A Monthly Volatility Index for the US Economy

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Abstract

We estimate the monthly volatility of the US economy from 1968 to 2006 by extending the coincident index model of Stock and Watson (1991). Our volatility index, which we call VOLINX, has four applications. First, it sheds light on the Great Moderation. VOLINX captures the decrease in the volatility in the mid-80s as well as the different episodes of stress over the sample period. In the 70s and early 80s the stagflation and the two oil crises marked the pace of the volatility whereas 09/11 is the most relevant shock after the moderation. Second, it helps to understand the economic indicators that cause volatility. While the main determinant of the coincident index is industrial production, VOLINX is mainly affected by employment and income. Third, it adapts the confidence bands of the forecasts. In and out-of-sample evaluations show that the confidence bands may differ up to 50% with respect to a model with constant variance. Last, the methodology we use permits us to estimate monthly GDP, which has conditional volatility that is partly explained by VOLINX. These applications can be used by policy makers for monitoring and surveillance of the stress of the economy.

Keywords: Great Moderation, temporal disaggregation, volatility, dynamic factor models, Kalman filter.


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1 Introduction

Timely information, say monthly, on the state of the economy is of paramount importance for economic policy decision making. This information is typically summarized by the level of the economy, measured by synthetic indexes or observed indicators, and its volatility. Given a set of monthly indicators sampled from 1968 to 2006, we extend the seminal model of Stock and Watson (1991) and we estimate a monthly index for the volatility of the US economy, which we denote by VOLINX. This index helps us to better understand the periods during which the US economy has been under stress, possible structural changes (such as the Great Moderation), determine the macroeconomic indicators that create uncertainty in the economy, and dynamically adjust the forecasts’ confidence intervals.

Additionally, the model includes quarterly GDP, generally considered as the major indicator of the state of the economy, as an additional source of information to the monthly indicators. The econometric model we rely on (state space models) and its treatment of missing of observations allows us to obtain monthly US GDP estimates -similarly to Mariano and Murasawa (2003). GDP presents time-varying volatility that is partially captured by VOLINX. These estimates can be used for intra-quarterly decision making, surveillance of the economy, and quarterly nowcast and forecast.

There is an increasing number of articles that look at using coincident indexes and high frequency measures of the state of the economy to monitor its evolution. We identify two main approaches: studies that use monthly information to construct a composite coincident indicator, and studies that disaggregate quarterly GDP. The first group of works goes back to the seminal paper of Stock and Watson (1991) (S&W henceforth). They develop a model for the coincident index by using a reduced set of well chosen economic variables that are believed to contain the most relevant information about the state of the economy. More recently, Stock and Watson (2002a) opt for a large scale dynamic factor model, in the spirit of Forni et al (2000). By working with monthly observations, this approach does not consider GDP. This issue motivates the second strand of the literature. Chow and Lin (1971) first showed how monthly GDP series could be constructed from regression estimates using GDP-related monthly data. Several authors improved on this idea by using different multivariate disaggregation methods (see Harvey and Chung, 2000, Moauro and Savio, 2005 and Proietti and Moauro, 2006). Evans (2005) presents an innovative model for estimating daily GDP. Finally, Mariano and Murasawa (2003) (M&M henceforth) combine the two approaches - disaggregation and use of the coincident index - casting S&W in a linear state space set up defined at monthly frequency and including the quarterly GDP.

The literature mentioned above ignores the stylized evidence which shows that most of the US macroeconomic series have shown a change in the volatility pattern in the last 30 years, the so-called Great Moderation. As documented by Stock and Watson (2002b), the majority of the 168 US economic variables that they analyze experience a decline in volatility, characterized by a break in the fluctuations in the mid-80s. Figure 1 is evidence of this. It shows the squared demeaned log growth rate of quarterly GDP and monthly
employment, sales, industrial production and personal income, all in logs, from 1968 to 2006. Since first differences are demeaned, their squares are a good proxy for volatility. The straight lines are the sample variances, which are also shown in the top panel of Table 1. The change in the volatility pattern is clear. The Great Moderation led to a significant decrease in growth fluctuations. For instance, the sample variance of quarterly GDP prior to 1985 was 1.263 while it dropped to 0.231 for the period 1985-2006, over 5 times less than the previous value. D'Agostino et al. (2006) found that with the great moderation the time series properties, besides the change in the volatility, of GDP have changed. Second from the top panel of Figure 1 show the sample autocorrelations or order one for the different periods. Although there are differences, in particular for industrial production, a test of equality of autocorrelations among periods would not be rejected in almost all cases.

On the basis of this, Stock and Watson (2002b) estimate univariate dynamic conditional variances for the 168 series using Stochastic Volatility (SV henceforth) models. Figure 2, which shows the observed indicators, suggests that we use the same approach. Clearly we can see that growth is conditional heteroskedastic. High and low rates (in absolute value) are clustered together. A closer look also reveals the structural break in the fluctuations of the mid-80s. The second from the bottom panel of Table 1 shows the GARCH(1,1) estimates for each indicator. The estimated variances all present persistence, as measured by the GARCH term, and they are all affected by shocks, as measured by the ARCH term. To further support this evidence, we formally test the hypothesis of the ARCH structure of the residuals in the S&W model by using the unilateral Demos and Sentana (1998) test. Results are reported in the bottom panel of Table 1. For all monthly series, the hypothesis of conditional homoscedasticity could not be accepted at 5% level. For the quarterly GDP, the null of homoscedasticity is rejected only at 10% level although the test is very close to the critical value for 5%.

Stock and Watson (2006) further develop the SV model for inflation. Based on the fact that an ARIMA(0,1,1) model can be expressed as a local level model (i.e. a random walk plus noise), they separate volatility into permanent and transitory components, and consider time-varying moving average parameters. This flexibility allows them to explain a variety of recent univariate inflation forecasting puzzles. Other researchers have gone down a similar path, analysing the fluctuations of the US economy with particular focus on inflation and monetary policy. By using different models and theoretical approaches, they all find evidence of a decrease in volatility for inflation, which reflects a change of monetary regime (Cecchetti et al., 2006, and Primiceri, 2005), a change in government policy maker models (Cogley and Sargent, 2005) or a change in policy rule coefficients (Sims and Zha, 2005).

These are the same indicators used by S&W and M&M, among others. Employment is defined as employees on non agricultural payrolls (thousand, SA). Industrial Production is defined as an index of industrial production (2002=100,SA). Sales is measured by manufacturing and trade sales (millions of chained (2000) dollars,SA). Income is defined as personal income less transfer payments (billions of dollars, SA,AR). And finally, GDP stands for real GDP (billions of chained (2000) dollars, SA, AR). The source of our data is the NBER.

Recall that the autocorrelations have variance equal to the inverse of the number of observations.
In this paper we extend S&W in two directions that allow us to estimate monthly GDP and VOLINX. We estimate the monthly GDP by disaggregating the quarterly value using the technique for state space models developed by Harvey (1989). In brief, this technique consists of augmenting the state vector to include monthly GDP as a latent process that can be estimated with the Kalman filter. A similar approach has been used by Mariano and Murasawa (2003, 2004). However they express the model in first differences and make use of the EM algorithm as a preliminary step for the smoothing and filtering processes. Our model is formulated in levels and we use the multivariate treatment of the univariate series by Koopman and Durbin (2000). We also use the initialization by de Jong (1989), which is a very convenient tool when faced with missing observations and intervention variables (such as outliers and calendar effects).

