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Entry accommodation under multiple commitment strategies: judo economics revisited

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Abstract

We consider a stage-game where the entrant may simultaneously commit to its product's quality and the level of its production capacity before price competition takes place. We show that capacity limitation is more effective than quality reduction as a way to induce entry accomodation: the entrant tends to rely exclusively on capacity limitation in a subgame perfect equilibrium. This is so because capacity limitation drastically changes the nature of price competition by introducing local strategic substitutability whereas quality differentiation only alters the intensity of price competition.

Keywords: entry, quality, differentiation, Bertrand-Edgeworth competition.

JEL Classification: D43, L13, L51

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1 Introduction

The analysis of entry strategies is a recurring theme within Industrial Organization. Almost all textbooks devote at least one chapter to this question. The mere fact that industry dynamics relies on entry is of course sufficient to motivate this academic interest. Entry games also provide a benchmark for the analysis of industries subject to deregulation. Such games also allow a clearcut analysis of the strategic incentives firms may face in simple stage-games.

It is now common to model oligopoly competition as a two-stage game where a commitment stage is followed by market competition. More precisely, it is assumed that firms decide first on the economic environment and then compete in the resulting market.[1] This is typically the case when one considers the role of technology adoption, competition in research and development, barriers to entry or product differentiation. The interactions between these two stages are well-understood by now and are classically summarized by the "animal" taxonomy proposed by Fudenberg and Tirole (1984). Their basic idea is that the mode of competition at the market stage, summarized by the distinction between strategic complementarity and substitutability, is put in relation with the direction of the strategic commitments in terms of over or under-investment in the corresponding variable. As shown by Fudenberg and Tirole (1984), this taxonomy is particularly illuminating when one considers the scope for deterrence or accomodation in entry games.

Notice however that, to the best of our knowledge, this strand of the literature most often does not allow firms to combine commitment tools. A noticeable exception is the literature on multi-dimensional product differentiation where one could argue that each dimension of product differentiation is a particular form of commitment. It is striking then to notice that firms tend to concentrate over a single dimension of differentiation in equilibrium (see in particular Irmen and Thisse (1998)).[2] Other exceptions such as Rosenkranz (2003) and Lin and Saggi (2002) explored the links between process and product innovation commitments. Our contribution enriches previous analysis by letting firms optionally combine drastically different commitment tools before the market competition stage. In particular, we consider a game where an entrant can commit to a quality *and* a capacity level before price competition takes place.

As a matter of fact, many industries feature one or few dominant firms and a fringe of small competitors. A rationale for this can be found in Gelman and Salop (1983) who claim that in order to relax price competition and make entry profitable, an entrant can use a "stick–and–carrot" strategy. She voluntarily limits her production capacity to guarantee a large residual

[1] Obviously, one could consider more stages, but the two-stages synthesis has gained wide acceptance.

[2] Obviously, many other contributions allow for a dichotomous choice between various commitment strategies, see for example Belleflamme (2001) and establish the conditions under which one tool dominates the other. However these tools cannot be combined.

demand for the incumbent but she names a low price that would prove dear to undercut. In their discussion of possible means to achieve this credible commitment, the authors claim that "producing a product with limited consumer appeal is analogous to capacity limitation" i.e., they identify capacity limitation with inferior quality differentiation. It is indeed true that a similar strategic commitment is at work in the models of quality differentiation of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) where the entrant optimally chooses a low quality and offers a substantial rebate on her product in order to induce the incumbent not to fight too aggressively in prices. The incumbent therefore prefers to accomodate entry although it is always possible for him to exclude the entrant from the market.

Our starting point then is to combine these two approaches by allowing the entrant to pick both a product quality and a production capacity. The question we raise is the following: does the entrant use product differentiation and capacity precommitment simultaneously? In other words, are capacity and quality choice *substitutes* or *complements* in softening price competition?

Our first result provides a negative answer to the above question. We show indeed in Proposition 1 that under efficient rationing, the entrant will typically choose quality imitation coupled with an optimal capacity limitation in a subgame perfect equilibrium. Although we cannot rule out the existence of subgame perfect equilibria where some differentiation prevails, we show that the entrant's payoff is bounded from above by the payoffs prevailing in the no-differentiation equilibrium. Capacity commitment thereofre seems to dominate quality differentiation. This result is established in a very specific model and may not be robust to perturbations. However, while establishing this result, we also shed a new light on two crucial components of the analysis of oligopoly competition.

First, we offer original developments for the analysis of Bertrand-Edgeworth competition games under product differentiation. Even though real life industries are most often characterized by firms selling differentiated products and facing various forms of quantitative constraints (at least in the short-run), the class of corresponding pricing games is probably the most understudied theoretical problem in Industrial Organization. While the case of homogeneous product has received much attention, in particular after Kreps and Scheinkman (1983), the case of differentiated products has been almost completely ignored. From a theoretical point of view, this is unfortunate: the robustness of virtually all oligopoly pricing models in which firms sell differentiated products is actually limited to those cases where marginal cost is constant. The analysis of Bertrand-Edgeworth games with product differentiation calls for a very specific analysis, some premices of which are laid out in this paper.

Lastly, our analysis sheds light on the analysis of commitment strategies. In particular we argue in the last section of the paper that the two instruments we consider, capacity limitation and product differentiation, display qualitatively different implications for the ensuing pricing game. These differences may provide a useful basis for a taxonomy of commitment strategies which is complementary to Fudenberg and Tirole (1984)'s "animal" one.

