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Competing for Ownership.

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Abstract

We develop a tractable model of the allocation of control in firms in competitive markets, which permits us to study how changes in the scarcity of assets, skills or liquidity in the market translate into control inside the organization. Firms will be more integrated when the terms of trade are more favorable to the short side of the market, when liquidity is unequally distributed among existing firms and following a uniform increase in productivity. We identify a multiplier effect of the first two moments of the distribution of liquidity on the moments of the distribution of ownership. The model identifies a price-like mechanism whereby local liquidity or productivity shocks propagate and lead to widespread organizational restructuring.

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1 Introduction

In the neoclassical theory of the firm, market signals affect choices of products, factor mixes, and production techniques. If labor becomes scarce, wages rise, and firms substitute machines for workers. Firm behavior, in turn, feeds back to the market, and through it, to other firms: if a labor-saving production process is introduced by some firms, wages will fall, and the other firms will reduce the capital intensity of their production. The neoclassical firm remains the backbone of much of economic analysis because it is so readily incorporated into the study of feedback effects like these.

The modern theory of the firm emphasizes contractual frictions and organizational design elements such as task allocations, asset ownership, and the assignment of authority and control. By augmenting economic analysis with this new set of variables, it has led to breakthroughs in our comprehension of institutions as different as the modern corporation and the sharecropped farm. But despite the theory’s formative purpose – to understand the nature of firms in market economies – as well as evidence that firms restructure themselves in response to market conditions or the behavior of other firms, there are few models that can take account of the effects of the neoclassical feedbacks on the modern variables of interest.

The purpose of this paper is to provide a tractable framework for this kind of analysis. We focus on the structure of ownership and control, understood

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1To mention just two examples, the wholesale restructuring of relations between US automakers and their suppliers in the 1980s was likely triggered by entry of Japanese firms into the US market (Dyer, 1996); on a smaller scale, decision rights over the outfitting of truck cabs or the accompaniment of drivers by their spouses during hauls have recently shifted from trucking firms to their drivers in response to the growth of wages in the construction industry (Urbina, 2006).
here, as in Grossman and Hart (1986), as an allocation of residual decision rights among a firm’s stakeholders. The model illuminates how scarcity in the market translates into control inside the firm and how changes in the fundamentals of some firms can spill over to economy-wide reorganizations.

The basic setup is a two-sided matching model, with the sides representing two types of production units, each one consisting of a manager and a collection of assets; when two units from opposite sides work together, they can produce marketable output that yields a surplus beyond what they can generate working on their own (or with units from the same side). Firms comprising one unit of each type form through a competitive matching process. Each matched pair is governed by a contract specifying its ownership structure.

After a firm has formed, a series of noncontractible management decisions, one for each asset, has to be taken. The organization must be designed to strike a compromise between productivity (managers share the firm’s profit) and the private costs of managing, and because of the noncontractibility, this can only be accomplished by a (re-)allocation of the rights to control the various assets (for simplicity, we abstract away from the adjustment of profit shares; few results reported here would change substantially if we allow for them: see Legros and Newman, 2007).

In general the more assets a manager owns, the better off he will be, since he will be able to ensure that more decisions go in his preferred direction. But because these decisions will impose both profit and private cost externalities on the other manager, different organizational designs generate different levels of total surplus for the firm as well as different divisions of
that surplus between its managers.

A crucial attribute of the environment we analyze is that liquidity—
instruments such as cash that can be transferred costlessly and without any
incentive distortions—is scarce. Managers have quasi-linear utility, so liquid-
dity transfers are the preferred means of reallocating surplus between them.
But when liquidity is in short supply, a large transfer of surplus will have to
be done through an organizational distortion, i.e., a reassignment of control.
This feature generates a key role for competitive analysis. The equilibrium
outcome can no longer be identified with the surplus-maximizing allocation of
ownership; instead, the market-determined division of the surplus is needed
to pin down the organizational outcome.

