = 1, 2$. For a given x_i, we have $\alpha_{ii}x_i + (1 - \alpha_{ii})e < a_i x_i + (1 - a_i)e < x_i$.
Third, in the anonymous auction for the experts and non-expert case, the weight that bidder 1 places on her signal once someone has dropped out is independent of the weight she places on x_2 and x_3 in her valuation. The intuition for this is that the probabilities that bidder 2 or bidder 3 is the first bidder to drop out are equal and valuations are linear in signals. Fourth, we cannot derive any revenue conclusion from the direct comparison of dropping strategies in Table (2) and Table (4) if $\alpha_{12} \neq \alpha_{13}$. Indeed, the next corollary establishes that $\frac{\alpha_{11}}{(1-\alpha_{12})} > b > \frac{\alpha_{11}}{(1-\alpha_{13})}$ when $\alpha_{12} > \alpha_{13}$. The reverse inequality holds when $\alpha_{12} < \alpha_{13}$. Thus depending on the state of the world, bidder 1 is both more aggressive or less aggressive in the transparent auction than in the anonymous auction.

Corollary 2. *Consider the experts and non-expert environment and suppose, without loss of generality, that* $\alpha_{12} > \alpha_{13}$. *Then* $\frac{\alpha_{11}}{(1-\alpha_{12})} > b > \frac{\alpha_{11}}{(1-\alpha_{13})}$

Proof: From 2 and 4, consider $\frac{\alpha_{11}}{(1-\alpha_{12})} > b = \frac{2\alpha_{11}}{(1+\alpha_{11})}$. Simplifying and rearranging the inequality in the α's, it follows that $1 + \alpha_{11} > 2 - 2\alpha_{12}$ and thus $\alpha_{11} + 2\alpha_{12} > 1$. Similarly, consider $\frac{\alpha_{11}}{(1-\alpha_{13})} < b = \frac{2\alpha_{11}}{(1+\alpha_{11})}$. Simplifying, it follows that $\alpha_{11} + 2\alpha_{13} < 1$. As $\sum\limits_{j=1}^{n} \alpha_{ij} = 1$, it follows that $\alpha_{11} + 2\alpha_{12} > \alpha_{11} + \alpha_{12} + \alpha_{13} > \alpha_{11} + 2\alpha_{13}$. $\square$

Finally, we note that the equilibrium in the transparent auction is ex-post efficient following Dasgupta and Maskin (2000). By contrast, the equilibrium in the anonymous auction is generically inefficient ex-post.

4 Revenue ranking

We now turn to ranking the expected revenue in the two auction formats. The next proposition describes the environments where the expected revenue in the anonymous auction is larger than or equal to the expected revenue in the transparent auction.

Proposition 3. *Let R_A and R_T be the revenue in the anonymous and transparent auctions respectively.*

(i) Suppose the environment with experts and non-expert bidders

$$\begin{aligned} v_1(X_1,X_2,X_3) &= \alpha_{11}X_1+\alpha_{12}X_2+\alpha_{13}X_3 \\ v_2(X_1,X_2,X_3) &= X_2 \\ v_3(X_1,X_2,X_3) &= X_3, \end{aligned}$$

then, $E[R_A] \leq E[R_T]$. The inequality is strict when $\alpha_{12} \neq \alpha_{13}$ and the result holds with equality when $\alpha_{12} = \alpha_{13}$.

(ii) Consider the environment with a fringe of one-time bidders

$$\begin{aligned} v_1(X_1,X_2,X_3) &= \alpha_{11}X_1+\alpha_{12}X_2 \\ v_2(X_1,X_2,X_3) &= \alpha_{22}X_2+\alpha_{21}X_1 \\ v_3(X_1,X_2,X_3) &= X_3, \end{aligned}$$

then $E[R_A] > E[R_T]$.

Proof: (i) When $\alpha_{12} = \alpha_{13}$, Corollary 2, together with Propositions 1 and 2, implies that bidder 1's dropping strategies in the transparent auction and in the anonymous auction are the same. Moreover, Table 2 and Table 4 show that bidder 2 and bidder 3's dropping strategies are the same in both implementations. Thus, there is no difference in the expected revenue. Without loss of generality, consider now the asymmetric case in which $\alpha_{12} > \alpha_{13}$. We can partition the set of signal realizations according to the identity of the winner; The figure below describes the possible realizations of (X_2, X_3) for a given realization of X_1.

Bidder 3 wins:

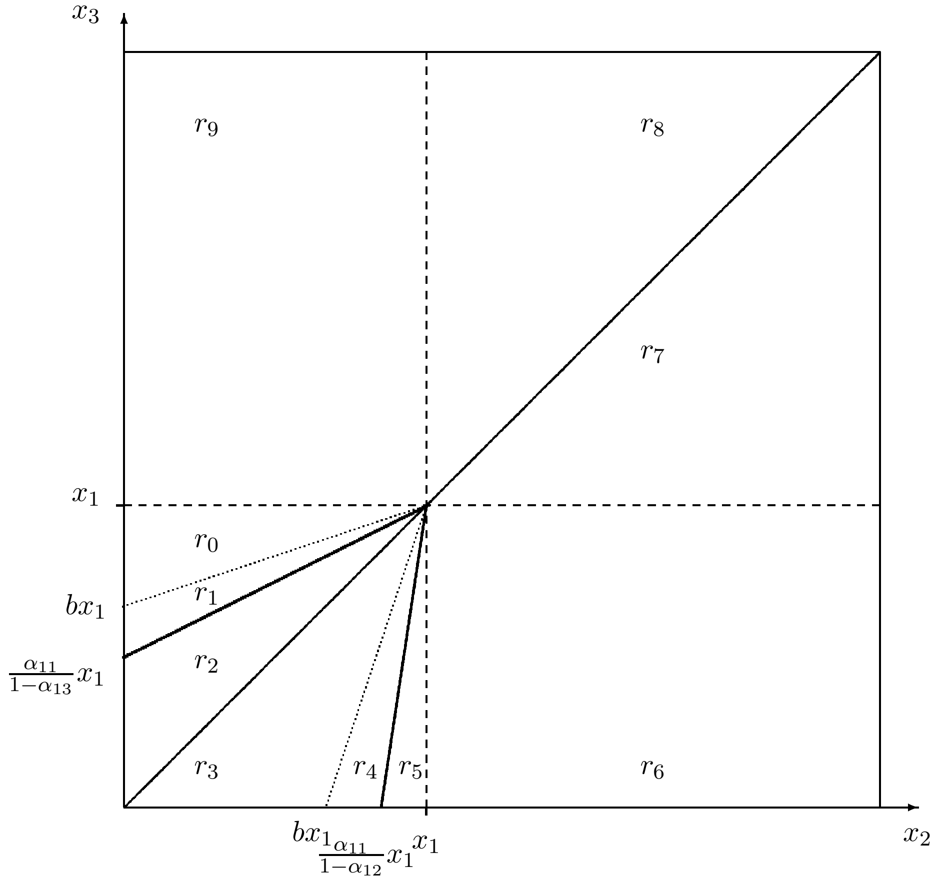
In the transparent auction, bidder 3 wins against bidder 2 in the area above the 45 degree line and for values of (x_2, x_3) such that $x_1 < x_2$ (region r_8 in Figure 2). The auctioneer's revenue is equal to x_2. Bidder 3 wins against bidder 1 when $x_2 < min\{x_1, x_3\}$ and $\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2 < x_3$ (region $r_9 \cup r_0 \cup r_1$). The auctioneer's revenue is then $\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2$.

