ECORE
INTERNATIONAL ASSOCIATION FOR RESEARCH AND TEACHING
Economics, Finance, Operations Research, Econometrics and Statistics

++ research ++ teaching ++

ECORE DISCUSSION PAPER

2009/86

A Note on Price Competition in Product Differentiation Models

Jean GABSZEWICZ

CORE
Université catholique de Louvain

ECARES
Université Libre de Bruxelles

CORE DISCUSSION PAPER
2009/58

A note on price competition in product differentiation models

Jean J. GABSZEWICZ[1]

September 2009

Abstract

We define a two-variant model of product differentiation which, depending on the number of consumers prefering one variant to the other, provides equilibrium prices reflecting the natural valuation of these variants by the market.

[1] Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium.
E-mail: jean.gabszewicz@uclouvain.be. This author is also member of ECORE, the association between CORE and ECARES.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.

1 Introduction

In this note, we define a model of product differentiation with two-variants of the same good which, depending on the number of consumers prefering, at equal price, one variant to the other, provides equilibrium prices reflecting a natural valuation of these variants by the market. By natural valuation, we mean that these prices share the following properties: *(i)* when one variant is prefered at equal price by a larger number of consumers than the other, its price is larger at equilibrium;*(ii)* when the number of those prefering one variant to the other at equal price is equal to the number of consumers in the whole population, we obtain the equilibrium prices corresponding to a "pure" vertical product differentiation model ; *(iii)* when this number is exactly equal to a half of the consumers' population, we get the "pure" horizontal product differentiation model with equal price at equilibrium for both variants; and, finally *(iv)* when this number tends to the number of consumers in the whole population, the corresponding equilibrium prices converge to the equilibrium prices of a simple duopoly model in which all consumers prefer one variant to the other when they are sold at equal price.

In the analysis of price competition with differentiated products, it is traditional to distinguish between *horizontal* and *vertical* product differentiation (see for instance, Beath and Katsoulacos (1991). Two products are horizontally differentiated when, sold at the same price, some consumers prefer to buy one variant while the remaining consumers prefer to buy the other one. Two products are vertically differentiated when, sold at the same price, all consumers prefer to buy one variant than the other. The paradigm of horizontal differentiation corresponds to the model proposed by Hotelling (1929) to analyze spatial competition. Two firms located at the extremities of a linear market sell the same homogeneous product. If the firms quote the same price, all consumers located at the right of the middle point of this linear market buy from the firm located at the right extremity of it, while those located at its left buy from the firm located at its left extremity. Vertical product differentiation corresponds to differentiation by quality: when sold at the same price, all consumers would prefer to buy the high quality variant of a product rather than its standard counterpart. For instance, when a TV-set broadcasting black and white images has to be bought at the same price than a colour TV, all consumers would prefer to buy at that price the latter than the former.

The two concepts defined above cover extreme cases of product differentiation. In many real situations, the variants effectively offered in the industry do not correspond exactly to the conditions required to fall into one of these two categories. For instance, in the case of spatial competition, firms can be located in the linear market in such a manner that almost all consumers would prefer to buy from one of the two firms, in spite of the fact that these firms set the same price. Suppose indeed that firm 2 is located at the right extremity of the linear market while firm 1 stands very close to it. Due to transportation costs, almost all consumers buy from firm 1 when it quotes the same price as firm 2. Thus, this situation corresponds closely to the definition of vertical differentiation, even if, *sensu stricto*, it should fall into the alternative category. Similarly, it is rather uncommon that two variants of the same good can be unanimously ranked in terms of their quality attributes. In most situations, when two variants of the same good are offered to consumers,and even when a very large number of them agree on their ranking, one can almost surely find some of them who prefer the reverse ordering. Think of cars for example. One should *a priori* expect consumers' utility to increase with the size of the cockpit or the number of seats. However some of them, even few, might prefer a car with a smaller cockpit or a smaller number of seats, simply because it provides more intimacy. This is often the case when the product is identified by several attributes, with utility increasing along one of them, and possibly decreasing for some consumers along another.