Preliminaries are developed in the next section. Section 3 is devoted to the equilibrium analysis while the last section discusses the implications of our analysis.

2 Preliminaries

2.1 The model

We follow Mussa and Rosen (1978) and (Tirole, 1988, sec. 2.1) to model quality differentiation. A consumer with personal characteristic x is willing to pay xs for one unit of quality s and nothing more for additional units. He maximizes surplus and when indifferent between two products, select his purchase randomly. Types are uniformly distributed in $[0; 1]$ and the mass of consumers is normalized to 1.

In agreement with most observed real cases, the incumbent is committed to the best available quality (normalized to unity) before entrants get an opportunity to pick their own, without however the ability to leapfrog him. We also assume that quality is not costly for firms[3] and that the marginal cost of production is nil (up to the capacity limit and equal to $+\infty$ otherwise). These considerations lead us to study the following stage game[4] G:

- At $t = 0$, an incumbent i enters the market and selects the top quality $s_i = 1$ and a large capacity $k_i = 1$.
- At $t = 1$, an entrant e selects its quality $s_e = s \leq 1$ and capacity $k_e = k \leq 1$.
- At $t = 2$, firms compete simultaneously in prices.

We denote $G(s, k)$ the pricing game occurring at the last stage. Our solution concept for the game G is Subgame Perfect Nash Equilibrium. Observe that two classes of price subgames might be generated by choices made at $t = 1$: either $k = 1$ and we face a standard game of vertical differentiation or $k < 1$ and we face a Bertrand-Edgeworth game with (possibly) product differentiation.

Consumers make their choice at the last stage by comparing the respective surpluses they derive when buying from the incumbent, the entrant or nobody i.e., $x - p_i$, $xs - p_e$ and 0. In the absence of differentiation ($s = 1$), demands are as in the standard Bertrand game. In the presence of differentiation ($s < 1$), it is a straightforward exercice to show that demands are

[3] An upper bound on the admissible qualities is required to ensure that firms' payoffs are bounded.

[4] Recall that Gelman and Salop (1983)'s model is of the Stackelberg type where the entrant commits to capacity and price before the incumbent is able to respond in price.

given by

$$D_i(p_i, p_e) \;=\; \begin{cases} 0 & \text{if} \quad p_e + 1 - s < p_i \\ 1 - \frac{p_i - p_e}{1-s} & \text{if} \quad \frac{p_e}{s} \leq p_i \leq p_e + (1-s) \\ 1 - \frac{p_i}{s_i} & \text{if} \quad p_i \leq \frac{p_e}{s} \end{cases} \tag{1}$$

$$D_e(p_i, p_e) \;=\; \begin{cases} 0 & \text{if} \quad p_e \geq p_i s \\ \frac{p_i - p_e}{1-s} - \frac{p_e}{s} & \text{if} \quad p_i - 1 + s \leq p_e \leq p_i s \\ 1 - \frac{p_e}{s} & \text{if} \quad p_e \leq p_i - 1 + s \end{cases} \tag{2}$$

Firms' profits in the pricing game are

$$\Pi_e(p_i, p_e) = p_e D_e(p_i, p_e) \quad \text{and} \quad \Pi_i(p_i, p_e) = p_i D_i(p_i, p_e) \tag{3}$$

When capacity is not an issue ($k = 1$) and products are differentiated ($s < 1$), Choi and Shin (1992) show that firms best replies are continuous and given by:

$$\phi_i(p_e) \;=\; \begin{cases} \frac{p_e + 1 - s}{2} & \text{if} \quad p_e \leq \frac{1-s}{2-s} s \\ \frac{p_e}{s} & \text{if} \quad \frac{1-s}{2-s} s \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{if} \quad p_e \geq \frac{s}{2} \end{cases} \tag{4}$$

$$\phi_e(p_i) \;=\; \begin{cases} \frac{p_i s}{2} & \text{if} \quad p_i \leq \frac{2(1-s)}{2-s} \\ p_i - 1 + s & \text{if} \quad \frac{2(1-s)}{2-s} \leq p_i \leq 1 - \frac{s}{2} \\ \frac{s}{2} & \text{if} \quad p_i \geq 1 - \frac{s}{2} \end{cases} \tag{5}$$

These best replies are displayed on Figure 1 and the equilibrium is summarized in Lemma 1 below.

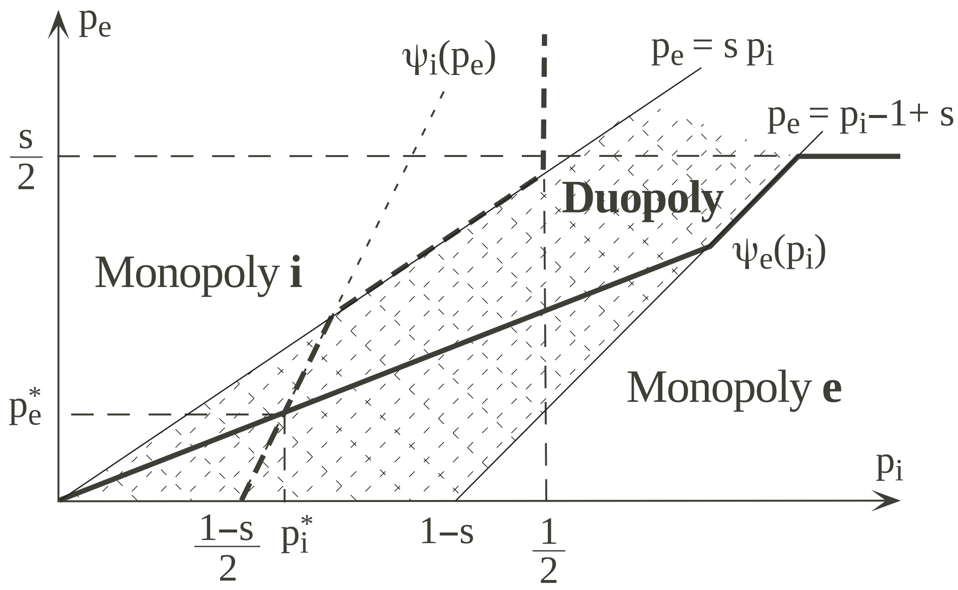

Figure 1: The price space with unlimited capacity

Lemma 1 *For $s < 1$, the game $G(s, 1)$ has a unique pure strategy equilibrium:*