The model highlights two distinct effects that arise from a change in
fundamentals such as liquidity endowments or technology. The first is an
“internal effect,” various forms of which have been studied in the literature
on ownership: the surplus that each partner obtains from a given contract is
a function of the characteristics of the partners in a relationship, in particular
the amount of liquidity they have and the production technology available.
Specifically, in our model, more liquidity in the firm enlarges the set of feasible
payoffs for the two managers by increasing transferability, though it does
not enlarge their set of production possibilities, since there is no need to
acquire productive assets from outside the partnership. Higher productivity
not only enlarges the payoff sets by expanding production possibilities, but
like liquidity also “flattens” the payoff frontier, because it induces managers
to increase the weight of profit (which can be shared) relative to private cost
(which cannot) in their decisions. Hence, a positive shock to a firm’s liquidity
or productivity will enable it to accomplish surplus division more efficiently and reduce organizational distortions.

But that same shock can have wider effects than on the firm that first experiences it. The internal effect implies that a manager has effectively a higher “ability to pay” for a partner after a positive shock than before. He may therefore bid up the terms of trade in the matching market: in order to meet the new price, firms which have not benefited from the shock will have to restructure. Thus the shock may have an external effect: “local” shocks may propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium of our model turns out to be amenable to a Marshallian supply-demand style of analysis, making the role of the external effect especially transparent. Suppose for instance, that one side of the market represents automobile manufacturers selling in the U.S. market and the other side represents their suppliers. An increase in the number of manufacturers due to entry from abroad will reduce the share of surplus accruing to the auto makers. This will entail a transfer of control to the suppliers, and many manufacturer-supplier relationships will become less integrated in the sense that a smaller fraction of the assets will be controlled by the auto maker’s manager.

Furthermore, while the internal effects of positive shocks to liquidity and technology are similar – they both decrease integration – the external effects differ. A uniform increase in the liquidity level of all agents lowers the degree of integration in all firms (the internal effect dominates the external effect). By contrast, a uniform shock to productivity increases the degree of integration in all firms (the external effect dominates the internal effect). These
effects can be quite pronounced: there is an “organizational multiplier” effect of shocks, with, for instance, a unit change in mean liquidity producing a larger than unit change in the mean degree of integration. As we show in Section 3, the model can also capture the effects of more complex changes in the liquidity endowments, in productivity, or in managers’ outside options.

Our model of the determination of ownership structure is inspired by Grossman and Hart (1986). However, we depart from their analysis in three respects. First, as in Hart and Moore (1990), we allow for a richer set (in fact, a continuum) of ownership structures than the two (integration and non-integration) discussed by Grossman and Hart. This feature yields a tractable model amenable to competitive analysis. Second, as have a few recent papers (e.g. Hart and Holmström, 2002; Aghion, Dewatripont, and Rey, 2004; Baker, Gibbons, and Murphy, 2006), we abstract away from the hold up problem by dropping ex-ante investments and assuming instead that ex-post decisions are not contractible. Our purpose in doing so is to make the surplus transfer role of ownership especially transparent: the set of feasible decisions is unaffected by who owns an asset, and therefore awarding ownership of more assets to one manager unambiguously raises his payoff.

The third and most important departure is the assumption that liquidity is scarce. The corporate finance literature beginning with Aghion and Bolton (1992) has already highlighted what we have termed the internal effect of limited liquidity on the allocation of control: given the division of surplus, raising a contractual party’s liquidity endowment will tend to give him more control and increase the efficiency of the relationship. What is new here is the identification and analysis of the external effect: limited liquidity implies
that a firm may modify its control right allocation, at a possible efficiency
cost, in response to changes in the liquidity (or technology) of another firm.
This effect would also be present for many other specific models of ownership
and organizational design: all that is important is that the payoff frontier not
reflect transferable utility, which in our formulation scarce liquidity helps to
guarantee.

2 Model

We consider an economy in which production requires the cooperation of two
production units, indexed by 1, 2. Each unit consists of a risk-neutral manager
and a collection of assets that he will have to work with in order to produce.
We have in mind competitive outcomes, and so we suppose that there is a
large number of production units: each side of the market is a continuum
with Lebesgue measure. The type 1’s are represented by \( i \in I = [0, 1] \) while
the type 2’s are represented by \( j \in J = [0, n] \), where \( n < 1 \); thus, the 2’s are
relatively scarce.

Many interpretations are possible: the two types of manager might be supplier and manufacturer, and the assets plant and equipment; a chain restaurateur and franchising corporation (in which case some of the assets are reputational); or as a firm and its workforce, for which the assets might be interpreted as tasks.