In all these intermediate cases, it is difficult to assert whether these situations correspond closerly to vertical than horizontal differentiation, or *vice-versa*. However, given two variants of the same product, the number of consumers who at equal price prefer one variant to the other provides an interesting information about how the corresponding variants are differentiated. First, notice that the extreme cases of pure vertical or horizontal differentiation correspond to specific values of this number. When the *total* number of consumers prefer at equal price variant 1 (resp. variant 2) to variant 2 (resp. variant 1), we obtain the "pure" vertical product differentiation, with variant 1 (resp. 2) of higher quality than variant 2 (resp. 1). Similarly, when this number is exactly equal to a half of the consumers' population, we get the "pure" horizontal product differentiation paradigm. Then, in the intermediate cases, the number of consumers who at equal price prefer one variant to the other does not correspond to the whole population of consumers, or to half of it. However, if a strict majority ranks at equal price one variant higher than the other, it can be viewed as if the society as a whole prefers that variant. Then, it is natural to think that the market should value higher the former than the latter since it is prefered by a majority of consumers.This would require that, at equilibrium, the price of the former should exceed the price of the latter. Notice that this property generally holds in the models capturing the extreme cases of pure vertical product differentiation: most price competition models considered in the literature lead to a higher price at equilibrium for the high quality firm (see for instance Gabszewicz and Thisse (1979) or Shaked and Sutton (1982)). Similarly, in spatial competition models *à la Hotelling*, one must naturally require that equilibrium prices are equal when firms are located at the extremities of the linear market, which is indeed the case in the Hotelling's location model[1].

We provide in this note a simple model of product differentiation, parametrized by the number of consumers prefering one variant to the other at equal price. The corresponding equilibrium prices share all the properties listed above for a natural market valuation of the variants.

[1]In this model, equilibrium prices are also equal when firms are located symmetrically around the center of the linear market : at such pairs of location, an equal number of consumers prefer one firm to the other, and *vice-versa*.

The model is introduced in section 1 while the equilibrium analysis is provided in section 2. We end up with a short conclusion.

2 The model

Two firms sell each a variant of the same good, produced at zero cost. There are two types of consumers. Consumers of type 1 prefer variant 1 to variant 2 at equal price and, similarly, those of type 2 prefer variant 2 to variant 1 under the same condition. Consumers make mutually exclusive single unit purchases. Consumers of each type are ranked in $[0, 1]$ according to the amount θ they would be willing to pay to accept to consume their least prefered variant when both these variants are sold at the same price. .Consumers of type 1 are uniformly distributed on $T_1 = [0, 1]$, with density equal to μ , and similarly for consumers of type 2 distributed on T_2 =$[0, 1]$, with density $1 - \mu$. Utility $U_1^2(\theta)$ for good 2 for consumers of type $\theta \in T_1$ defines as

$$U_1^2(\theta) = U - (\theta + p_2),$$

with U a constant sufficiently large to guarantee that at equilibrium, all the market is served by one or the other variant. The utility for good 1 $U_1^1(\theta)$ for consumers in T_1 is given by

$$U_1^1(\theta) = U - p_1.$$

Utility for good 2 $U_2^2(\theta)$ for consumers of type $\theta \in T_2$ defines as

$$U_2^2(\theta) = U - p_2,$$

while utility for good 1 $U_2^1(\theta)$ for consumers in $\theta \in T_2$ is given by

$$U_2^1(\theta) = U - (\theta + p_1).$$

3 Equilibrium analysis

Suppose $\mu < \frac{1}{2}$.Then no consumer in T_1 buys variant 2 when $p_2 > p_1$ while there exists a type of consumer $\theta(p_1, p_2)$ in $[0, 1] = T_2$ who is indifferent between the two variants, namely, $\theta(p_1, p_2)$ defined by the equality

$$U - p_2 = U - (\theta + p_1),$$

or

$$\theta(p_1, p_2) = p_2 - p_1.$$

It follows that demands to the firms at prices (p_1, p_2), with $p_2 > p_1$, are defined by

$$\begin{aligned} D_1(p_1, p_2) &= \mu + (1-\mu)\theta(p_1, p_2) \\ &= \mu + (1-\mu)(p_2 - p_1) \end{aligned}$$

for firm 1, and

$$\begin{aligned} D_2(p_1, p_2) &= (1 - \theta(p_1, p_2))\,(1-\mu) \\ &= (1 - (p_2 - p_1))(1-\mu). \end{aligned}$$

for firm 2, respectively. The resulting profits are concave and given by

$$\pi_1(p_1, p_2) = (\mu + (1-\mu)(p_2 - p_1))\,p_1$$

and

$$\pi_2(p_1, p_2) = ((1 - (p_2 - p_1))(1-\mu))\,p_2.$$

The first order conditions obtain as

$$\frac{\partial}{\partial p_1}\left((\mu + (1-\mu)(p_2 - p_1))\,p_1\right) = \mu - 2p_1 + p_2 + 2\mu p_1 - \mu p_2 = 0,$$

$$\frac{\partial}{\partial p_2}\left(((1 - (p_2 - p_1))(1-\mu))\,p_2\right) = p_1 - \mu - 2p_2 - \mu p_1 + 2\mu p_2 + 1 = 0.$$

Accordingly, the price equilibrium is given by

$$p_1^* = \frac{\mu + 1}{3(1-\mu)}, p_2^* = \frac{2-\mu}{3(1-\mu)},$$

First we have assumed that $p_1 < p_2$. That this condition holds at the candidate equilibrium follows from the direct comparison of equilibrium prices, namely, $p_1^*(\mu) = \frac{\mu+1}{3(1-\mu)} < \frac{2-\mu}{3(1-\mu)} = p_2^*(\mu) \Leftrightarrow \mu \in (0, \frac{1}{2})$.