$$p_i^* = \frac{2(1-s)}{4-s} \text{ and } p_e^* = \frac{s(1-s)}{4-s} \tag{6}$$

Plugging (6) into (3), we obtain the entrant's first stage payoff as a function of his quality: $\Pi_e = \frac{s(1-s)}{(4-s)^2}$. Straightforward computations yield the following corollary.

Corollary 1 *The optimal quality for the entrant in the class of pricing games $\{G(s, 1), s < 1\}$ is $s^* = \frac{4}{7}$, yielding the profit $\pi_e^* = \frac{1}{48}$.*

Notice that the pricing game $G(1, 1)$ is a classical Bertrand game with linear demand $D(p) = 1 - p$. In case of a price tie, demand is shared equally by the two firms.

2.2 Sales Functions in the presence of rationing

Whenever the entrant has unlimited capacity ($k = 1$), sales are equal to demand as characterized by equations (1) and (2). However, if the entrant has built a limited capacity ($k < 1$), there are prices leading up to more demand than can be served i.e., $D_e(p_e, p_i) > k$. In such cases, some consumers will be rationed and possibly report their purchase on the incumbent. In order to characterize firms' sales in that situation, we assume *efficient rationing*: rationed consumers are those exhibiting the lowest willingness to pay for the good. The limited k units sold by the entrant will be contested by potential buyers,[5] the price p_e paid for them will rise to the level ρ_e where the excess demand vanishes. In the case of duopoly competition, we solve

$$D_e(\rho_e, p_i) = \frac{p_i - \rho_e}{1-s} - \frac{\rho_e}{s} > k \Leftrightarrow p_e < \rho_e \equiv (p_i - k(1-s))s \tag{7}$$

while in the case of monopoly,

$$D_e(p_e, p_i) = 1 - \frac{p_e}{s} > k \Leftrightarrow p_e < s(1-k) \tag{8}$$

Using (7) and (8), the entrant is capacity constrained i.e., $S_e(p_e, p_i) = k$ whenever

$$p_e \leq \min\{\rho_e, s(1-k)\} \tag{9}$$

Now, using (1), we obtain the residual demand addressed to the incumbent firm as

$$D_i^r(p_i) \equiv 1 - ks - p_i. \tag{10}$$

[5]We implicitely assume that a secondary market opens where consumers may take advantage of the arbitrage possibilities at no cost.

The expressions for the sales functions are therefore:

$$S_e(p_i,p_e) = \begin{cases} 0 & \text{if } \; p_e \geq p_i s & (a) \\ \frac{p_i-p_e}{1-s} - \frac{p_e}{s} & \text{if } \; p_e \in [\max\{p_i-(1-s),\rho_e\};p_i s] & (b) \\ 1-\frac{p_e}{s} & \text{if } \; p_e \in [s(1-k);p_i-(1-s)] & (c) \\ k & \text{if } \; p_e \leq \min\{\rho_e, s(1-k)\} & (d) \end{cases} \quad (11)$$

$$S_i(p_i,p_e) = \begin{cases} 0 & \text{if } \; p_i \geq p_e+1-s & (a) \\ 1-ks-p_i & \text{if } \; p_i \in \left[\frac{p_e}{s}+k(1-s);p_e+1-s\right] & (b) \\ 1-\frac{p_i-p_e}{1-s} & \text{if } \; p_i \in \left[\frac{p_e}{s};\frac{p_e}{s}+k(1-s)\right] & (c) \\ 1-\frac{p_i}{s_i} & \text{if } \; p_i \leq \frac{p_e}{s} & (d) \end{cases} \quad (12)$$

where branch (11:c) is void if $p_i < 1-ks$.

3 Equilibrium analysis

The analysis proceeds in three steps. First, we characterize firms' best responses in subgames $G(s,k)$. A key result of this subsection consists in showing that the presence of a capacity constraint leads to a discontinuity in the incumbent's best reponses. Second, we characterize firms' payoffs in the price equilibria of $G(s,k)$. The key result here is the following: although we do not explicitely characterize equilibrium strategies, we establish an upper bound for the entrant's payoff over the whole set of price subgames $G(s,k)$. This enables us to characterize the set of subgame perfect equilibria of G in the third section.

3.1 Price best responses

Whenever $k<1$, the analysis of $G(k,s)$ must take into account the possibility that firms sales are respectively given by equations (12:b) and (11:d) where the entrant's capacity is binding and the incumbent recovers all rationed consumers. It is immediate to see that the best the entrant can do is to sell her capacity at the highest price, which is ρ_e. On the other hand, whenever the incumbent plays along segment (12:b), he maximizes profits by trying to set $\bar{p}_i \equiv \frac{1-ks}{2}$, and, if successful, obtains a minmax profit equal to $\overline{\pi}_i \equiv \frac{(1-ks)^2}{4}$.