In an individual production unit, an asset’s contribution to profit depends
on a planning decision made by one of the managers, not necessarily the one
who will have to operate it. Planning decisions are not contractible, but the
right to make them can be allocated via contract to either manager. For simplicity we assume that planning choices (e.g., choosing the background music for a retail store) are costless. But while potentially beneficial for profits (some music is likely to induce consumers to make impulse purchases), those choices affect the private cost of later operations (such music may be unpleasant for the store’s floor manager).

The $i$-th type-1 manager will have at her disposal a quantity $l_1(i) \geq 0$ of cash (or “liquidity”) which may be consumed at the end of the period and which may be useful in contracting with managers of the opposite type; for the type 2’s, the liquidity endowment is $l_2(j)$. The indices $i$ and $j$ have been chosen in order of increasing liquidity.

When discussing a generic production unit or its manager, we shall usually drop the indices.

2.1 The Basic Organizational Design Problem

2.1.1 Technology and Preferences

Managers seek to maximize their expected income (including the initial liquidity) less the private costs of operating the enterprise.

The collection of assets in the type-1 production unit is represented by a continuum indexed by $k \in [0, 1)$; the type-2 assets are indexed by $k \in [1, 2)$. An asset’s contribution to profit is proportional to the planning level $q(k)$, where $q(k) \in [0, 1]$.

Planning decisions contribute to the firm’s performance as follows. The firm either succeeds, generating profit $R > 0$, with probability $p(q)$; or it
fails, generating 0, with probability $1 - p(q)$, where $q : [0, 2) \to [0, 1]$ are the planning decisions. The success probability functional is

$$p(q) = \frac{A}{R} \int_0^2 q(k) dk,$$

where $A$ is a technological parameter. Obviously, for $p(\cdot)$ to be well-defined, we also need $A/R < 1/2$.

Either manager is capable of making planning decisions. There is no cost to making a plan, but there is a (private) operating cost to the manager who subsequently works with an asset: the 1-manager bears cost $c(q(k)) = \frac{1}{2}q(k)^2$ for $k \in [0, 1)$, and zero for $k \in [1, 2]$; similarly for 2, the cost is $c(q(k))$ on $[1, 2)$ and zero on $[0, 1)$. For brevity we write

$$C_1(q) = \int_0^1 c(q(k)) dk, \quad C_2(q) = \int_1^2 c(q(k)) dk.$$

This is the cost externality we alluded to: the cost to the manager operating the asset is increasing in $q(k)$, whether or not he has chosen it. For instance in a manufacturing enterprise, $q$ could index choices of possible parts or material inputs, ordered by the value they contribute to the final product, while $c(q)$ could represent the cost of managerial attention devoted to over-

\footnote{Note that we are assuming symmetry in the technology and cost between the two managers; any difference that emerges between the two sides will be only due to a difference in scarcity. An obvious extension would be to examine asymmetries at the level of individual enterprises such as different cost functions or different contributions to productivity. For instance if we assume that $C_2 = 0$, a firm is basically a principal-agent model. If one interprets the asset of type 2 as "capital," the model could be viewed a static version of a financial contracting problem, like Aghion and Bolton (1992). If instead we assume that one type is more productive that the other, we get closer to the type questions asked in papers like Hart and Moore (1990) concerning on who should rather who does own the asset. Such extensions are straightforward.}
seeing assembly, supervising workers, and so on. Each input choice requires solving a number of manufacturing process problems; we are supposing that higher value inputs require greater learning and adaptation effort on the part of the manufacturer’s management.3

Or, in a relationship between a fast food chain and a franchisee, numerous operating decisions, from the content of an advertising campaign, color scheme in the restaurant, participation in discount coupon promotions, or franchise relocation could in principle be delegated to the franchisee or retained by the company. Most of these decisions will have significant effects on the franchisee’s welfare, though it is hard to see most of them having much impact on the chain apart from their influence on expected profit. And while one might imagine a complete contingent contract on relocation might be drawn up (for instance, relocation might be dependent on location of a competing chain’s stores, local income levels and crime rates, etc.), in practice this remains a residual decision of the chain (Blair and Lafontaine, 2005).