Now let us consider this model "at the limit", when μ is exactly equal to zero. The system of demands then reduces to

$$\begin{aligned} D_1(p_1, p_2) &= p_2 - p_1 \\ D_2(p_1, p_2) &= 1 - (p_2 - p_1), \end{aligned}$$

with corresponding profits

$$\begin{aligned} \pi_1(p_1, p_2) &= (p_2 - p_1)p_1 \\ \pi_2(p_1, p_2) &= (1 - (p_2 - p_1))p_2. \end{aligned}$$

This system of demands corresponds to the limit model of vertical differentiation, in which everybody prefers good 2 to good 1 ($\mu = 0$).The corresponding price equilibrium is easily derived from the first order conditions, namely,

$$p_1^* = \frac{1}{3}, p_2^* = \frac{2}{3}.$$

We notice that $\lim_{\mu\to 0}\{p_1^*(\mu)\} = \frac{1}{3}$ and $\lim_{\mu\to 0}\{p_2^*(\mu)\} = \frac{2}{3}$: when μ tends to zero, the model gets closer and closer to a situation of vertical differentiation, in which a larger and larger majority prefers variant 2 to variant 1 ($1-\mu$ tends to 1), and the corresponding equilibrium prices converge to the equilibrium prices in the limit model.

Finally, notice that when $\mu = \frac{1}{2}$, equilibrium prices are equal to each other and equal to one.

It is easy to check that the equilibrium analysis covering the case when $\frac{1}{2} < \mu < 1$ is, *mutatis mutandis*, identical to the preceding one: firm 2 now plays the role of firm 1 in the definition of demands and profits, firm 1 selling now the variant prefered by the majority.

Thus we may summarize the above results in the following

Proposition 1 *(pure vertical differentiation model): when the set of consumers prefering variant i to variant j at equal price,$i \neq j$, coïncides with the whole population ($\mu = 0$ or $\mu = 1$), equilibrium prices are given by $p_i^* = \frac{2}{3}$, $p_j^* = \frac{1}{3}$.*

Proposition 2 *(pure horizontal differentiation model): when the number of consumers prefering variant i to variant j at equal price is exactly equal to a half of the consumers' population ($\mu = \frac{1}{2}$) , both prices are equal to 1 at equilibrium;*

Proposition 3 *(intermediate models): when the number of consumers prefering variant i to variant j at equal price satisfies $0 < \mu < 1$, (i) the variant with a smaller number of consumers who prefer it at equal price has a lower price at equilibrium than the other; (ii) the higher the number of consumers prefering one variant to the other, the higher the price at equilibrium for that variant; (iii) when $\mu \to 0$ or $\mu \to 1$, equilibrium prices in the intermediate models converge to the equilibrium prices in the pure vertical differentiation model .*

Thus we conclude that, whatever the number of consumers who prefer one variant to the other, the corresponding equilibrium prices share all the properties listed above for a natural market valuation of the variants.

4 Conclusion

We have provided in this note a simple model of product differentiation allowing a natural valuation of the variants by the market. The demand system, and the corresponding equilibrium prices, depend on the number of consumers prefering one variant to the other in the population of consumers. The analog for this model of pure vertical differentiation leads to equilibrium prices which are the limit of the equilibrium prices corresponding to situations in which the density of the population which prefers one variant to the other tends to one. Similarly, the analog of the pure horizontal product differentiation model has equilibrium prices which are equal. Finally, the larger the density prefering one variant to the other at equal price, the higher the price of that variant at equilibrium.

It would be interesting to identify, in an extended model of product differentiation, similar properties at equilibrium for the case where the number of variants exceeds two.

References

[1] Beath, John and Yannis Katsoulacos (1991). *The Economic Theory of Product Differentiation*, Cambridge U. Press, 1991.

[2] Gabszewicz, Jean .J. and Jacques Thisse (1979). Price competition, quality and income disparities. *Journal of Economic Theory, 20 (3), 340-359.*

[3] Hotelling, Harold (1929). Stability in competition. *The Economic Journal, 39 (153), 41-57.*

[4] Shaked, Avner and John Sutton (1982). Relaxing price competition through product differentiation. *The Review of Economic Studies, 49 (1), 3-13*