In the anonymous auction, bidder 3 wins against bidder 2 in the area above the 45 degree line and for values of (x_2, x_3) such that $x_1 < x_2$ (region r_8). The auctioneer's revenue is equal to x_2. Bidder 3 wins against bidder 1 when $x_2 < min\{x_1, x_3\}$ and $bx_1+(1-b)x_2 < x_3$ (region $r_9 \cup r_0$). The auctioneer's revenue is then $bx_1 + (1 - b)x_2$.

Bidder 1 wins:

In the transparent auction, bidder 1 wins against bidder 3 when $x_2 < min\{x_1, x_3\}$ and $\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2 > x_3$ (region r_2). The auctioneer's revenue is then x_3. Bidder 1 wins against bidder 2, when $x_3 < min\{x_1, x_2\}$ and $\frac{\alpha_{11}}{1-\alpha_{12}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{12}})x_3 > x_2$ (region $r_3 \cup r_4$). The auctioneer's revenue is x_2.

In the anonymous auction, bidder 1 wins against bidder 3 when $x_2 < min\{x_1, x_3\}$ and $x_3 < bx_1 + (1-b)x_2$ (region $r_1 \cup r_2$). The auctioneer's revenue is x_3. Bidder 1 wins against bidder 2 when $x_1 < min\{x_1, x_2\}$ and $bx_1 + (1-b)x_3 > x_2$ (region r_3). The auctioneer's revenue is x_2.



Note that the dotted lines represent the anonymous auction and the 2 thick lines exist only if $\alpha_{12} > \alpha_{13}$. When $\alpha_{12} = \alpha_{13}$, $r_4 \cup r_1 = \phi$ and the separating lines in Transparent and anonymous auctions coincide. Comparing the auctioneer's revenue region by region yields the following for (x_1, x_2, x_3) in the interior of their support $[0, 1]^3$ (where we have used Corollary 2 to compare expressions for the revenue):

Table 5: Revenue comparison between the two auctions

	R_T [w]	$\alpha_{12} = \alpha_{13}$	$\alpha_{12} > \alpha_{13}$	R_A [w]
r_0	$\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2$ [3]	$=$	$<$	$bx_1 + (1-b)x_2$ [3]
r_1	$\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2$ [3]	n/a	$<$	x_3 [1]
r_2	x_3 [1]	$=$	$=$	x_3 [1]
r_3	x_2 [1]	$=$	$=$	x_2 [1]
r_4	x_2 [1]	n/a	$>$	$bx_1 + (1-b)x_3$ [2]
r_5	$\frac{\alpha_{11}}{1-\alpha_{12}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{12}})x_3$ [2]	$=$	$>$	$bx_1 + (1-b)x_3$ [2]
r_6	$\frac{\alpha_{11}}{1-\alpha_{12}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{12}})x_3$ [2]	$=$	$>$	$bx_1 + (1-b)x_3$ [2]
r_7	x_3 [2]	$=$	$=$	x_3 [2]
r_8	x_2 [3]	$=$	$=$	x_2 [3]
r_9	$\frac{\alpha_{11}}{1-\alpha_{13}}x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}})x_2$ [3]	$=$	$<$	$bx_1 + (1-b)x_2$ [3]

*where [w] stands for the winning bidder in the region.

In the following claim, we compare regions r_6 and r_9.

Claim 3. *$E[R_A - R_T|\ (x_1, x_2, x_3) \in r_6 \cup r_9] = \frac{1}{Pr(r_6 \cup r_9)} \frac{-\alpha_{11}(\alpha_{12}-\alpha_{13})^2}{2(1+\alpha_{11})(\alpha_{11}+\alpha_{12}\alpha_{13})} K < 0$ when $\alpha_{12} > \alpha_{13}$ where $K = \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2) dx_2 dx_1 dx_3$*

Proof. In r_9, bidder 3 wins against bidder 1. Thus

$$E[R_A|(x_1, x_2, x_3) \in r_9] = \frac{1}{Pr(r_9)} \int_0^1 \int_0^{x_3} \int_0^{x_1} (bx_1 + (1-b)x_2) dx_2 dx_1 dx_3$$

$$E[R_T|(x_1, x_2, x_3) \in r_9] = \frac{1}{Pr(r_9)} \int_0^1 \int_0^{x_3} \int_0^{x_1} (\frac{\alpha_{11}}{1-\alpha_{13}} x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{13}}) x_2) dx_2 dx_1 dx_3$$

Therefore, the difference is

$$E[(R_A - R_T)|(x_1, x_2, x_3) \in r_9] = \frac{1}{Pr(r_1)} (b - \frac{\alpha_{11}}{1-\alpha_{13}}) \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2) dx_2 dx_1 dx_3 \quad (5)$$

In r_6, bidder 2 wins against bidder 1. Thus

$$E[R_A|(x_1, x_2, x_3) \in r_6] = \frac{1}{Pr(r_6)} \int_0^1 \int_0^{x_2} \int_0^{x_1} (bx_1 + (1-b)x_3) dx_3 dx_1 dx_2$$

$$E[R_T|(x_1, x_2, x_3) \in r_6] = \frac{1}{Pr(r_6)} \int_0^1 \int_0^{x_2} \int_0^{x_1} (\frac{\alpha_{11}}{1-\alpha_{12}} x_1 + (1 - \frac{\alpha_{11}}{1-\alpha_{12}}) x_3) dx_3 dx_1 dx_2$$

Therefore, the difference is

$$E[(R_A - R_T)|(x_1, x_2, x_3) \in r_6] = \frac{1}{Pr(r_1)}(b - \frac{\alpha_{11}}{1-\alpha_{12}}) \int_0^1 \int_0^{x_2} \int_0^{x_1} (x_1 - x_3) dx_3 dx_1 dx_2 \quad (6)$$