Given the incumbent's price p_i, the entrant's payoff function remains concave in own prices (over the domain where $D_e(.) \geq 0$). The best response function is now given by

$$BR_e(p_i,k) = \begin{cases} \frac{p_i s}{2} & \text{if } \; p_i \leq 2k(1-s) \\ \rho_e & \text{if } \; 2k(1-s) \leq p_i \leq \min\left\{1-\frac{s}{2}, 1-ks\right\} \\ p_i-1+s & \text{if } \; 1-ks \leq p_i \leq 1-\frac{s}{2} \\ \max\left\{\frac{s}{2}, s(1-k)\right\} & \text{if } \; p_i \geq \min\left\{1-\frac{s}{2},1-ks\right\} \end{cases} \quad (13)$$

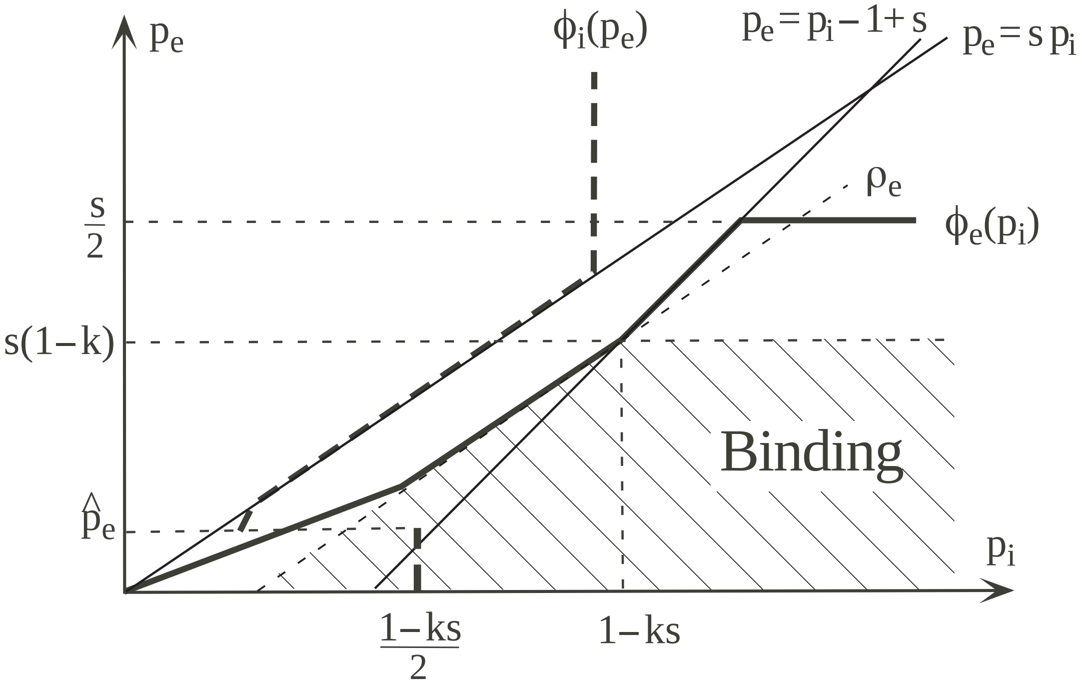

Figure 2: The price space with binding capacity

On Figure 2 we illustrate the case $k > \frac{1}{2}$ (in the other case, the third branch of (13) vanishes).

As should appear from the inspection of $S_i(p_e)$, the payoff of the incumbent is likely to be non-concave when his sales switch from segment (12:b) to (12:c). Accordingly, the best response to p_e might be non-unique. Solving $\pi_i\left(\frac{p_e+1-s}{2}, p_e\right) = \overline{\pi}_i$ for p_e, we obtain:

$$\hat{p}_e(s,k) \equiv \sqrt{1-s}\left(1 - ks - \sqrt{1-s}\right) \tag{14}$$

which is represented on Figure 2. Yet, if $\overline{\pi}_i > \pi_i\left(\frac{p_e+1-s}{2}, p_e\right)$ over the whole domain where $\phi_i(p_e) = \frac{p_e+1-s}{2}$ is defined by equation (4:a), we must compute the incumbent's payoff along segment (4:b). Solving $\frac{p_e}{s}\left(1 - \frac{p_e}{s}\right) = \overline{\pi}_i$ for p_e, we obtain:

$$\tilde{p}_e(s,k) \equiv \frac{s}{2}\left(1 - \sqrt{ks(2-ks)}\right) \tag{15}$$

Last, to know which case applies, we solve $\hat{p}_e = \tilde{p}_e$ to obtain:

$$h(s) \equiv \frac{1}{s}\left(1 - \frac{2\sqrt{1-s}}{2-s}\right) \tag{16}$$

Depending on the value of the capacity k, we might therefore obtain two different shapes for the best response of the incumbent firm in the pricing game:

- if $k \geq h(s)$, then

$$BR_i(p_e) = \begin{cases} \frac{1-ks}{2} & \text{if} \quad p_e \leq \hat{p}_e \\ \frac{p_e+1-s}{2} & \text{if} \quad \hat{p}_e < p_e \leq \frac{1-s}{2-s}s \\ \frac{p_e}{s} & \text{if} \quad \frac{1-s}{2-s}s \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{if} \quad p_e \geq \frac{s}{2} \end{cases} \tag{17}$$