To estimate VOLINX, we rely on a new type of GARCH model. It differs from the traditional version by two aspects. First, it is a conditional volatility model for unobserved components and thus the errors are also unobserved, as pointed out by Harvey, Ruiz and Sentana (1991). Second, we replace the past square error by a linear combination of the past standardized forecasting errors of the economic indicators. The forecasting errors can be seen as the non-anticipated information, or the surprise. The weights attached to each past surprise are estimated endogenously and allow us to determine which indicators most explain the volatility of the economy. Moreover, VOLINX is the common volatility for all the indicators up to an idiosyncratic scale, which also allows us to study the contemporaneous causality between the volatility of the coincident index and the indicators.

It is worthwhile to note that these two issues, disaggregation and dynamic conditional volatility, are strongly related. We know that the lower the frequency, the more homoscedastic and the less dependent the time series becomes. In an ARMA-GARCH setting, Drost and Nijman (1993) show theoretically the link between the parameters of ARMA-GARCH models at different frequencies. The parameters that capture the dynamics are a positive function of the aggregation frequency, i.e. the lower the frequency the closer the parameters are to zero and therefore the more memoryless and the more homocedastic the process becomes.

Our results show that the US economy suffered regular episodes of stress prior to the Great Moderation, for instance during the stagflation and the oil crisis. After the mid-80s, the volatility decreased substantially. During the Clinton administration, the economy had a long period of steady growth without significant stress. Only two peaks of uncertainty show up after the Great Moderation. The first is from mid-1986 to mid-1987. This is probably due to uncertainty that the tax reform act of 1986 produced in income and corporate taxes. The second is 09/11 and the subsequent months, though the sudden increase in the uncertainty disappeared as quickly as it appeared. Although the date chosen to mark the beginning of the Great Moderation is exogenous, we compute, as a robustness check, VOLINX for an array of breaking dates, from January 1984 to December 1985. VOLINX is robust to this choice, giving similar results prior and after the Great Moderation regardless of the chosen date.

Another usefulness of VOLINX is that it helps us to understand the determinants of uncertainty in
the US economy. As previously mentioned, we use the same set of macroeconomic variables as in S&W and M&M. They are also among the most important indicators used by the NBER dating committee. By constraining ourselves to this set of macro-indicators, we can investigate if the variables that are thought to be the main drivers of the economy are also the main drivers of volatility. The answer is no. While we find that, consistent with the literature, industrial production is the most important determinant for the level of the US economy, employment and income are the most important determinants of the volatility, with industrial production taking a backseat. This leads to the policy recommendation that, in order to boost smooth growth, policy makers should increase industrial production and minimize unexpected changes in employment and income.

Monthly GDP growth presents heteroscedasticity and volatility clustering with a clear cut in the fluctuations prior and after the Great Moderation in the mid-80s. This finding is also relevant for forward-looking exercises, namely nowcasting and forecasting. By allowing for time-varying volatility, the in- and out-of-sample forecasts of GDP have confidence intervals that shrink and expand with the degree of uncertainty in the economy. We show that in-sample prior to the Great Moderation the confidence intervals are much wider (up to 50% more) with respect to the case of constant variance and, moreover, they change substantially over time. We observe a similar phenomenon after the Great Moderation, with confidence intervals reduced by around 40%. In other words, by assuming constant volatility, the model underestimates the fluctuations prior to the Great Moderation and exacerbates them after the mid-80s. Additionally, we show that out-of-sample confidence intervals also substantially differ from those of the constant volatility scenario. We perform a one-month ahead rolling forecast experiment for the periods January 1983 - December 1986 (around the Great Moderation) and January 2000 - December 2003 (around 09/11). In both cases the forecasted confidence intervals vary over time, with differences of up to 50% compared to the constant volatility case. This is a useful tool for economic policy decision makers, who may adapt their decisions based on the width of the confidence intervals.

The structure of the paper is as follows. Section 2 presents the model in simple terms, with the more technical aspects featured in the Appendix. The empirical results, the forecasting exercise and the sensitivity to the breaking date of the great moderation are detailed in Section 3. Finally, in Section 4 we present our conclusions.

2 A model for mean-variance coincident index and for monthly GDP

Let \( y_t, \) for \( t = 1, \ldots, T, \) denote an \( N \times 1 \) vector of time series, which we assume to be integrated of order one, so that \( \Delta y_{it}, i = 1, \ldots, N, \) has a stationary and invertible representation. This vector contains both the monthly macro indicators and the quarterly GDP.\(^3\) Following S&W, \( y_t \) is expressed as a linear

\(^3\)For the sake of clarity, we relegate the technical exposition of the disaggregation of quarterly GDP to the Appendix. A brief non-technical discussion is provided at the end of the Section.
combination, with $N \times 1$ loadings $\vartheta_0$ and $\vartheta_1$, of a coincident index (denoted by $\mu_t$) and $N$ idiosyncratic components (denoted by $\mu^*_t$) specific for each series

$$y_t = \theta_0 \mu_t + \theta_1 \mu_{t-1} + \mu^*_t + X_t \beta.$$  

which can be written as

$$y_t = \vartheta_0 \mu_t + \vartheta_1 \Delta \mu_{t-1} + \mu^*_t + X_t \beta,$$  

where $\vartheta_0 = \theta_0 + \theta_1$ and $\vartheta_1 = -\theta_1$. The $N \times K$ matrix $X_t$ contains $K$ exogenous variables that are used to incorporate calendar effects and intervention variables. The coincident index follows an autoregressive difference stationary process of order one:

$$(1 - \phi L) \Delta \mu_t = \eta_t,$$  

and the idiosyncratic components follow an autoregressive difference stationary process of order one with drift

$$D(L) \Delta \mu^*_t = \delta + \eta^*_t,$$  

where $\delta$ is a $N \times 1$ vector of intercepts and $D(L)$ is a diagonal polynomial matrix:

$$D(L) = \text{diag} \left[ (1 - d_1 L), (1 - d_2 L), \ldots, (1 - d_N L) \right].$$  

If the disturbances $\eta_t$ and $\eta^*_t$ are Gaussian, mutually uncorrelated at all leads and lags, the variance of $\eta_t$ is 1, and the idiosyncratic disturbances have constant variances, this model boils down to S&W. However, given the stylized facts shown in the introduction, the assumption of constant variances is not realistic.

We propose a time-varying volatility framework that assumes a common variance. We apply to the variances a decomposition similar to that of the conditional mean. The fundamental difference is that the variance decomposition is multiplicative rather than additive. The variance of the disturbance of the coincident index is heteroskedastic:

$$\eta_t \sim \text{NID}(0, \sigma^2_t),$$  

where $\sigma^2_t$ is a GARCH type of model to be defined below. This is the coincident index for the volatility, VOLINX, and it multiplies, or re-scales, the variances of the idiosyncratic disturbances:

$$\eta^*_t \sim \text{NID}(0, \sigma^2_t \Sigma^*).$$  

These disturbances are mutually uncorrelated at all leads and lags and have constant variances: $\Sigma^* = \text{diag}(\sigma^2_{\eta^*_1}, \ldots, \sigma^2_{\eta^*_N})$. The time varying factor $\sigma^2_t$ is therefore responsible for the fluctuations of the coincident index and of the idiosyncratic components.