The integral expressions in (5) and (6) are identical (as long as the random variables have same support, just relabel x_2, x_3 and vice-versa). Let $K = \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2) dx_2 dx_1 dx_3$. Thus

$$\begin{aligned} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_6 \cup r_9] &= \frac{Pr(r_6)}{Pr(r_6 \cup r_9)} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_6] \\ &+ \frac{Pr(r_9)}{Pr(r_6 \cup r_9)} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_6] \\ &= \frac{1}{Pr(r_6 \cup r_9)} \left(2b - (\frac{\alpha_{11}}{1-\alpha_{12}} + \frac{\alpha_{11}}{1-\alpha_{13}}) \right) K \\ &= \frac{1}{Pr(r_6 \cup r_9)} \left(2\frac{2\alpha_{11}}{1+\alpha_{11}} - \frac{\alpha_{11}(1+\alpha_{11})}{\alpha_{11} + \alpha_{12}\alpha_{13}} \right) K \end{aligned}$$

As we imposed $\sum\limits_{j=1}^{n} \alpha_{ij} = 1$ and $\alpha_{ii} > \alpha_{ij}$ for all $j \neq i$, it follows from the comparison of the dropping prices that

$$\begin{aligned} 2\frac{2\alpha_{11}}{1+\alpha_{11}} - \frac{\alpha_{11}(1+\alpha_{11})}{(\alpha_{11} + \alpha_{12}\alpha_{13})} &= \frac{\alpha_{11}(4\alpha_{11} + 4\alpha_{12}\alpha_{13} - (1+\alpha_{11})^2)}{2(1+\alpha_{11})(\alpha_{11} + \alpha_{12}\alpha_{13})} \\ &= \frac{\alpha_{11}(4\alpha_{12}\alpha_{13} - (1-\alpha_{11})^2)}{(1+\alpha_{11})(\alpha_{11} + \alpha_{12}\alpha_{13})} \\ &= \frac{\alpha_{11}(4\alpha_{12}\alpha_{13} - (\alpha_{12}+\alpha_{13})^2)}{(1+\alpha_{11})(\alpha_{11} + \alpha_{12}\alpha_{13})} \\ &= -\frac{\alpha_{11}(\alpha_{12}-\alpha_{13})^2}{(1+\alpha_{11})(\alpha_{11} + \alpha_{12}\alpha_{13})} < 0 \end{aligned}$$

□

Claim 4. *Consider* $\alpha_{12} > \alpha_{13}$ *and where* $K = \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2) dx_2 dx_1 dx_3$. *Then*
$E[R_A - R_T|(x_1, x_2, x_3) \in r_0 \cup r_1] < \frac{1}{6}[(1 - \frac{\alpha_{11}}{1-\alpha_{13}})(b - \frac{\alpha_{11}}{1-\alpha_{13}})]K$
$E[R_A - R_T|(x_1, x_2, x_3) \in r_4 \cup r_5] < \frac{1}{6}[(1 - \frac{\alpha_{11}}{1-\alpha_{12}})(b - \frac{\alpha_{11}}{1-\alpha_{12}})]K.$

Proof. In this claim, we need to take into account the probabilities of being in a particular region as they differ. Consider the triangular region r_0 that can be seen as a triangle in the figure above. Thus, the probability of being in this region is

$$Pr(r_0) = \frac{1}{2} \int_0^1 (1-b) x_1^2 dx_1 = \frac{1-b}{6}$$

Consider the triangular region r_1. The probability of being in this region is

$$Pr(r_1) = \frac{1}{2}\int_0^1 ((1 - \frac{\alpha_{11}}{1-\alpha_{13}}) - (1-b))x_1^2 dx_1 = \frac{1 - \frac{\alpha_{11}}{1-\alpha_{13}}}{6}$$

Let us rewrite revenues in those regions taking into account that the probability of being in the region depends on the area of the triangles. The difference in expected revenues is

$$\begin{aligned}\frac{1}{Pr(r_0 \cup r_1)} E[R_A - R_T | r_0 \cup r_1] \quad &\leq \quad [\underbrace{\frac{1-b}{6}(b - \frac{\alpha_{11}}{1-\alpha_{13}})}_{r_0} + \underbrace{\frac{1 - \frac{\alpha_{11}}{1-\alpha_{13}}}{6}(\overline{b} - \frac{\alpha_{11}}{1-\alpha_{13}}) - \frac{1-b}{6}(\overline{b} - \frac{\alpha_{11}}{1-\alpha_{13}})}_{r_1}]K \\ &= \quad \frac{1}{6}[(1 - \frac{\alpha_{11}}{1-\alpha_{13}})(b - \frac{\alpha_{11}}{1-\alpha_{13}})]K\end{aligned}$$

Similarly, we derive the difference in expected revenue for regions r_4 and r_5

$$\begin{aligned}\frac{1}{Pr(r_4 \cup r_5)} E[R_A - R_T | r_4 \cup r_5] \quad &\leq \quad [\underbrace{\frac{1-b}{6}(b - \underline{b}) - \frac{1 - \frac{\alpha_{11}}{1-\alpha_{12}}}{6}(b - \underline{b})}_{r_4} + \underbrace{\frac{1 - \frac{\alpha_{11}}{1-\alpha_{12}}}{6}(b - \frac{\alpha_{11}}{1-\alpha_{12}})}_{r_5}]K \\ &= \quad \frac{1}{6}[(1 - \frac{\alpha_{11}}{1-\alpha_{12}})(b - \frac{\alpha_{11}}{1-\alpha_{12}})]K\end{aligned}$$

where the weights $\overline{b}$ and $\underline{b}$ represent respectively the upper bound in $R_A|r_1$ and the lower bound in $R_T|r_4$ respectively. Precisely, the winning (and the losing) bidder is swapping into those two regions with the environment change. Therefore, we rewrite revenues as function of x_1. These substitutions characterize an upper bound for for the expected revenue difference but allow us to perform a direct comparison. Note that the existence of these bounds is sufficient to perform the calculation as they cancel out.