- if $k \leq h(s)$, then

$$BR_i(p_e) = \begin{cases} \frac{1-ks}{2} & \text{if} \quad p_e \leq \tilde{p}_e \\ \frac{p_e}{s} & \text{if} \quad \tilde{p}_e < p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{if} \quad p_e \geq \frac{s}{2} \end{cases} \tag{18}$$

The critical values $\hat{p}_e$ and $\tilde{p}_e$ therefore identify the price level at which firm i is indifferent between naming the security price $\bar{p}_i = \frac{1-ks}{2}$ or naming a lower price which ensures a larger market share. The resulting discontinuity is likely to destroy the existence of a pure strategy equilibrium.[6]

A critical comment is in order at this step. Comparative statics analysis on the best responses functions indicate that changes in s, i.e. in the degree of product differentiation essentially affect the shape of best replies, it affects in particular the slope of the entrant's one and the position of the incumbent's. However, as long as installed capacities cannot be binding, the pricing game retains strategic complementarity as its main defining characteristic. By contrast, whenever the entrant decides to limit its capacity, the nature of the strategic interaction is altered. The incumbent's best repsonses exhibits a discontinuity, but even more importantly, for some critical level of price $\hat{p}_e$, the best response jumps *down*, i.e. at this cut-off price, the game locally exhibits strategic substitutability. When choosing the level of installed capacity, the entrant is actually choosing the highest price level for which the incumbent is willing to really accommodate entry, in the sense of "being soft" in the pricing game. Unsurprisingly, the qualitative difference between the product differentiation device and the capacity one, as exemplified by their effect on price best responses will play a crucial role in determining subgame perfect equilibrium strategies.

3.2 Price Equilibrium

We analyze the Nash equilibria for each price subgame $G(s,k)$. Let us first deal with imitation whereby the entrant chooses top quality ($s = 1$). In this case, the vertical differentiation model degenerates into a Bertrand-Edgeworth competition for a homogenous product. Levitan

[6] In order to avoid any misunderstanding, let us stress that it is only the existence of a *pure* strategy equilibrium which is problematic here. Since payoffs are continuous as long as products are differentiated, the existence of a mixed strategy equilibrium is ensured by Glicksberg (1952)'s theorem (Dasgupta and Maskin (1986) is not needed).

and Shubik (1972) analyze this game under the efficient rationing hypothesis and derive the following result whose proof is given in Appendix A.[7] Notice that applying Gelman and Salop (1983)'s Stackelberg sequentiality to the current demand yields exactly the same optimal capacity (cf. Appendix B).

Lemma 2 *$G(1,k)$ has a unique price equilibrium in which the entrant earns exactly $k\tilde{p}_e(1,k)$. Furthermore the maximum of this payoff is $\pi_e^\dagger \equiv \frac{3}{4} - \frac{1}{\sqrt{2}} \simeq 0.043$ and is reached for $k^\dagger \equiv 1 - \frac{1}{\sqrt{2}} \simeq 0.293$.*

When products are differentiated and one firm faces a capacity constraint, the existence of a price equilibrium is not problematic since payoffs are continuous (cf. Footnote 6). Besides, there exists quality-capacity constellations where a pure strategy equilibrium exists. More precisely, the pure strategy equilibrium prevailing in the limiting case where $k = 1$ is preserved. Let us define $g(s) \equiv 1 - \frac{4\sqrt{1-s}}{4-s} > h(s)$.[8]

Lemma 3 *For $s < 1$, $p_i^* = \frac{2(1-s)}{4-s}$ and $p_e^* = \frac{s(1-s)}{4-s}$ is a pure strategy equilibrium of $G(s,k)$ whenever $k \geq g(s)$.*

Proof The candidate equilibrium is (p_i^*, p_i^*) characterized in Lemma 1 (cf. eq. (6) and Figure 1). The price p_i^* remains a best response to p_e^* only if $p_e^* \geq \hat{p}_e$; straightforward computations yield the condition $k \geq g(s)$ and since $g(s) > h(s)$, we check that $\hat{p}_e$ was indeed the benchmark to use. ■

Whenever $k < g(s)$, a pure strategy equilibrium fails to exist. For intermediate capacities, it is easy to identify a particular equilibrium in which the incumbent randomizes over two atoms while the entrant plays the pure strategy $\hat{p}_e$. However, there also exists a domain of small capacities where even this equilibrium fails to exist. When this is the case, both firms use non-degenerate mixed strategy in equilibrium. The equilibrium strategy used by firm $j = i, e$ in equilibrium of $G(k,s)$ is denoted F_j; the lower bound and upper bound of the support of F_j are denoted respectively by p_j^- and p_j^+.[9] With these notations in hand, we now establish a set of lemmata which allow us to identify an upper bound for the entrant's equilibrium payoffs in pricing subgames.

Lemma 4 *Let $k < g(s)$ and $s < 1$. In equilibrium of $G(k,s)$, $p_i^+ \leq \frac{1-ks}{2}$ and $p_e^+ \leq BR_e\left(\frac{1-ks}{2}\right)$.*

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[7] Since $h(1) = 1$, the relevant benchmark is $\tilde{p}_e$.

[8] Indeed, $g(s) > h(s) \Leftrightarrow 16s^2(1-s) + s^4(3+s) > 0$ which is always true (over the relevant domain $0 \leq s \leq 1$).

[9] W.l.o.g. pure (price) strategies belong to the compact $[0; v]$. A mixed strategy is $F \in \Delta$, the space of (Borel) probability measures over $[0; 1]$, its support $\Gamma(F)$ is the set of all points for which every open neighbourhood has positive measure. We then have $\underline{p} = \inf(\Gamma(F))$ and $\overline{p} = \sup(\Gamma(F))$.