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4We prefer the specification of the model in levels, instead of in differences. This formalization gives us a convenient way of dealing with missing observations and temporal disaggregation.

5This is more convenient representation for estimation in terms of state space models. We refer to the Appendix for further details.

6Fixed variance and equal to 1 for $\eta_t$ is a consequence of identification, which is reached by concentrating it out of the log-likelihood function. Other possibilities are feasible by rescaling all the variances. We prefer to stick to the original model presentation of S&W.
There are several possibilities for the specification of VOLINX. In light of the empirical evidence presented in the introduction, one may think of a variance mechanism that considers two regimes, prior and after the beginning of the Great Moderation, in line with Stock and Watson (2002b) and the top panel of Table 1:

$$\sigma^2_t = \omega_1 I_{[1]} + \omega_2 I_{[2]},$$  

(7)

where $I_{[1]}$ ($I_{[2]}$) is an indicator function that takes value 1 for $t$ prior (posterior) to the Great Moderation.\(^7\) This is our first model, which we will refer as "two regimes variance" model and can be considered as naive in the sense that it does not capture the conditional heteroskedasticity found in the data.

A GARCH-type model provides a finer parametrization. However, when working with unobserved components, such models become quite troublesome as the residuals are no longer equal to the difference between the observed component and the prediction but, rather, between the unobserved and its prediction.\(^8\) Harvey, Ruiz and Sentana (1991) introduce a GARCH model for unobserved components defined in a state space framework. Modified to fit our case:

$$\sigma^2_t = (1 - \alpha_0 - \alpha_1) + \alpha_0 \sigma^2_{t-1} + \alpha_1 \psi^2_t,$$  

(8)

where $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$. The second component in the RHS is standard in traditional GARCH literature and captures the persistence: the expected variance is expressed as a function of its past expectations. Since the residual is based on unobserved components, we follow Harvey, Ruiz and Sentana (1991) and replace it in the third term of the RHS with its conditional expectation, provided by the Kalman filter. The intercept term $(1 - \alpha_0 - \alpha_1)$ is such that the unconditional variance of the coincident index is 1, as in S&W. While this method is rather appealing from an econometric point of view, it lacks an economic interpretation. As an indicator of the underlying volatility that drives the economy, it should be caused, in some sense, by the past fluctuations of the macroeconomic indicators. Note however that in this model $\sigma^2_t$ explains the contemporaneous volatility of the indicators, as their conditional volatility given by $\sigma^2_t \Sigma_{\nu^\prime}$.

In order to establish the link between past fluctuations of the indicators and the volatility of the coincident index, we consider the following model:

$$\sigma^2_t = (1 - \alpha_0 - \alpha_1) + \alpha_0 \sigma^2_{t-1} + \alpha_1 \sum_{i=1}^{N} \varphi_i \nu^2_{it},$$  

(9)

where $0 \leq \alpha_0 < 1$, $\alpha_1 \geq 0$, $\varphi_i \geq 0$, and $\sum_{i=1}^{N} \varphi_i = 1$. The third term accounts for shocks and is expressed as a linear combination of past forecasting errors of the indicators, which can be seen as the non-anticipated information or surprise. Following the literature of macroeconomic news announcements and their effect on financial markets, we assume that the uncertainty of the economy increases when the growth rates of the main economic indicators vary more than expected. In other words, it is the magnitude of the

\(^7\)The choice of the breaking date is discussed later.

\(^8\)Another possibility is to opt for SV models, where the aforementioned drawback is not present. But estimation of this models is even more difficult.
forecasting error what causes the uncertainty to increase, rather than the growth rate itself. The standardized surprises for each component are gathered in the vector $\nu_{t,i}$, where each component is $\nu_{t,i} = (y_{t,i} - \mathbb{E}(y_{t,i}|y_1, \ldots, y_{t-1}))/\text{Std}(\nu_{t,i})$, and is provided by the Kalman filter. A criticism of this approach is that the non-anticipated information of the indicators is model-based although different agents in the economy may use different models to make predictions. In the empirical section we proceed with a robustness analysis of our model-based forecasting errors, substituting $\nu_{t,i}$ by the median surprise of the survey data on analyst forecasts, provided by Standard and Poor's Global Markets (MMS) and its successor Informa Global Markets.

The weights of the linear combination, estimated endogenously, have a similar interpretation to the factor loadings for the conditional mean. We can infer which are the most relevant variables that explain the volatility of the economy. Note that the unconditional variance of this model is still 1 as the sum of the weights is 1 and the standardized innovations have second moment equal to 1 as well. Moreover, if we set $\alpha_0$ and $\alpha_1$ equal to zero, we come back to S&W. This is our second model, which we will refer as "GARCH" model.

But the constraint on the unconditional variance is not, in this context, technical. Rather, it can lead to misspecification. For instance, while (9) accounts for the stylized facts of the middle section of Table 1, it cannot explain the change in level of the volatility circa 1984. The literature on the Great Moderation features extensive debates as to the date when it actually began. Stock and Watson (2002b) attempt to date the Great Moderation using different methods. They find a breaking date for most of the components series of GDP in February 1983 with a 67% confidence interval from April 1982 to March 1985.

We follow their suggestion and consider a GARCH-type model with two regimes for the unconditional variance:

$$\sigma_t^2 = \omega_1 I_{[1]} + \omega_2 I_{[2]} + \alpha_0 \sigma_{t-1}^2 + \alpha_1 \sum_{i=1}^N \phi_i \nu_{t,i}^2$$

where $\omega_1 \geq 0, \omega_2 \geq 0, 0 \leq \alpha_0 < 1, \alpha_1 \geq 0, \phi_i > 0$ for $i = 1, \ldots, N$, and $\sum_{i=1}^N \phi_i = 1$. As earlier, $I_{[1]} (I_{[2]})$ is an indicator function taking value 1 for $t$ prior (after) the Great Moderation. We consider the breaking date of January 1984, which is roughly the middle date of the confidence interval determined by Stock and Watson (2002b). In our subsequent analysis we check the robustness of the estimates and of VOLINX to changes in this date. This model combines (7) and (9), and captures the stylized facts of change in the level of volatility, a decrease in volatility after the Great Moderation occurs if $\omega_2 < \omega_1$, and conditional heteroskedasticity. We will refer to this model as "two regime GARCH".

Regardless of the choice for $\sigma_t^2$, the model (1)-(6) has a linear state space form (SSF). The unknown parameters, which for (10) are $(\theta_{0,i}, \theta_{1,i}, d_i, \delta_1, \phi, \varphi_i, \sigma_{n_i^2}, \beta_k, \omega_1, \omega_2, \alpha_0, \alpha_1)$ for $i = 1, \ldots, N$ and $k = 1, \ldots K$, can be estimated by maximum likelihood. Given the parameter values, the Kalman filter and smoother provide the minimum mean square estimates of the coincident index and the idiosyncratic components. The technical aspects are explained in detail in the Appendix and we refer the reader to Proietti
and Frale (2006) for a deeper statistical treatment. Yet, there are two issues concerning the estimation process that are worth commenting them.