$\square$

Putting the two previous claims together and using the fact that $E[R_A - R_T | r_0 \cup r_2 \cup r_3 \cup r_7 \cup r_8] = 0$, we directly focus on the weights in front of the signals of $E[R_A - R_T | r_0 \cup r_1 \cup r_4 \cup r_5 \cup r_6 \cup r_9]$ and obtain as an upper bound

$$\begin{aligned}&\frac{1}{6}[4b - (2+b)(\frac{\alpha_{11}}{1-\alpha_{12}} + \frac{\alpha_{11}}{1-\alpha_{13}}) + (\frac{\alpha_{11}}{1-\alpha_{13}})^2 + (\frac{\alpha_{11}}{1-\alpha_{12}})^2]K \\ = \quad & -\frac{\alpha_{11}}{6}\frac{(\alpha_{12}-\alpha_{13})^2(1-\alpha_{11}+\alpha_{12}^2+\alpha_{13}^2)}{(-1+\alpha_{12})^2(-1+\alpha_{13})^2(-1-\alpha_{11})^2}K < 0\end{aligned}$$

We know that the weight under the Anonymous implementation is lower than the mean of the two weights under the Transparent implementation. Intuitively, the asymmetry implies more uncertainty for the non-expert player in the first round. It involves an expected revenue difference that vanishes when α_{13} tends to α_{12}.

(ii) Suppose in the following claim that $\alpha_{11} = \alpha_{22}$.[4] We use Propositions 1 and 2 to partition the space of signal realizations as a function of the identity of the winning bidder and to determine the revenue. The figure below represents such a partition for a given realization of X_3. The full lines correspond to the partition in the transparent auction. The dotted lines correspond to the partition in the anonymous auction. The dashed lines correspond to the equal realizations of signals between bidder 3 and bidder 1 (vertical) and between bidder 3 and bidder 2 (horizontal). We will concentrate in the description of the following figure on the area below the 45 degree line given that bidders 1 and 2 are symmetric. We have that

Bidder 3 wins:

In the transparent auction

Bidder 3 wins against bidder 1 in the area below the 45 degree line and for values of (x_1, x_2) such that $\alpha_{11}x_1 + (1 - \alpha_{11})x_2 < x_3$ (regions $r_1 \cup r_2 \cup r_3$ in the figure below). The auctioneer's revenue is equal to $\alpha_{11}x_1 + (1 - \alpha_{11})x_2$.

In the anonymous auction

Recalling from Corollary 1 that $a_1 > \alpha_{11}$, the corresponding area is below the 45 degree line and for values of (x_1, x_2) such that $a_1x_1 + (1 - a_1)x_2 < x_3$ (regions $r_1 \cup r_2$). The auctioneer's revenue is equal to $a_1x_1 + (1 - a_1)x_2$.

Bidder 1 wins:

In the transparent auction

Bidder 1 wins against bidder 3 when $x_2 < x_3 < \alpha_{11}x_1 + (1 - \alpha_{11})x_2$ (region r_4). The auctioneer's revenue is then x_3. Bidder 1 wins against bidder 2, when $x_1 > x_2 > x_3$ (region r_5). The auctioneer's revenue is x_2.

In the anonymous auction

Bidder 1 wins against bidder 3 when $x_2 < x_3 < a_1x_1 + (1 - a_1)x_2$ (regions $r_3 \cup r_4$). The auctioneer's revenue is x_3. Bidder 1 wins against bidder 2 when $x_1, x_2 > x_3$ and $a_1x_1+(1-a_1)x_3 > a_2x_2+(1-a_2)x_3$ (region r_5) The auctioneer's revenue is $a_2x_2+(1-a_2)x_3$.

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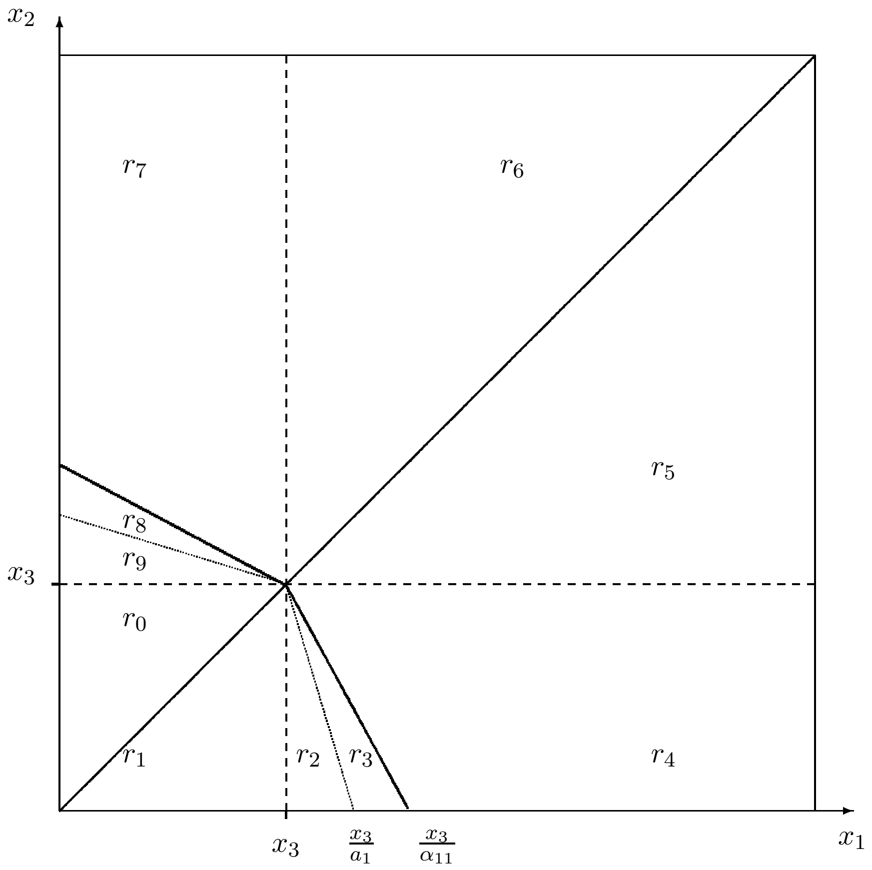
[4] Without loss of generality, the case where $\alpha_{22} > \alpha_{11}$ is provided in Appendix C.



Comparing the auctioneer's revenue region by region yields Table 6 (for x_1, x_2, x_3 in the interior of their support). Note that conditional on being in r_4 or r_7, the revenue in the Transparent and the anonymous auction is the same. Thus, to rank the expected revenue of the two auctions, we can abstract from signal realizations in r_4 and r_7.