Proof: The proof proceeds by iteration; Figure 2 is helpful to follow the argument. Observe firstly that $p_i^+ \leq \frac{1}{2}$, the monopoly price because at any $p_i > \frac{1}{2}$, $\pi_i(p_i, p_e)$ is decreasing in p_i, thus the average $\pi_i(p_i, F_e)$ is also decreasing in p_i which proves that such a price cannot belong to the support of F_i. Next, since $BR_e(p_i)$ is increasing and $p_i^+ \leq \frac{1}{2}$, $BR_e(\frac{1}{2})$ is the largest best reply for the entrant to consider. This means that for $p_e > BR_e(\frac{1}{2})$, $\pi_e(p_i, p_e)$ is decreasing in p_e whatever $p_i \leq \frac{1}{2}$, thus the average $\pi_e(p_e, F_i)$ is also decreasing in p_e which proves that $p_e^+ \leq BR_e(\frac{1}{2})$.

Referring to Figure 2, one observes that because $BR_i(p_e)$ for $p_e > \hat{p}_e$ and $BR_e(p_i)$ are both increasing, they cannot cross. Reiterating the previous reasoning, we can sequentially reduce the upper price played by each firm in a Nash equilibrium. This tendency to lower prices comes to a stop at $\bar{p}_i = \frac{1-ks}{2}$ because there is no reason to exclude the incumbent from putting mass on that price. We thus end up with $p_i^+ \leq \frac{1-ks}{2}$ and $p_e^+ \leq BR_e\left(\frac{1-ks}{2}\right)$. ■

Lemma 5 *Let $k < g(s)$. In equilibrium of $G(k, s)$, $p_i^+ = \frac{1-ks}{2}$ and the equilibrium payoff is the minimax $\overline{\pi}_i$.*

Proof We may check by algebra that when $k < g(s)$, it is true that $2k(1-s) < \bar{p}_i = \frac{1-ks}{2}$. This implies that $BR_e(\bar{p}_i) = \rho_e$ and by the previous lemma, that $p_e^+ \leq \rho_e$. Hence, for p_i in a neighborhood of $\bar{p}_i$, the incumbent's sales are the residual ones D_i^r so that we have $\pi_i(p_i, F_e) = p_i(1 - ks - p_i)$.

If $2k(1-s) \leq p_i^+ < \bar{p}_i$, then $\pi_i(p_i, F_e)$ is strictly increasing over $\left]p_i^+; \bar{p}_i\right[$ which implies that p_i^+ cannot be part of an equilibrium strategy for the incumbent.

If, on the contrary, $p_i^+ < 2k(1-s)$, then the previous argument does not apply because the incumbent's sales might vary. However, if this case occurs then the entrant's demand, when facing F_i, is always of the duopolistic kind without capacity constraint, hence his best reply is the pure strategy ϕ_e computed at the average of p_i. Since the pure strategy equilibrium does not exist over the present domain, the incumbent must be playing a mixed strategy and the only candidate when the entrant plays a pure strategy involves playing the security price $\bar{p}_i$, a contradiction with $p_i^+ < \bar{p}_i$.

We have thus shown that $p_i^+ = \frac{1-ks}{2}$ and since the equilibrium payoff can be computed at any price in the support of F_i, we have $\pi_i(p_i^+, F_e) = p_i^+(1 - ks - p_i^+) = \frac{(1-ks)^2}{4} = \overline{\pi}_i$. ■

Lemma 6 *Let $k < g(s)$. In equilibrium of $G(k, s)$, $p_e^- \leq \hat{p}_e$ if $k \geq h(s)$ and $p_e^- \leq \tilde{p}_e$ if $k \leq h(s)$. The entrant's equilibrium payoff is bounded from above by $k\hat{p}_e(s, k)$ if $k \geq h(s)$ and by $k\tilde{p}_e(s, k)$ if $k \leq h(s)$.*

Proof: Let us consider first the case $k < h(s)$. If $p_e^- > \tilde{p}_e$ then for any $p_i < \frac{p_e^-}{s}$, the incumbent's demand is monopolistic whatever $p_e \geq p_e^-$. Hence, $\pi_i(p_i, F_e) = p_i(1 - p_i)$ is strictly increasing, which means the lowest price of the mixed strategy F_i cannot belong to this area. We have thus shown that $p_i^- \geq \frac{p_e^-}{s}$ holds true. If $p_i^- = \frac{p_e^-}{s}$, then at p_i^-, the incumbent is a monopoly whatever $p_e \geq p_e^-$, thus $\pi_i(p_i^-, F_e) = p_i^-(1-p_i^-) = \frac{p_e^-}{s}\left(1-\frac{p_e^-}{s}\right) > \frac{\tilde{p}_e}{s}\left(1-\frac{\tilde{p}_e}{s}\right) = \frac{(1-ks)^2}{4} = \overline{\pi}_i$ by definition of $\tilde{p}_e$ and by the previous lemma. This inequality is a contradiction with p_i^- being in the support of F_i. The last case is thus $p_i^- > \frac{p_e^-}{s}$. Then, $\pi_i(p_i^-, F_e) \geq \pi_i\left(\frac{p_e^-}{s}, F_e\right)$ since p_i^- is an optimal price and $\frac{p_e^-}{s}$ is not; observing that $\pi_i\left(\frac{p_e^-}{s}, F_e\right) = \frac{p_e^-}{s}\left(1-\frac{p_e^-}{s}\right)$, the previous argument applies and we obtain again a contradiction. This proves $p_e^- > \tilde{p}_e$ is not true i.e., our claim.