First, the model mixes different frequency data, e.g. monthly indicators and quarterly GDP. Following Harvey (1989), the state vector in the SSF is suitably augmented by using an appropriately defined cumulator variable. In particular, the cumulator variable at times $t = 3\tau, \tau = 1, \ldots, \lfloor T/3 \rfloor$ coincides with the (observed) aggregated series, and otherwise contains the partial cumulative value of the aggregate in the months up to the quarter. In our analysis, the series in the model are expressed in logarithms. Thus, in order to work with a linear constraint we use the approximation of Mariano and Murasawa (2003, 2004) to disaggregate the quarterly log GDP into three unobserved monthly values by the geometric mean (see Appendix for more details). The missing observations of the cumulator are provided by the Kalman filter and smoother and the monthly estimates of the GDP are obtained by decumulation.

Second, we treat the multivariate model in terms of univariate models, similarly to Anderson and Moore (1979) and Koopman and Durbin (2000). The later show that it is a flexible and convenient method for filtering, smoothing and handling missing values. The multivariate vectors of indicators, where some elements can be missing, are stacked one on top of the other to yield a univariate time series, whose elements are processed sequentially. This allows us to include intervention variables in the model, which is especially useful when dealing with outliers. In fact, Figure 2 suggests the presence of an outlier for the series of employment in August 1983, which will be incorporated in the model through the vector $X_t$.

3 Results

We first discuss the estimated parameters under different volatility specifications. We next discuss the coincident index and VOLINX, which is interpreted in terms of the mayor economic and political events of the last decades. Next, we analyze the estimated monthly GDP and evaluate the goodness of its conditional volatility for interval prediction both in and out-of-sample. Last, we study the robustness of the index to different breaking dates for the Great Moderation.

Tables 2-5 show the estimated parameters for model (1)-(6) under different specifications for VOLINX. Estimated parameters of the conditional mean are similar under all specifications. This is expected as they are estimated under Gaussianity and hence QML theory applies. All indicators are explained contemporaneously by the coincident index and only industrial production is affected by the index one month later. Estimated $\vartheta_{0,i}$ and $\vartheta_{1,i}$ show that industrial production presents the largest loading. This confirms the general consensus that it is one of the most relevant indicators for the growth of the economy. The outlier for employment in August 1983, which appears in the top left corner of Figure 2, is significantly different

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9 Although there are more sophisticated solutions to this problem, e.g. Proietti (2006), we chose this one for its simplicity.
10 Preliminary estimations confirm that other indicators are not caused by past values of the indicator.
11 Yet, as M&M point out, these loadings are not comparable across indicators as they are measured differently. Following M&M we divide the loadings by the standard error of the corresponding indicators. Our conclusions do not change qualitatively.
from zero and the autoregressive parameters of the indicators are also consistent across models.\textsuperscript{12} However, the significance level varies across parameters. For the most complete models - VOLINX given by (10) - some of the parameters are not statistically different from zero. Finally, the autoregressive parameter of the coincident index is in all case around 0.9, indicating that the economy, at monthly level, presents a high degree of inertia.

The bottom panels of Tables 3-5 show the estimated parameters for various specifications of VOLINX other than the standard S&W model. It is intriguing that the estimated intercepts of (7) in Table 3 are not statistically different from zero. This could indicate that the model is misspecified and requires a richer structure for the variance. The estimates for the GARCH and the two regime GARCH models are all significant. The volatility of the economy is conditional on past fluctuations, as $\alpha_0$ is significant and takes values in line with what is found in time series with similar characteristics. Past shocks of the indicators also matter to explain the current uncertainty of the economy, as $\alpha_1$ is significant at 5%. The estimated weights $\varphi_i$ indicate that the magnitude of past forecasting errors of the indicators significantly contribute to explaining today’s uncertainty in the economy. Results for both models show that the past forecasting errors with the largest impact on the uncertainty of the economy are those of employment, followed by income and sales.\textsuperscript{13} On the other hand, the contemporaneous effect of VOLINX in the fluctuations of the indicators is given by the scaling factors $\sigma_{ni}^*$. In all models the ranking across indicators is the same. An increase in the uncertainty of the economy implies, above all, an increase in the volatility of income, followed by GDP and sales.

This recursive sequence (where the magnitude of the past forecasting errors cause contemporaneous VOLINX which, in turn, causes contemporaneous volatility in the indicators) leads to interesting policy implications. To limit volatile movements of the economy, policy makers should focus first on reducing the unexpected movements in employment and income, followed closely by sales. If the economy incorrectly predicts these indicators, policy makers can expect an increase in the uncertainty of income mainly but also of GDP and sales. Note that, of all the indicators, industrial production appears to play the least important role. This is in sharp contrast with the results for the conditional mean, where industrial production is the indicator most caused by the coincident index. Overall, these results indicate that in order to boost smooth economic growth, policy makers should focus on increasing industrial production and minimizing surprises.

\textsuperscript{12}The autoregressive parameter for GDP has been set to one. Preliminary estimations have show that its estimated value is very close to this. Moreover, estimation of the unconstrained model turns out to be difficult. Similar problems have been found by other authors (Proietti and Mouauro, 2006).

\textsuperscript{13}GDP surprise is not a regressor in the volatility model, as it is available only every quarter - a surprise is by definition the difference between the observation and its conditional expectation. Nevertheless, we have estimated the model including the quarterly surprise (monthly surprises that are not frequency of the quarter are treated as missing observations). Its effect on volatility is rather small (0.082 for the GARCH model and 0.012 for the two regime GARCH model). It indicates that surprises in quarterly GDP have much less impact in the volatility than employment, sales or income (their estimated coefficients don’t change significantly). Results are available under request.
As mentioned in the previous Section, a reasonable criticism of this analysis is that the non-anticipated information is based on our model, as they are given by the Kalman filter. Therefore a different model may lead to different conclusion. As to a robustness check, we estimate the model (1)-(6) and (10) where we substitute the past forecasting errors \( \nu_{t,i} \) by the surprise of the survey data on analyst forecasts, provided by Standard and Poors Global Markets (MMS) and its successor Informa Global Markets. The surprise is defined as the difference between initial announcements of the indicators (change in percentage points with respect to the previous months) and the median of the analysts’ forecasts. Because MMS only started to release survey data of employment in January 1985, we can only estimate the model from that date up to December 2005. The last row of Table 5 shows the estimated weights. Though some estimates change slightly, they are in concordance with our results. Employment and income remain the most important determinants of uncertainty, closely followed by sales and leaving industrial production to as the least important.

Finally, the level of volatility before and after the Great Moderation, measured by \( \omega_1 \) and \( \omega_2 \), differ substantially. As expected, \( \omega_1 \) is much larger than \( \omega_2 \). This is an expected result that cannot be measured by (9). The fact that \( \omega_1 \) is 95 times larger than \( \omega_2 \) does not imply that the unconditional variance before the Great Moderation is 95 times larger than after. The unconditional variances prior and after the Great Moderation are \( \omega_1 + \alpha_1/1 + \alpha_0 \) and \( \omega_2 + \alpha_1/1 + \alpha_0 \) respectively. Substituting the parameters by their estimates gives 0.191 and 0.088 respectively, or a ratio of 2.19. This number is similar to 2.45, the average of the ratio of the sample variances in the top panel of Table 1 (except GDP, as it doesn’t enter in VOLINX).