Table 6: Revenue comparison between the two auctions when $\alpha_{11} = \alpha_{22}$*

region	transparent auction [w]		anonymous auction [w]
r_0	$(1-\alpha_{22})x_1 + \alpha_{22}x_2$ [3]	$<$	$(1-a_2)x_1 + a_2x_2$ [3]
r_1	$\alpha_{11}x_1 + (1-\alpha_{11})x_2$ [3]	$<$	$a_1x_1 + (1-a_1)x_2$ [3]
r_2	$\alpha_{11}x_1 + (1-\alpha_{11})x_2$ [3]	$<$	$a_1x_1 + (1-a_1)x_2$ [3]
r_3	$\alpha_{11}x_1 + (1-\alpha_{11})x_2$ [3]	$<$	x_3 [1]
r_4	x_3 [1]	$=$	x_3 [1]
r_5	x_2 [1]	$>$	$a_2x_2 + (1-a_2)x_3$ [1]
r_6	x_1 [2]	$>$	$a_1x_1 + (1-a_1)x_3$ [2]
r_7	x_3 [2]	$=$	x_3 [2]
r_8	$(1-\alpha_{22})x_1 + \alpha_{22}x_2$ [3]	$<$	x_3 [2]
r_9	$(1-\alpha_{22})x_1 + \alpha_{22}x_2$ [3]	$<$	$(1-a_2)x_1 + a_2x_2$ [3]

*where [w] stands for the winning bidder in the region.

In the following claim, we compare regions r_1 and r_6.

Claim 5. $E[R_A - R_T|\ (x_1, x_2, x_3) \in r_1 \cup r_6] = \frac{1}{Pr(r_1 \cup r_6)}\alpha_{12}\frac{(\alpha_{11}-\alpha_{22})}{(2\alpha_{22}-\alpha_{12})}K$ *where* $K = \int_0^1\int_0^{x_3}\int_0^{x_1}(x_1 - x_2)dx_2dx_1dx_3$

Proof: In r_1, bidder 3 wins against bidder 1. Thus

$$\begin{aligned} E[R_A|(x_1,x_2,x_3) \in r_1] &= \frac{1}{Pr(r_1)}\int_0^1\int_0^{x_3}\int_0^{x_1}(a_1x_1 + (1-a_1)x_2)dx_2dx_1dx_3 \\ E[R_T|(x_1,x_2,x_3) \in r_1] &= \frac{1}{Pr(r_1)}\int_0^1\int_0^{x_3}\int_0^{x_1}(\alpha_{11}x_1 + (1-\alpha_{11})x_2)dx_2dx_1dx_3 \end{aligned}$$

Therefore, the difference is

$$E[(R_A - R_T)|(x_1,x_2,x_3) \in r_1] = \frac{1}{Pr(r_1)}(a_1 - \alpha_{11})\int_0^1\int_0^{x_3}\int_0^{x_1}(x_1 - x_2)dx_2dx_1dx_3 \quad (7)$$

In r_6, bidder 2 wins against bidder 1. Thus:

$$\begin{aligned} E[R_A|(x_1,x_2,x_3) \in r_6] &= \frac{1}{Pr(r_6)}\int_0^1\int_0^{x_2}\int_0^{x_1}(a_1x_1 + (1-a_1)x_3)dx_3dx_1dx_2 \\ E[R_T|(x_1,x_2,x_3) \in r_6] &= \frac{1}{Pr(r_6)}\int_0^1\int_0^{x_2}\int_0^{x_1}(x_1)dx_3dx_1dx_2 \end{aligned}$$

Therefore, the difference is

$$E[(R_A - R_T)|(x_1,x_2,x_3) \in r_6] = \frac{1}{Pr(r_6)}(a_1 - 1)\int_0^1\int_0^{x_2}\int_0^{x_1}(x_1 - x_3)dx_3dx_1dx_2 \quad (8)$$

The integral expressions in (7) and (8) are identical (as long as the random variables have same support, just relabel x_2, x_3 and vice-versa). Let $K = \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2) dx_2 dx_1 dx_3$. Thus

$$\begin{aligned} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_1 \cup r_6] &= \frac{Pr(r_1)}{Pr(r_1 \cup r_6)} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_1] \\ &+ \frac{Pr(r_6)}{Pr(r_1 \cup r_6)} E[(R_A - R_T)|(x_1, x_2, x_3) \in r_6] \\ &= \frac{1}{Pr(r_1 \cup r_6)} (2a_1 - \alpha_{11} - 1) K \end{aligned}$$

To complete the proof, we simply substitute a_1 for its expression in terms of α_{ij}'s

$$\begin{aligned} (2a_1) - (1 + \alpha_{11}) &= \frac{4\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}{2\alpha_{22} - \alpha_{12}} - (1 + \alpha_{11}) \\ &= \frac{2\alpha_{22}(\alpha_{11} - 1) - \alpha_{12}(\alpha_{21} - 1 - \alpha_{11})}{(2\alpha_{22} - \alpha_{12})} \\ &= \frac{2\alpha_{22}(-\alpha_{12}) - \alpha_{12}(-\alpha_{22} - \alpha_{11})}{(2\alpha_{22} - \alpha_{12})} \\ &= \alpha_{12} \frac{(\alpha_{11} - \alpha_{22})}{(2\alpha_{22} - \alpha_{12})} \end{aligned}$$

□

This completes the proof of Claim 5 and it follows that $E[(R_A - R_T)|(x_1, x_2, x_3) \in r_1 \cup r_6] = 0$. Regions r_0 and r_5 following the same pattern as described above, we finally check the inequalities for revenues in regions r_2, r_3, r_8 and r_9 in Table 6. It immediately follows that $E[R_A] > E[R_T]$ in the symmetric case for $\alpha_{ii} \in [\frac{1}{2}, 1)$. In addition, claim 6 in Appendix C completes the proof for the asymmetric case.

Interpretation and Bidding Behavior. Proposition 3 highlights the notion of the relevance of information. In the first part of Proposition 3, the knowledge of the identity of the dropping bidder is irrelevant and does not affect the interdependent values bidders' behavior. This suggests that the relevance of information in the symmetric case can be extended to one interdependent values bidder and many symmetric private values bidders. It also suggests that an environment where the effect of revelation is neutral is limited to the case of a single interdependent values bidder.

The second part of the Proposition is the result of the effect of information. On the one hand, information about dropping bidders' identities helps interdependent values bidders to evaluate the value of the object. On the other hand, information works through a change in the resulting allocation. This is absent in the symmetric Milgrom and Weber (1982) model. In the standard common values model, revealing information reduces the impact of the private information on the bidding behavior. In our case, when an interdependent values bidder learns that the signal of the dropping bidder has no impact on her valuation, this bidder will place more weight on her private information (e.g. compare $p_1^T(x_1, (\{1,2\}, e))$ and $p_1^T(x_1, (\{1,3\}, e))$ with $p_1^A(x_1, (2, e))$).

In practice, the proof of Proposition 3 shows that the overall effect favors Anonymity (see Table 6, regions r_0, r_1, r_2, r_5, r_6, r_9). In addition, the information revealed can also change the identity of the winner, a phenomenon that never happens in the symmetric case (regions r_3 and r_8 in Table 6). In our setting, this effect further increases the revenue.