The second claim is a simple consequence of the fact that the equilibrium payoff can be computed at any price in the support of F_e, hence

$$\pi_e(p_e^-, F_i) = p_e^- \int S_e(p_e^-, p_i)dF_i(p_i) \leq kp_e^- \leq k\tilde{p}_e$$

since sales are bounded by the capacity. The case for $k \geq h(s)$ is identical since the benchmarks $\tilde{p}_e$ and $\hat{p}_e$ play a symmetric role. ■

3.3 Optimal Selection of Capacity and Quality

Although we do not have a full characterization of the mixed strategy equilibrium in all possible subgames, we have derived enough to state:

Proposition 1 *An optimal quality-capacity pair is $s = 1$ and $k = k^\dagger$. Other optimal pairs necessarily satisfy $s \geq \bar{s} \equiv 2(\sqrt{2}-1) \simeq 0.83$ and $sk = k^\dagger$.*

Proof For $k < h(s)$, $\pi_e(F_e, F_i) \leq k\tilde{p}_e(s,k) = \frac{ks}{2}\left(1-\sqrt{ks(2-ks)}\right)$ which is a function of the product $x = ks$, whose maximum is reached for $x = k^\dagger$ and yields an overall maximum $\pi_e^\dagger$. It then remains to observe that this is precisely the optimal quality and the maximum entrant's payoff for $s = 1$ and $k = k^\dagger$ as shown in Lemma 2. The pair $\left(1, k^\dagger\right)$ is shown as a diamond on Figure 3. The maximum payoff over the domain $s < 1$ and $k < h(s)$ is therefore dominated by that in $G(1, k^\dagger)$.

A similar analysis applies for $s < 1$ and $h(s) \leq k \leq g(s)$. The upper bound $k\hat{p}_e(s,k) = k\sqrt{1-s}\left(1-ks-\sqrt{1-s}\right)$ reaches its maximum for $k = \frac{1-\sqrt{1-s}}{2s}$. Replacing by the optimal value and simplifying, the objective is now $\frac{\sqrt{1-s}\left(1-\sqrt{1-s}\right)^2}{4s}$. The maximum is achieved at $\bar{s}$ (previously defined) and leads to the optimal capacity $k^\dagger/\bar{s} \simeq 0.35$ and profit $\pi_e^\dagger$. The pair $\left(\bar{s}, \frac{k^\dagger}{\bar{s}}\right)$ satisfies $k = h(s)$ and is shown as a dot on Figure 3. We have thus shown that the entrant's profit for $h(s) \leq k \leq g(s)$ is lower than a function whose maximum is $\pi_e^\dagger$.

Finally, for $s < 1$ and $k \geq g(s)$, the optimum strategy is to differentiate with $s^* = \frac{4}{7}$ to earn $\pi_e^* = \frac{1}{48} \simeq 0.021 < \pi_e^\dagger \simeq 0.043$. Overall, the pair $\left(1, k^\dagger\right)$ is an optimal strategy; there might be other optimal strategies satisfying $ks = k^\dagger$ but they all give the same final payoff. ■

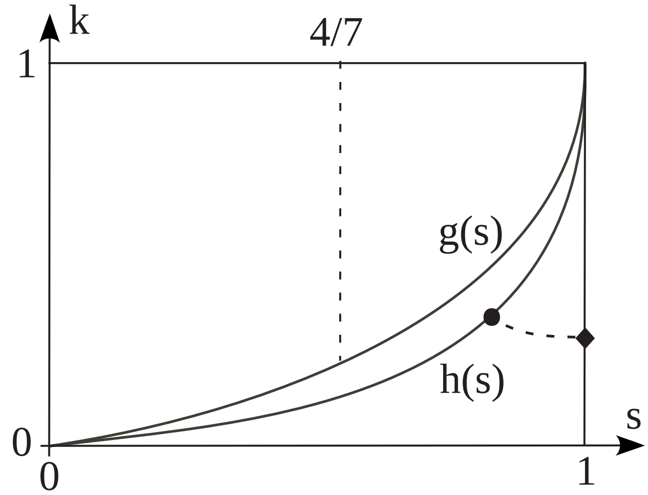

Figure 3: Strategy Space

4 Comments

Proposition 1 has been obtained in a highly stylized model. Notice however that it is also built on a set of new theoretical results pertaining to the analysis of Bertrand-Edgeworth models with product differentiation. Obviously, the efficient rationing rule and the fact that quality is not costly are instrumental in obtaining such clearcut results. It is our belief however, that our analysis actually illustrates a more general moral.

The first lesson is merely a reminder, though an important one. Oligopoly pricing games with product differentiation are almost always analyzed under the assumption of constant returns to scale. Pure strategy equilibria are then the rule. This assumption is quite restrictive: casual observation suggest that most of the time, firms install limited production capacities (and most often produce in the vicinity of these capacity limits), and sell differentiated products. The nature of equilibria in pricing games with differentiated products and (various forms of) decreasing returns to scale should be investigated further. Our present analysis suggests indeed that the presence of capacity constraints carries dramatic implications in models where product differentiation is endogenous. Wihtin the limited scope of our model, the supposedly ubiquitous "principle of maximum differentiation" does not hold! A more general analysis of Bertrand-Edgeworth games with differentiated products is definitely called for.