Given these results, we focus hereafter on VOLINX given by (10) and plotted in middle panel of Figure 3, jointly with the estimated coincident index (top panel) and a smooth version of VOLINX (bottom panel). The vertical dashed lines represent the different presidential mandates. The coincident index is similar to those computed for any other specification of VOLINX and which other authors have found (e.g. M&M).\(^{15}\)

The start to the biggest recessions and the largest peaks of stress have occurred during Republican presidential mandates.\(^{16}\) In fact, around the beginning of a Republican mandate, the economy enters into recession. When President Nixon took office in January 1969, the coincident index began to feel the effects of stagflation. President Ford took office in August 1974, at the beginning of the oil crisis. The first few months of the Reagan administration in January 1981 were followed by a sharp decrease in the coincident index. Approximately one year after the start of George Bush presidency, in January 1989, the coincident index detects the beginning of another period of recession. And finally, the index dropped

---

\(^{14}\)These results fit nicely into the literature surrounding the effect of macro announcement in the volatility of financial markets. Some have shown (Balduzzi, Elton and Green, 2001, Hautsch and Hess, 2002, and Andersen, Bollerslev, Diebold and Vega, 2003, among others) that one of the most important macroeconomic indicators to explain financial assets’s volatility is employment.\(^{15}\)

\(^{15}\)Results are available at request.

\(^{16}\)This statement should be interpreted carefully as during the 25 of the 37 years of the sample (or 67.5%), there was a Republican president in the White House.
sharply just a few months before the start of George W. Bush mandate in January 2001. Whether this correlation between Republican mandates and recessions is merely anecdotic or actually the result of their policy (which could have exacerbated the decline) remains an open question. By contrast, both the Carter and Clinton administrations are characterized by long steady growth, in particular during the eight years of President Clinton’s mandate, without the sign of any significant stress (compared to the Republican presidencies). Only at the end of their respective mandates do we observe a decrease in the coincident index: in 1979 following the second oil crisis, due to the Iranian revolution, and in the second semester of 2000 due to the burst of the internet bubble.\(^{17}\)

A glance at VOLINX in the middle panel gives rise to the following three conclusions. First of all, the degree of uncertainty of the economy is not constant. Since the late 60s, the US economy has suffered regular periods of stress and tranquility. A likelihood ratio test for \(\alpha_0 = \alpha_1 = 0\) equals 44, which considerably exceeds the critical value of a \(\chi^2\). Second, its value almost always lies below 1, which indicates that setting the unconditional volatility to unity was too constraining. Thirdly, the structural change in the mid-80s is evident.

In order to better analyze VOLINX in terms of economic and political events, the bottom panel shows a smoothed version. Late 1970-early 1971, under the Nixon administration, the US social agenda was marked by the conflict in Vietnam and the economic agenda by the stagflation (sluggish growth and the cost of living rising by 15% every year) that eventually led in August 1971 to the end of the Bretton-Woods system. Late 1974-1975 is also a period of stress. Uncertainty peaks around this time, however stress continued over the next three years, a consequence of the oil crisis. Although the oil embargo by the members of OPEC only lasted from October 1973 to May 1974, the increase in oil prices led to sudden inflation and economic recession. Another unusual period detected by VOLINX is late 1984-beginning 1985, followed by a fast transition to tranquility. This is the so-called Great Moderation. There are three often-cited explanations for this phenomenon, which is not yet fully understood. The first is the structural change in economic institutions, technology and business practices which have improved the ability of the economy to absorb shocks. The second is better monetary policy. The final explanation is that nothing changed internally in the US economy, nor was the monetary policy more effective. Simply, during the late 80s and 90s a smaller number of shocks, of little intensity, hit the economy. Only three peaks of uncertainty worth mentioning appear after the Great Moderation. The first is from mid-1986 to mid-1987. This is probably due to uncertainty from the tax reform act of 1986 which hit income and corporate taxes. The second is during 1990-1992, due to the invasion of Kuwait by Iraq and the subsequent first Gulf war. The third is 09/11. This attack, followed by the US government response with the invasion of

\(^{17}\)We have done a similar exercise with the Governors of the Fed. No particular insights have been found, besides what is known. The moments of higher uncertainty took place under Burns as Governor, the Great Moderation happened in the Volcker’s era and Greenspan period was marked by tranquility. As it happens with the Presidents, analyze if Volckers and Greenspan policy helped to reduce uncertainty remains an open question.
Afghanistan undermined both consumer and business confidence, and lead to a drop in payroll employment, which translated into increased stress for the economy. Interestingly enough, the degree of uncertainty has decreased considerably in the last few years, reaching unprecedented low levels, even lower than during the Bush and Clinton administrations.

As mentioned in the introduction, the indicators we consider are among the most important that are used by the NBER’s Business Cycle Dating Committee. It is therefore worth comparing their dating of the turning points with our indexes. Vertical lines Figure 4 show the peaks (dashed lines) and troughs (dotted lines) of the business cycle dated by the NBER. Prior to the Great Moderation the coincident index detects the peaks and troughs almost at the same periods than the NBER. But this concordance does not hold anymore after, in particular for the troughs. It is however interesting to note that the troughs announced by the NBER match with the peaks of VOLINX for the whole sample period. In other words, whenever the NBER considers that the economy has been in the trough of the cycle, VOLINX indicates that it has also been in a period of stress.

Figure 5 presents the analysis of the monthly GDP estimates. The top left panel shows the estimated monthly series. It displays heteroskedasticity and volatility clustering, with a clear cut in the fluctuations prior and after the Great Moderation. We expected this as the same features were found for quarterly GDP. However, the monthly estimates present a higher degree of clustering as some shocks that are smoothed out at quarterly frequency. In fact, the univariate GARCH estimates are 0.57 and 0.47 for the shock and persistence respectively, indicating that shocks are more present at higher frequencies.

We compare the forecasting ability of the model with that of the S&W model which assumes constant variance. The top right panel shows the in sample ratio of the confidence intervals, a measure of their accuracy. The horizontal dashed line at one indicates whether the S&W model provides the same confidence interval as our model. Intuitively, during periods of high (low) volatility, confidence bands should be wider (more narrow). Ignoring this fact could bring economic policy makers to underestimate (overestimate) the uncertainty. In fact, prior to the Great Moderation, the confidence intervals are much wider (up to 50% more) with respect to the case with constant variance and, moreover, they change substantially over time. We observe a similar phenomenon after the Great Moderation, with confidence intervals reduced by around 40%. In other words, if we assume constant volatility, the model underestimates the fluctuations prior to the Great Moderation and exacerbates them after the mid-80s. This is a useful tool for economic policy decision makers, who can adapt their decisions depending on how wide the confidence intervals are.

The bottom panels show a similar exercise but for the out-of-sample. They are the one month ahead predictions of the confidence intervals for two periods of special interest. The first are the two years around the Great Moderation, from January 1983 to December 1986. The second are the years around 09/11 from January 2000 to December 2002. In both cases the forecasted confidence intervals vary over time, with differences up to 50% to the constant volatility case. It is worth noticing that while the decrease in the volatility in the great moderation is steady over two years, the uncertainty caused by 09/11 is resolved.
relatively quickly. After six months, the volatility comes back to previous levels.

In our model, the breaking date for the Great Moderation is exogenous. Based on Stock and Watson (2002b), we choose it to be January 1984. We now check the robustness of the results. We estimate the model for a grid of breaking dates from January 1984 to December 1985. The top and middle panels of Figure 6 show the estimated parameters of the conditional volatility for the different breaking dates. In general, the parameters are stable and there are no significant changes. Not even \( \alpha_0 \), which appears to change for the breaking date of March 1985. This is just a visual artifact as, for representation purposes, it is plotted on the right axis. The change due to the breaking date of March 1985 is very minor when compared with the values of the other parameters. The same applies to the middle panel that presents the stability analysis for the weights. The bottom panel shows the overlapped VOLINX for all the breaking dates. All the indexes are very similar except during the period of the Great Moderation. The bottom line of this robustness check is that the degree of uncertainty over the 37 years is robust to the choice of the breaking date.