Corollary 3. *In the fringe of one-time bidders case, interdependent values bidders and the auctioneer prefer to hide the identities of dropping bidders. The private values bidder prefers the identities to be revealed.*

Proof: From Proposition 3, the auctioneer prefers the anonymous auction. From Table 6, interdependent values bidders win more often under Anonymity (region r_3 and r_8) and when they win, they pay less than under Transparency (regions r_5 and r_6). Because the anonymous auction is inefficient ex-post, it must be that private values bidders are worse off. $\square$

Corollary 3 emphasizes the benefit of being a regular bidder as opposed to being a one-time bidder. Both the auctioneer and regular bidders prefer the anonymous auction which provides an incentive for both parties to collude on such a design. To summarize, the experts and non-expert environment result in the fact that Proposition 3 remains valid when the non-expert trusts one expert more. In that case, the non-expert bidder faces no uncertainty on the type of bidder she will be facing in the last round of the auction. For the fringe of one-time bidder environment, the difference in expected revenue can be interpreted as a monotonic decreasing function in the difference of signals. More precisely, the more regular bidders are similar the larger the difference in expected revenue since the part of private information is largest when regular bidders are symmetric and the weight they put on private information remains higher.

5 Conclusion

In this paper, we consider a model of an ascending price auction with asymmetric bidders and interdependent values. An important technical contribution is that we derive the unique subgame perfect ex-post equilibrium of our model and show that revealing or not the identity of a dropping bidder has an impact on revenue and should taken into account by the auctioneer. The sole constraint related to this derivation is the use of a uniform distribution of signals. Our results suggest the classes of environments in which it is in the interest of the auctioneer to hide the identity of the dropping bidder. The key mechanism behind this result is that more information does not mean that bidders place less weight on their private signals when bidders are asymmetric and have interdependent values.

In terms of policy implications, our proof shows that more interdependence makes the revenue higher under Anonymity. This is useful for an auctioneer who does not perfectly know the environment and the type of those who bid. It is profitable to him to encourage this interdependence. This is probably what most important salesrooms intuitively do by organizing previews of important auctions, where competing bidders meet.

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A Appendix

Proof of Proposition 1 : First, we note that private values bidders have a unique weakly dominant strategy at all stages of the auction: $p_i^T(x_i, s) = x_i$ for all s and $i \in N_s$. Because $v_i - p \geq 0$ iff $p \leq p_i$, these are also the only candidate strategy for an ex-post equilibrium meeting the ex-post feature.

Then, fix s. From Lemma 1, the candidates for an ex-post equilibrium in the resulting subgame are strictly increasing in their own signal. Hence, we can invert the strategies such that $x_i(p)$ is the value of the signal for bidder i such that bidder i drops at p. The proof will be given separately for each Tables considering each possible state of the world in turn.

(i) Proof of Table 1

$s = (\{1, 2, 3\}, +\infty)$: We first consider the state of the world where no bidder has dropped out so far. Bidder 3 has a unique weakly dominant strategy $p_3(x_3, s) = x_3$. Bidders 1 and 2's expected payoff conditional on p being the final price and conditional on winning is :

$$\begin{aligned} & \alpha_{ii}x_i + \alpha_{ij}E[X_j|(\text{bidder j drops at } p)] - p \\ = \; & \alpha_{ii}x_i + \alpha_{ij}x_j(p) - p \end{aligned}$$

Thus, at equilibrium, $x_1(p)$ and $x_2(p)$ must solve the following system of equations with $x_i(p) \in [0, 1]$:

$$\begin{cases} \alpha_{11}x_1(p) + \alpha_{12}x_2(p) - p = 0 \; \forall p \\ \alpha_{22}x_2(p) + \alpha_{21}x_1(p) - p = 0 \; \forall p \end{cases} \tag{9}$$

Note that Lemma 1 ensures at equilibrium that the price coincides with the type and therefore $x_i = x_i(p)$. Rewriting $x_2(p)$ as a function of $x_1(p)$ and p from the first equation of (9), we have

$$x_2(p) = \frac{p - \alpha_{11}x_1(p)}{\alpha_{12}} \tag{10}$$

Substituting (10) in the second equation of (9) :

$$\alpha_{22}\left(\frac{p - \alpha_{11}x_1(p)}{\alpha_{12}}\right) + \alpha_{21}x_1(p) - p = 0$$

Rearranging the equation,

$$p = \frac{(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}{(\alpha_{22} - \alpha_{12})}x_1(p)$$

Using the fact that $\sum\limits_{j=1}^{n} \alpha_{ij} = 1$, we get $p = x_i(p)$

$$p_1^T(x_1, (\{1, 2, 3\}, +\infty)) = x_1$$

Likewise, $p = x_2$. These are the unique solution for equations (9) and (10).

$s = (\{1,2\}, e)$: Second, consider the state of the world where bidder 3 has dropped out first. Because bidder 3's signal does not affect bidders 1 and 2's valuation, $x_1(p)$ and $x_2(p)$ must solve the same system of equations (9). Thus

$$p_i^T(x_i, (\{1,2\}, e)) = x_i$$

constitutes the unique equilibrium strategies.
$s = (\{1,3\}, e)$: Third, we consider state of the world $(\{1,3\}, e)$ where bidder 2 has dropped at price e. Given the equilibrium strategies in $s = (\{1,2,3\}, e)$ bidder 1 can infer that $x_2 = e$. Thus $v_1 = \alpha_{11}x_1 + \alpha_{12}e$. Her best response is to set $p_1^T(x_1, (\{1,3\}, e)) = \alpha_{11}x_1 + (1-\alpha_{11})e$ which constitutes the unique ex-post equilibrium strategy of this subgame. The argument for $p_2^T(x_2, (\{2,3\}, e))$ is similar.