A second lesson pertains to the analysis of commitment strategies. It may indeed seem a priori that capacity limitation and quality differentiation are two faces of the same coin; in both cases indeed, the entrant chooses a low profile aimed at making upfront competition costly for the incumbent. As a result, the incumbent optimally chooses to accommodate entry. Our analysis reveals however that these strategies have qualitatively different implications for the market competition stage. In the original pricing game, prices are strategic complements. This property is fully preserved when a product differentiation strategy is retained. Product differentation smoothes price competition by introducing continuity in demand and by enlarging the set of prices where the market is shared by the two firms. Capacity limitation works differently by introducing two different strategic profiles into the pricing game. In our model, because of the entrant's limited capacity, the incumbent can choose to price high and somehow retreat on a protected market or to price aggressively and cover the whole market. Even though prices remain strategic complements in each of these two profiles, prices are locally strategic substitutes at the critical level of the entrant's prices where the incumbent switches from one regime to the other. In other words, capacity constraints introduce a qualitative change in the nature of strategic interaction at the market stage. To some extent, it partially changes the mode of market competition and this is instrumental in explaining why firms tend to rely exclusively on capacity limitation in order to relax price competition. In particular, under vertical differentiation, opting for a low quality level automatically implies that the potential surplus to be extracted from consumers decreases. This is costly for the entrant. The capacity limitation strategy does not have this drawback while carriyng with it the benefits of a relaxed competition.

The, now classical, animal taxonomy of Fudenberg and Tirole (1984) relies on a relationship between the nature of strategic interaction at the market stage and the direction of strategic commitments made at earlier stages. Our analysis suggests that a complementary classification of commitments could be considered. One might indeed distinguish those tools which alter the intensity of the strategic interaction from those which change its nature. In our example, quality differentiation belongs to the first class whereas capacity limitiation belongs to the second. Other examples are easy to identify. Lock-in models for instance also have this property of separating the markets into different segments for later stages, thereby allowing for strategic complementarity within segments but strategic substitutability between segments. We plan to explore this line of reasoning further in future research.

Appendix

A: Proof of Lemma 2

Let F_e and F_i be the equilibrium cumulative distributions, assuming no mass except at the end points. Due to the nature of demand, the entrant gets all demand if her price p is the lowest i.e., with probability $1 - F_i(p)$, her payoff is thus $\pi_e = p(1 - F_i(p)) \min\{k, 1-p\}$. Likewise the incumbent's is $\pi_i = p\,(1 - p - F_e(p) \min\{k, 1-p\})$. Bottom prices have to be the same because otherwise one profit would be strictly increasing in between (all prices are lesser than the monopoly one) and this would contradict the equilibrium definition.

At the common bottom price p_l, $F_i = 0$ and $1 - p_l > k$, thus $\pi_e = kp_l$. The entrant's top price cannot be greater than the incumbent's one because π_e would be zero, hence at the top price p_h, $F_e = 1$. If there was no rationing at p_h then π_i would be zero, thus $1 - p_h > k$ and

$\pi_i = p_h(1-p_h-k)$. Furthermore the right derivative must be negative to make sure than no other greater price is better, hence $p_h \geq \frac{1-k}{2}$. We also have $F_e(p) = \frac{1-p-\pi_i/p}{k}$ (recall that $1-p > k$ over the whole interval) thus the density must be $f_e(p) = \frac{1}{k}(\pi_i/p^2 - 1)$. Being positive, we derive $p^2 \leq \pi_i = p_h(1-p_h-k)$ and applying this inequality at the top price, we get $p_h \leq \frac{1-k}{2}$. Combining with the reverse inequality, we obtain $p_h = \frac{1-k}{2}$, so that $\pi_i = \frac{(1-k)^2}{4}$. Now, at the bottom price $\pi_i = p_l(1-p_l)$, thus $p_l = \frac{1}{2}\left(1-\sqrt{k(2-k)}\right)$ which is $\tilde{p}_e(1,k)$ so that $\pi_e = k\tilde{p}_e(1,k)$ as claimed. ■

B: Optimal Capacity in Judo Economics

In Gelman and Salop (1983)'s setting, the challenger enters with capacity k and commited price p_e to which the incumbent later responds with p_i. The incumbent's payoff with the aggressive price-cutting strategy is $(1-p_e)p_e$. By accommodating and serving the residual demand, his profit is $(1-k-p_i)p_i$. The optimal price is $p_i^* = \frac{1-k}{2}$ yielding profit $\frac{(1-k)^2}{4}$.

Playing on the possibility of inducing accommodation, the entrant can maximize her profit kp_e under the constraints $p_i^* > p_e$ (undercut the incumbent) and $(1-p_e)p_e \leq \frac{(1-k)^2}{4}$ (leave the incumbent happy). We obtain two conditions on the entrant's capacity that must be satisfied simultaneously i.e.,

$$k \leq \min\left\{1-2p_e, 1-2\sqrt{(1-p_e)p_e}\right\} \Leftrightarrow k \leq \min\left\{\frac{1}{2}, 1-2\sqrt{(1-p_e)p_e}\right\}$$

Since the entrant's profit is increasing with capacity, he will choose a value that saturates the constraint i.e., $k = 1-2\sqrt{(1-p_e)p_e}$. The profit of the entrant is thus $p_e\left(1-2\sqrt{(1-p_e)p_e}\right)$ and is maximum for $p_e^* = \frac{1}{2} - \frac{1}{2\sqrt{2}} \simeq 0.15$, leading to $k^* = 1 - \frac{1}{\sqrt{2}} = k^\dagger$. ■

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