4 Conclusions

Given a set of monthly indicators between 1968 and 2006, we estimate a monthly coincident index for the level and the volatility of the US economy. We rely on a new type of GARCH model where we replace the past square error by a linear combination of the past standardized forecasting errors of the economic indicators. The weights of the linear combination allow us to infer which are the most relevant indicators that explain the volatility of the economy. We estimate the monthly GDP by disaggregating its quarterly value using a technique for state space models developed by Harvey (1989).

Our results show that the US economy suffered regular episodes of stress prior to the Great Moderation (stagflation, first and second oil crisis). Only two peaks of uncertainty appear after the Great Moderation. The first runs from mid-1986 to mid-1987 (which corresponds to the tax reform act of 1986) and the second is 09/11. We also find that, consistent with the literature, industrial production is the most important determinant for the level of the US economy. However, the most important determinants of the volatility are employment and income, while industrial production falls behind.

Monthly GDP growth presents heteroscedasticity and volatility clustering with a clear cut in the fluctuations prior and after the Great Moderation in the mid-80s. These in and out of sample forecasts of GDP have confidence intervals that shrink and expand with the degree of uncertainty in the economy. This is a useful tool for economic policy decision makers, who may adapt their decisions depending on how wide the confidence intervals are.

Further research goes in two directions. First, one may want to include more indicators in the analysis, in the same spirit as Giannone et al. (2007). As we mentioned in the introduction, we have restricted ourselves to the same set of indicators as in S&W and M&M. One of our aims was to determine whether
the indicators that are usually believed to contribute most to the economy also explain its uncertainty and if so, to what extent. Moreover, the introduction of a large number of indicators imply a large number of parameters, which render the model untractable. An extension of Forni et al. (2000) to the volatility is worth looking at. Another potential extension of this analysis would be to estimate endogenously the breaking date of the Great Moderation. This would entail the use of regime switching GARCH models where the regimes would be treated endogenously. The literature does exist for traditional volatility settings (i.e. observed components) but its adaptation to state space models deserves further research.

References


A Appendix: The model in State Space Form

We show how to cast the model (1)-(6) in the SSF. We first consider the case of homogeneous frequency of the series. We also explain how to extend it to include the temporal disaggregation constraint. To make our exposition clear, we present the state space of each component separately, first for the coincident index, followed by the idiosyncratic components, and finally we put it all together to get the complete form. Further details can be found in Proietti and Frale (2006).

The coincident index \((1 - \phi L)\Delta\mu_t = \eta_t\) can be written as

\[
\Delta\mu_t = [0, 1] g_t, \\
g_t = \begin{pmatrix} \phi_1 & 1 \\ \phi_2 & 0 \end{pmatrix} g_{t-1} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sigma_t^2 \eta_t,
\]

and the Markovian representation of the model for \(\mu_t\) becomes

\[
\mu_t = [1, 0, 0] \alpha_{\mu, t} \\
\alpha_{\mu, t} = T_\mu \alpha_{\mu, t-1} + [1, 1, 0]' \sigma_t^2 \eta_t, \quad T_\mu = \begin{bmatrix} 1 & \phi_1 & 1 \\ 0 & \phi_1 & 1 \\ 0 & \phi_2 & 0 \end{bmatrix}
\]

A similar representation holds for each individual \(\mu_{it}^*, \phi_j\) replaced by \(d_{ij}\), so that, if we assume
that

\[ \mu_{\star t}^{\ell} = [1, 0, 0] \alpha_{\mu t} \]

\[ \alpha_{\mu t} = T_{1} \alpha_{\mu t-1} + [\delta_{i}, \delta_{i}, 0]^t + [1, 1, 0]^t \sigma_{\eta t}^{\star 2}, \quad T_{1} = \begin{bmatrix} 1 & d_{11} & 1 \\ 0 & d_{11} & 1 \\ 0 & d_{12} & 0 \end{bmatrix} \]

where \( \delta_{i} \) is the drift of the \( i \)-th idiosyncratic component, and thus of the series.

Combining all the blocks, we obtain the SSF of the complete model by defining the state vector \( \alpha_{t} \), with dimension \( 2 + 2N \), where \( N \) is the number of indicators, as follows:

\[ \alpha_{t} = [\alpha_{\mu t}^{'}, \alpha_{\mu_{1,t}}^{'}, \ldots, \alpha_{\mu_{N,t}}^{'},] \]

(11)

The measurement and the transition equations of the S&W model in levels are:

\[ y_{t} = Z\alpha_{t} + X_{t} \beta, \quad \alpha_{t} = T\alpha_{t-1} + W \beta + H \epsilon_{t}, \]

(12)

where \( \epsilon_{t} = [\eta_{t}, \eta_{1,t}^{\star}, \ldots, \eta_{N,t}^{\star}]^{t} \) and the system matrices are given below:

\[ Z = \begin{bmatrix} \theta_{0} & 0 & \vdots & \theta_{1} \\ 0 & \vdots & \vdots & \vdots \end{bmatrix} \quad \text{diag}(e_{2}^{\prime}, \ldots, e_{2}^{\prime}) \]

\[ T = \text{diag}(T_{\mu}, T_{1}, \ldots, T_{N}) \]

\[ H = \text{diag}(h_{\mu}, h_{1}, \ldots, h_{N}). \]

(13)

where \( e_{k}^{\prime} = [1, 0, \ldots, 0] \) is a \( 1 \times k \) vector and \( h_{\mu}, \ldots, h_{N} = [1, e_{2}^{\prime}] \)

The two matrices \( X_{t} \) and \( W \) are used to incorporate regression effects and to initialize the system, whereas the \( (2N + k) \) vector \( \beta \) contains the pairs \( \{ \mu_{00}, \delta_{i}, i = 1, \ldots, N \} \), the starting values at time \( t = 0 \) of the idiosyncratic components and the constant drifts \( \delta_{i} \).

In particular, the regression matrix \( X_{t} = [0, X_{t}^{*}] \), where \( X_{t}^{*} \) is a \( N \times k \) matrix containing the values of exogenous variables that are used to incorporate \( k \) calendar effects (trading day regressors, Easter, length of the month) and intervention variables (level shifts, additive outliers, etc.). The zero block in the matrix \( X \) has dimension \( N \times 2N \) and corresponds to the elements of \( \beta \) that are used for the initialisation and other fixed effects.