(ii) Proof of Table 2

$s = (\{1,2,3\}, +\infty)$: Bidders 2 and 3 have a unique weakly dominant strategy which constitutes the only candidate for an ex-post equilibrium as this strategy is the only one to meet the ex-post requirement. Thus, the best response of bidder 1 is

$$E[v_i|s, i \text{ wins at price } p] - p = \alpha_{11}x_1(p) + \alpha_{12}p + \alpha_{13}p - p = 0$$

Using the fact that $\sum_{j=1}^{n} \alpha_{ij} = 1$, we get $p_i^T = x_i$.
$s = (\{1,2\}, e)$ and $s = (\{1,3\}, e)$: Given that private values bidders have a dominant strategy, we focus on the strategies of bidder 1. Consider state of the world $(\{1,2\}, e)$ that is bidder 3 has dropped at price e. Again, we show that $p_1^T(x_1, (\{1,2\}, e)) = \frac{\alpha_{11}}{1-\alpha_{12}}x_1 + \frac{\alpha_{13}}{1-\alpha_{12}}e$ is a best response if bidder 2 adopts strategy $p_2^T(x_2, (\{1,2\}, e)) = x_2$.[5] Once again, we have no uncertainty on the dropping bidder and bidder 1's expected payoff if the final price is p and conditional on winning is given by solving the following system of equations :

$$\left\{ \begin{array}{r} \alpha_{11}x_1(p) + \alpha_{12}x_2(p) + \alpha_{13}e - p = 0 \\ x_2(p) - p = 0 \end{array} \right.$$

Therefore, her best response is to set $p_1^T(x_1, (\{1,2\}, e)) = \frac{\alpha_{11}}{1-\alpha_{12}}x_1 + \frac{\alpha_{13}}{1-\alpha_{12}}e$. It constitutes the unique ex-post equilibrium of these subgames. The argument for $p_1^T(x_1, (\{1,3\}, e))$ is similar.

$\square$

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[5] remember that bidder 2 is private values in this environment.

B Appendix

Proof of Proposition 2 : Dropping when $p_i^A(x_i, .) = x_i$ is the unique weakly dominant strategy for private values bidders. Moreover, the exact format of the auction is irrelevant as long as all bidders are in, so the equilibrium strategies when the state of the world is $(3, +\infty)$ are as in the transparent auction. For the remaining case $s = (2, e)$, the proof will be divided for both Tables considering this state of the world. *(i) Proof of Table 3*
$\underline{s = (2, e)}$: We prove that $p_1^A(x_1, (2, e)) = a_1 x_1 + (1 - a_1)e$. Given that X_2 and X_3 are identically distributed and given the strategies in the first stage of the auction, bidder 1 assigns equal probabilities to the event that bidder 2 or bidder 3 is the dropping bidder. Thus, if p is the final price, the expected valuation of interdependent values bidders, given the dropping strategies of all bidders, is given by

$$\begin{aligned} \pi_i(p, x, s|s = (2, e)) &= \frac{1}{2}\left(\alpha_{ii} x_i + \alpha_{ij} E[X_j|(\text{bidder j drops at } p)] + \alpha_{ii} x_i + \alpha_{ij} e\right) - p \\ &= \alpha_{ii} x_i + \frac{1}{2}\alpha_{ij}\left(x_j(p) + e\right) - p \end{aligned}$$

We then arrive at a system of equations of the following form

$$\begin{cases} x_1(p) = \frac{p}{\alpha_{11}} - \frac{\alpha_{12}}{2\alpha_{11}}\left(x_2(p) + e\right) \\ x_2(p) = \frac{p}{\alpha_{22}} - \frac{\alpha_{21}}{2\alpha_{22}}\left(x_1(p) + e\right) \end{cases}$$

By substitution, we obtain for bidder 1

$$x_1(p) = \frac{p}{\alpha_{11}} - \frac{\alpha_{12}}{2\alpha_{11}}\left(x_2(p) + e\right) = \frac{p}{\alpha_{11}} - \frac{\alpha_{12}}{2\alpha_{11}}\left(\frac{p}{\alpha_{22}} - \frac{\alpha_{21}}{2\alpha_{22}}\left(x_1(p) + e\right) + e\right)$$

Solving for p, we have

$$p = \frac{4\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}{4\alpha_{22} - 2\alpha_{12}} x_1 + \frac{\alpha_{12}(2\alpha_{22} - \alpha_{21})}{4\alpha_{22} - 2\alpha_{12}} e$$

define $\frac{4\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}{4\alpha_{22} - 2\alpha_{12}} = a_1$. Note that $1 - a_1 = \frac{\alpha_{12}(2\alpha_{22} - \alpha_{21})}{4\alpha_{22} - 2\alpha_{12}}$. Thus $p_1^A(x_1, (2, e)) = a_1 x_1 + (1 - a_1)e$ constitutes the unique ex-post equilibrium. Moreover this strategy involves no regret. In addition, the case for bidder 2 is symmetric. *(ii) Proof of Table 4*
$\underline{s = (2, e)}$: We prove that $p_1^A(x_1, (2, e)) = bx_1 + (1 - b)e$. Given that X_2 and X_3 are identically distributed and given the strategies in the first stage of the auction, bidder 1 assigns equal probabilities to the event that bidder 2 or bidder 3 is the dropping bidder. Thus, her expected payoff in equation (4), given the dropping strategies of private values