The SSF is complete with the equation for the GARCH-type volatility:

\[ \sigma_{t}^{2} = (1 - \alpha_{0} - \alpha_{1}) + \alpha_{0} \sigma_{t-1}^{2} + \alpha_{1} \sum_{i=1}^{N} \omega_{it} \nu_{it}^{2} \]

(14)

where the standardized innovations \( \nu_{t,i} \) are defined as follow:

\[ \nu_{t,i} = (y_{t,i} - E(y_{t,i}|y_{1}, \ldots, y_{t}))/\text{Std}(\nu_{t,i}) \]

(15)

In order to deal with mixed frequency we follow Harvey (1989) operating a suitable augmentation of the state vector (11) using an appropriately defined cumulator variable.
Let us partition the set of indicators, $y_t$, into two groups, $y_t = [y'_{1,t}, y'_{2,t}]'$, of dimension $N = (N1 + N2)$, where $y_{1,t}$ contains the series observable every period (monthly) and the second block the series partially observable (quarterly). We consider an underlying random sequence $y^*_{2,t}$ such that

$$\ln(y_{2,\tau}) = \frac{1}{3} \sum_{i=0}^{2} \ln(y^*_{2,\tau-i}), \quad \tau = 1, 2, \ldots, \lfloor T/3 \rfloor.$$ 

or $y_{2,t}$ is the geometric mean of the unobserved three monthly values $y^*_{2,t-1}, y^*_{2,t-2}, y^*_{2,t-3}$. Then define a $N_2 \times 1$ vector $y^c_{2,t}$, as follows

$$y^c_{2,t} = \psi_t y^c_{2,t-1} + \frac{1}{3} \ln(y^*_2).$$

where $\psi_t$ is the cumulator variable, defined by:

$$\psi_t = \begin{cases} 
0 & t = 3(\tau - 1) + 1, \quad \tau = 1, \ldots, \lfloor n/3 \rfloor \\
1 & \text{otherwise}.
\end{cases}$$

In other words, the cumulator is equal to the (observed) aggregated series at times $t = 3\tau$, and otherwise it contains the partial cumulative monthly underlying value $\frac{1}{3} \ln(y^*_2)$ making up the quarter, up to and including the current one. The monthly GDP in logarithms is obtained from $y^c_{2,t}$ applying an inverse function which is linear.

The original SSF is augmented by $y^c_{2,t}$, which is observed at times $t = 3\tau, \tau = 1, 2, \ldots, \lfloor n/3 \rfloor$, and is missing at intermediate times:

$$\alpha^*_t = \begin{bmatrix} \alpha_t \\ y^c_{2,t} \end{bmatrix}, \quad y^\dagger_t = \begin{bmatrix} y_{1,t} \\ y^c_{2,t} \end{bmatrix}.$$

The final measurement and transition equations are as follows:

$$y^\dagger_t = Z^* \alpha^*_t + X_t\beta, \quad \alpha^*_t = T^* \alpha^*_{t-1} + W^* \beta + H^* \epsilon_t, \quad (16)$$

with system matrices:

$$Z^* = \begin{bmatrix} Z_1 & 0 \\ 0 & I_{N_2} \end{bmatrix}, \quad T^* = \begin{bmatrix} T & 0 \\ \psi I & Z_2T \end{bmatrix}, \quad W^* = \begin{bmatrix} W \\ Z_2W + X_2 \end{bmatrix}, \quad H^* = \begin{bmatrix} I \\ Z_2 \end{bmatrix}H. \quad (17)$$

where $Z_2$ is the block of the measurement matrix $Z$ corresponding to the second set of variables (the cumulator for the quarterly GDP), $Z = [Z'_1, Z'_2]'$ and $y_{2,t} = Z_2\alpha_t + X_2\beta$, where we have partitioned $X_t = [X'_1 X'_2]'$.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Sample Variances</th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Employment</td>
<td>Industrial Production</td>
<td>Sales</td>
<td>Income</td>
<td>GDP</td>
</tr>
<tr>
<td>1968-2006</td>
<td>0.043</td>
<td>0.664</td>
<td>0.133</td>
<td>0.282</td>
<td>0.676</td>
</tr>
<tr>
<td>1968-1984</td>
<td>0.075</td>
<td>1.109</td>
<td>0.159</td>
<td>0.283</td>
<td>1.263</td>
</tr>
<tr>
<td>1985-2006</td>
<td>0.019</td>
<td>0.321</td>
<td>0.114</td>
<td>0.267</td>
<td>0.231</td>
</tr>
</tbody>
</table>

Sample autocorrelations of order one

|                  | Employment       | Industrial Production | Sales   | Income | GDP     |
| 1968-2006        | 0.497            | 0.470              | 0.387           | -0.12            | 0.247            |
| 1968-1984        | 0.643            | 0.095              | 0.409           | -0.37            | 0.250            |
| 1985-2006        | 0.541            | 0.362              | 0.366           | -0.17            | 0.251            |

Univariate GARCH estimates

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>ARCH</th>
<th>GARCH</th>
<th></th>
<th>Demos-Sentana ARCH test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.75E-07**</td>
<td>2.23E-05**</td>
<td>2.72E-06**</td>
<td>1.48E-06**</td>
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<td>0.464**</td>
<td>0.332**</td>
<td>0.142**</td>
<td>0.116**</td>
<td>0.096**</td>
</tr>
<tr>
<td></td>
<td>0.331**</td>
<td>0.281*</td>
<td>0.617**</td>
<td>0.831**</td>
<td>0.893**</td>
</tr>
</tbody>
</table>

Constant, ARCH and GARCH in the middle panel stand for the $\omega$, $\alpha_0$ and $\alpha_1$ of the model $\sigma_t^2 = \omega + \alpha_0 \varepsilon_{t-1}^2 + \alpha_1 \sigma_{t-1}^2$. The superscript "**" stands for significant parameters at 5% level, while "*" refers to 10% level (Bollerslev-Wooldridge robust standard errors). The test in the bottom panel is the Demos-Sentana (1998) ARCH LM test on the residuals of the SW standard model. The test considers 5 lags and is distributed according to a mixture of $\chi^2$ with critical value equal to 7.480 at 5% level and 5.835 at 10%.
Table 2: S&W model

<table>
<thead>
<tr>
<th>Measurement Equation</th>
<th>Employment</th>
<th>Industrial Production</th>
<th>Sales</th>
<th>Income</th>
<th>GDP</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 × ϑ₀,i</td>
<td>0.082**</td>
<td>0.317**</td>
<td>0.070**</td>
<td>0.043**</td>
<td>0.095**</td>
<td></td>
</tr>
<tr>
<td>100 × ϑ₁,i</td>
<td>0.426**</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Idiosyncratic conditional means parameters

| 100 × δᵢ            | 0.194**    | 0.018                  | 0.194** | 0.744** | 0.251** |    |
| dᵢ                  | -0.259**   | 0.925**                | 0.223** | -0.201** |        |    |

Idiosyncratic standard deviations parameters

| 100 × σηᵢ²          | 0.112**    | 0.130**                | 0.322** | 0.519** | 0.454** |    |

Coincident Indicator mean parameter

| φ                    | 0.883**    |                        |        |        |     |    |

Maximum likelihood estimates for model (1)-(6) with σᵡ² = 1. CI stands for coincident indicator. The superscript "**" stands for significant parameters at 5% level, while "*" refers to 10% level (Bollerslev-Wooldrige robust standard errors). To make them readable, some parameters are multiplied by 100, as it indicates the 100× in the front.