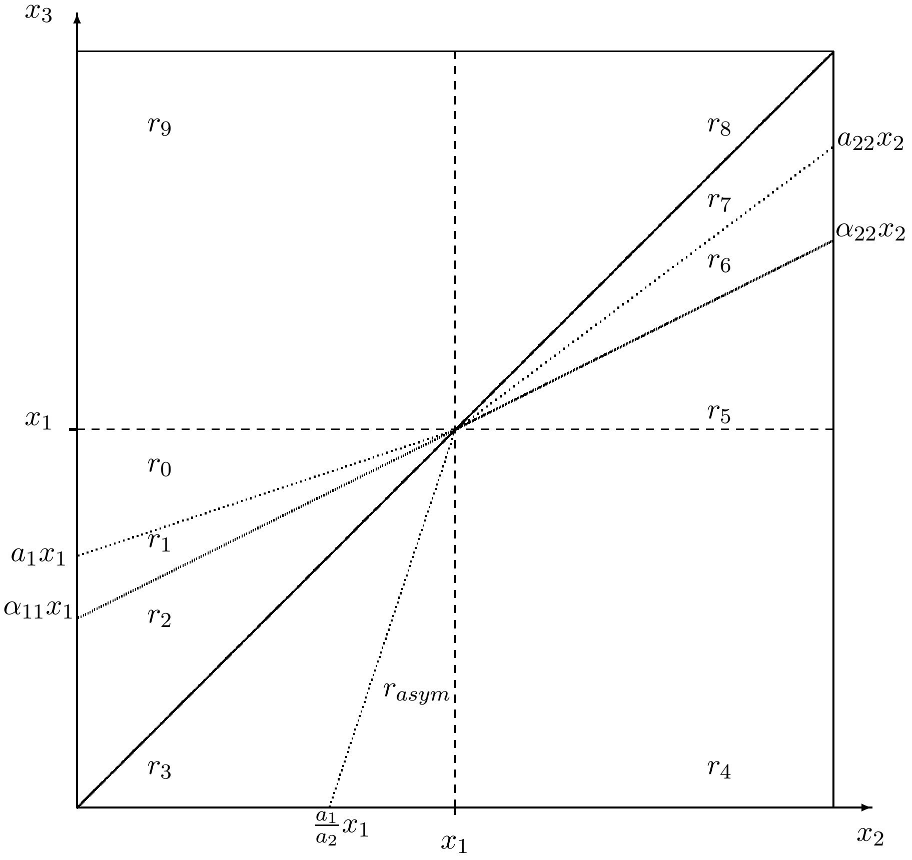
bidder 2 and bidder 3, is given by

$$\begin{aligned}\pi_1(p,x,s|s=(2,e)) &= \frac{1}{2}(\alpha_{11}x_1+\alpha_{12}E[X_2|(\text{bidder 2 drops at } p)]+\alpha_{13}e \\ &+ \alpha_{11}x_1+\alpha_{12}e+\alpha_{13}E[X_3|(\text{bidder 3 drops at } p)])-p \\ &= \alpha_{11}x_1+\frac{1}{2}\alpha_{12}\left(p+e\right)+\frac{1}{2}\alpha_{13}\left(p+e\right)-p \\ &= \alpha_{11}x_1+\frac{\alpha_{12}+\alpha_{13}}{2}e-\frac{2-(\alpha_{12}+\alpha_{13})}{2}p\end{aligned}$$

and $\frac{2\alpha_{11}}{2-(\alpha_{12}+\alpha_{13})}=\frac{2\alpha_{11}}{(1+\alpha_{11})}=b$. Thus, a best response for bidder 1 is $p_1^A(x_1,(2,e))=bx_1+(1-b)e$, the unique ex-post equilibrium and this strategy implies no regret. ☐

C Appendix

Without loss of generality, we consider the case where $1 > \alpha_{22} > \alpha_{11}$. We use Propositions 1 and 2 to partition the space of signal realizations as a function of the identity of the winning bidder and to determine the revenue. The space (x_2, x_3) allows us to consider the results in Claim 3 and Claim 4 in order to ease the understanding of this proof.



Note that if $\alpha_{11} = \alpha_{22}$, r_{asym} does not exist. Thus, in the Figure above, the region r_{asym} appears since interdependent values bidders are not symmetric anymore. Comparing the auctioneer's revenue region by region yields the following for $(x_1, x_2, x_3$ in the interior of their support):

Table 7: Revenue comparison between the two auctions when $1 > \alpha_{22} > \alpha_{11}$

region	R_T [w]	$\alpha_{11} = \alpha_{22}$	$\alpha_{22} > \alpha_{11}$	R_A [w]
r_0	$\alpha_{11}x_1 + (1 - \alpha_{11})x_2$ [3]	$<$	$<$	$a_1x_1 + (1 - a_1)x_2$ [3]
r_1	$\alpha_{11}x_1 + (1 - \alpha_{11})x_2$ [3]	$<$	$<$	x_3 [1]
r_2	x_3 [1]	$=$	$=$	x_3 [1]
r_3	x_2 [1]	$>$	$>$	$a_2x_2 + (1 - a_2)x_3$ [1]
r_{asym}	x_2 [1]	n/a	$>$	$a_1x_1 + (1 - a_1)x_3$ [2]
r_4	x_1 [2]	$>$	$>$	$a_1x_1 + (1 - a_1)x_3$ [2]
r_5	x_3 [2]	$=$	$=$	x_3 [2]
r_6	$\alpha_{22}x_2 + (1 - \alpha_{22})x_1$ [3]	$<$	$<$	x_3 [2]
r_7	$\alpha_{22}x_2 + (1 - \alpha_{22})x_1$ [3]	$<$	$<$	$a_2x_2 + (1 - a_2)x_1$ [3]
r_8	$\alpha_{22}x_2 + (1 - \alpha_{22})x_1$ [3]	$<$	$<$	$a_2x_2 + (1 - a_2)x_1$ [3]
r_9	$\alpha_{11}x_1 + (1 - \alpha_{11})x_2$ [3]	$<$	$<$	$a_1x_1 + (1 - a_1)x_2$ [3]

*where [w] stands for the winning bidder in the region.

Claim 6. *Consider $\alpha_{22} \geq \alpha_{11}$. Then $E[R_A - R_T] > 0$*

Proof. Following the strategy of the proof in Claim 4, we first substitute revenues under Anonymity in several regions with lower bounds in order to perform a direct comparison. Let $K = \int_0^1 \int_0^{x_3} \int_0^{x_1} (x_1 - x_2)dx_2dx_1dx_3$. It turns out that

$$\begin{aligned}
\frac{1}{Pr(r_0 \cup r_1)} E[R_A - R_T | r_0 \cup r_1] &\geq \frac{1}{6}[(1 - a_1)(a_1 - \alpha_{11})]K \qquad (11)\\
\frac{1}{Pr(r_3 \cup r_4)} E[R_A - R_T | r_3 \cup r_4] &= \frac{1}{6}[\frac{a_1}{a_2}(a_2 - 1)(a_1 - 1)]K = \frac{1}{6}[(\frac{a_1}{a_2} - 1)]K\\
\frac{1}{Pr(r_{asym})} E[R_A - R_T | r_{asym}] &\geq \frac{1}{6}[(a_1 + a_2 - 1)(1 - \frac{a_1}{a_2})]K
\end{aligned}$$

$$\frac{1}{Pr(r_6 \cup r_7 \cup r_8 \cup r_9)} E[R_A - R_T | r_6 \cup r_7 \cup r_8 \cup r_9] \geq \frac{1}{6}[(a_2 - \alpha_{22})(a_1 - \alpha_{22}) + (2 - a_2)(a_2 - \alpha_{22}) + (a_1 - \alpha_{11})]K$$

In regions r_1, r_{asym} and r_6, the lower bounds under Anonymity are respectively $\alpha_{11}x_1 + (1 - \alpha_{11}x_2)$, $a_1x_2 + (1 - a_1)x_3$ and $a_1x_2 + (1 - a_1)x_1$. Moreover, we abstract regions r_2 and r_5 because expected revenues are the same in the two implementations. Summing and simplifying the previous equations, expected revenue difference yields

$$[(a_1 - \alpha_{11})(2 - a_1) + (a_2 - \alpha_{22})(2 - a_2 - \alpha_{22} + a_1) + (1 - \frac{a_1}{a_2})(a_1 + a_2 - 2)] \geq 0 \qquad (12)$$

Using the fact that $E[R_A - R_T | r_2 \cup r_5] = 0$, we conclude that the expected revenue under Anonymity is strictly larger than under Transparency. $\square$