Table 3: Two regimes variance model

<table>
<thead>
<tr>
<th>Measurement Equation</th>
<th>Employment</th>
<th>Industrial Production</th>
<th>Sales</th>
<th>Income</th>
<th>GDP</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 × φ₀,i</td>
<td>0.069**</td>
<td>0.245**</td>
<td>0.074**</td>
<td>0.038*</td>
<td>0.072**</td>
<td></td>
</tr>
<tr>
<td>100 × φ₁,i</td>
<td>0.484**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Idiosyncratic conditional means parameters

| 100 × δᵢ            | 0.171**    | 0.020**                | 0.194** | 0.711** | 0.246** |    |
| dᵢ                  | -0.211**   | 0.914**                | 0.175   | -0.267** |        |    |

Idiosyncratic standard deviations parameters

| 100 × σηᵢ²          | 0.109**    | 0.133**                | 0.338** | 0.556** | 0.445** |    |

Coincident Indicator mean parameter

| φ                    | 0.909**    |                        |        |        |     |    |

Coincident Indicator variance parameters

| ω₁                   | 1.463      |                        |        |        |     |    |
| ω₂                   | 0.622      |                        |        |        |     |    |

Maximum likelihood estimates for model (1)-(6) with σᵩ² equal to (7). CI stands for coincident indicator. The superscript "**" stands for significant parameters at 5% level, while "*" refers to 10% level (Bollerslev-Wooldrige robust standard errors). To make them readable, some parameters are multiplied by 100, as it indicates the 100× in the front.
Table 4: GARCH model

<table>
<thead>
<tr>
<th>State Equation</th>
<th>Employment</th>
<th>Industrial Production</th>
<th>Sales</th>
<th>Income</th>
<th>GDP</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \times \theta_{0,i}$</td>
<td>0.069**</td>
<td>0.253**</td>
<td>0.071**</td>
<td>0.042**</td>
<td>0.074**</td>
<td></td>
</tr>
<tr>
<td>$100 \times \theta_{1,i}$</td>
<td></td>
<td>0.477**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$100 \times \beta_{h}$</td>
<td>-1.799**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Idiosyncratic conditional means parameters

| $100 \times \delta_{i}$ | 0.169** | 0.014** | 0.209** | 0.705** | 0.235** |   |
| $d_{i}$ | -0.159 | 0.929** | 0.142 | -0.222** | | |

Idiosyncratic standard deviations parameters

| $100 \times \sigma_{\eta_{i}}$ | 0.106** | 0.111** | 0.322** | 0.516** | 0.454** |   |

Coincident Indicator mean

| $\phi$ | 0.877** |

Coincident Indicator variance parameters

| $\alpha_{0}$ | 0.812** |
| $\alpha_{1}$ | 0.177* |
| $\varphi_{i}$ | 0.599** | 0.003** | 0.112** | 0.276** |

Maximum likelihood estimates for model (1)-(6) with $\sigma_{t}^{2}$ equal to (9). CI stands for coincident indicator. The superscript "**" stands for significant parameters at 5% level, while "*" refers to 10% level (Bollerslev-Wooldridge robust standard errors). To make them readable, some parameters are multiplied by 100, as it indicates the $100 \times$ in the front.
Table 5: Two regime GARCH model

<table>
<thead>
<tr>
<th>Employment</th>
<th>Industrial Production</th>
<th>Sales</th>
<th>Income</th>
<th>GDP</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$100 \times \theta_{0,i}$</td>
<td>0.090**</td>
<td>0.317**</td>
<td>0.096**</td>
<td>0.048*</td>
<td>0.094**</td>
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<tr>
<td>$100 \times \theta_{1,i}$</td>
<td>0.676**</td>
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<tr>
<td>$100 \times \beta_k$</td>
<td>-1.851 **</td>
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<td>Idiosyncratic conditional means</td>
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<tr>
<td>$100 \times \delta_i$</td>
<td>0.165**</td>
<td>0.018**</td>
<td>0.201**</td>
<td>0.685**</td>
<td>0.242**</td>
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<tr>
<td>$d_i$</td>
<td>-0.158</td>
<td>0.916**</td>
<td>0.132</td>
<td>-0.221</td>
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<tr>
<td>$100 \times \sigma_{\eta i}$</td>
<td>0.149**</td>
<td>0.158**</td>
<td>0.444**</td>
<td>0.700**</td>
<td>0.600**</td>
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<td>Coincident Indicator mean</td>
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<tr>
<td>$\varphi$</td>
<td>0.899**</td>
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<tr>
<td>$\omega_1$</td>
<td>0.095**</td>
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<tr>
<td>$\omega_2$</td>
<td>0.001**</td>
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<tr>
<td>$\alpha_0$</td>
<td>0.773**</td>
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<tr>
<td>$\alpha_1$</td>
<td>0.081**</td>
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<tr>
<td>$\varphi_i$</td>
<td>0.374**</td>
<td>0.002**</td>
<td>0.290**</td>
<td>0.333**</td>
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</tr>
<tr>
<td>$\varphi_{iIGM}^{}$</td>
<td>0.411**</td>
<td>0.002**</td>
<td>0.213**</td>
<td>0.374**</td>
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</tr>
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Maximum likelihood estimates for model (1)-(6) with $\sigma_i^2$ equal to (10). CI stands for coincident indicator and $\varphi_{iIGM}^{}$ for the weights of the survey data surprises in VOLINX. The superscript "**" stands for significant parameters at 5% level, while "*" refers to 10% level (Bollerslev-Wooldridge robust standard errors).

To make them readable, some parameters are multiplied by 100, as it indicates the $100 \times$ in the front.

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Figure 1: Squared log growth rates vs. sample variance

From top to bottom and from left to right, squared demeaned log growth of employment, industrial production, sales, income and GDP. The straight line shows the sample variance.
Figure 2: Monthly indicators and quarterly GDP in log levels and log growth rates

From top to bottom and from left to right, log growth rates in percentage of employment, industrial production, sales, income and GDP. Log levels (increasing lines) and log growth rates in percentage are represented in the right and left axes respectively.
Figure 3: Coincident index, VOLINX and Presidential mandates

Top panel shows the coincident indicator and middle panel VOLINX for model (1)-(4) and (10). Bottom panel is a smoothed version of VOLINX. Smoothing by Hodrick-Prescott filter with lambda set by Ravn-Uhlig frequency rule with power 1. Vertical dashed lines represent the mandates of the Presidents over the sample period.
Figure 4: Coincident index, VOLINX and NBER turning points

Top panel shows the coincident indicator and middle panel VOLINX for model (1)-(4) and (10). Bottom panel is a smoothed version of VOLINX. Smoothing by Hodrick-Prescott filter with lambda set by Ravn-Uhlig frequency rule with power 1. Vertical dashed and dotted lines represent the peaks and troughs of the economy according to the NBER business cycle reference dates.
Figure 5: Monthly GDP, and in sample and out of sample accuracy evaluation

Top left panel shows the monthly estimates of GDP for model for model (1)-(4) and (10). Log levels (increasing line) and log growth rate in percentage are represented in right and left axes respectively. Top right panel shows the in sample ratio of the monthly GDP uncertainty as measured by our model and M&M. A value of 1 (represented by the dashed straight line) means no difference. Bottom panels show the out-of-sample (one month ahead predictions) ratio of the monthly GDP uncertainty as measured by our model and S&W for two periods: January 1983 - December 1985 (left panel) and January 2000 - December 2002 (right panel).
Analysis of the change of the breaking date of the great moderation in the conditional variance parameters and VOLINX. For all panels, the breaking date is rolling from January 1984 to December 1985. The rolling breaking date is represented in the x-axis for the top and middle panel. Top panel shows the estimated parameters $\alpha_0$, $\alpha_1$, $\omega_1$ and $\omega_2$. All represented in the left y-axis except $\omega_1$. Middle panel shows the estimated weights of the forecasting errors of the indicators. All represented in the left axis expect the weight for employment. In bottom panel overlapped VOLINX for all the breaking dates.