2.1.2 Contracts

We have already made the following contractibility assumptions:4

3 In the 1960’s, W. Corporation owned an electronic systems division that manufactured airplane cockpit voice recorders, and a composite materials division that made various compounds suitable for heat-resistant recording tape, a critical input for recorders. The electronic systems division had perfected a manufacturing process that used mylar tape, but W. ordered them to use a new metal-oxide tape developed by its materials division. The new tape was less flexible than mylar, and therefore subject to kinking and breakage, which raised manufacturing problems that required nearly a year of process redevelopment to resolve. A former manager of the systems division admits that had it been up to him, his division would have stuck with the mylar tape, simply because the experimentation and retooling costs were not (and because of verifiability and incentive problems likely could never be) appropriately reimbursed, even if the metal oxide tape arguably had slightly better heat-resistance and recording properties.

4 See Aghion et al. (2004) for similar assumptions.
**Assumption**  (i) The right to decide \( q(k) \) is both alienable and contractible.

(ii) The decisions \( q \) are never contractible.

(iii) The costs \( C_i(q) \) are private and noncontractible.

A contract \((\omega, t)\) specifies the allocation of ownership \( \omega \) and liquidity transfers \( t \) made from 1 to 2 before any planning or production takes place. The liquidity levels of the two types being \( l_1 \) and \( l_2 \) respectively, we must have \( t \in [-l_2, l_1] \). The ownership allocation \( \omega \) is the fraction of assets re-assigned to one of the managers. The type-1 manager owns assets in \([0, 1 - \omega)\), where \(-1 < \omega \leq 1\), and the type-2 owns \([1 - \omega, 2)\).

Since we want to focus here on allocations of control rights, we will simplify matters by ignoring the usual effects related to variations in the sharing of profits. Instead, we simply assume that each manager gets half of the realized output, that is he gets \( R/2 \) if output is \( R \) and 0 if output is 0. Obviously, this is a rather stark representation of the constraints faced by real firms in the use of incentive pay. Similar kinds of assumptions have been used elsewhere in the literature (e.g., Holmström and Tirole, 1998). In Appendix I, we show that this restriction can be derived as a consequence of a simple moral hazard problem. It is straightforward, albeit algebra-intensive, to relax this assumption and allow for a rich set of budget-balancing sharing rules. The yield is some predictions on the interplay between ownership allocations and profit shares; the modified model of the firm can easily be embedded in our framework. The results in Section 3 will go through with only minor modification.\(^5\)

This leaves out a logical possibility: the managers might use a third party “budget breaker” who will pay the firm if there is success and will be paid out of the liquidity available in the firm if there is failure. Using third parties in this way may improve efficiency, but only if the third party gets more when the firm fails than when it succeeds. Apart from the undesirable incentive problems this creates (the third party may want the firm to fail), this modification would not change the basic message of this paper.

When \( \omega = 0 \), each manager retains ownership of his original assets, and, following the literature, we refer to this situation as non-integration. As \( \omega \) increases beyond 0, we have an increasing degree of integration (a growing fraction of the assets are owned by 2), until with \( \omega = 1 \) we have full integration. (The symmetric cases with \( \omega < 0 \) correspond to 1-ownership, but will not occur in any competitive equilibrium of our model, given the greater scarcity of 2’s.) Since \( \omega \) not only describes the ownership structure but also provides a scalar measure of the fraction owned by one party, we shall often refer to its (absolute) value as the degree of integration of the firm.

### 2.1.3 The Feasible Set for a Firm

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which \( q(k) = A \) for

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6There are three others. First, that the managers “swap” assets: in addition to \( \omega \), which indicates how many of 1’s assets are shifted to 2, the contract would have an additional variable \( \psi \) indicating how many of 2’s assets are shifted to 1. Second, that the managers pledge their liquidity to increase the total revenue available after the output is realized. Third that agents use external finance, i.e., sign debt contracts. We show in Appendix I that none of these possibilities can improve on contracts as we define them.

7It would, however, make the analysis more complex; in particular, we would lose the simple supply-demand analysis that we perform here. For an example of the use of third parties in the formation of firms when there are liquidity constraints see Legros and Newman (1996).

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all $k$) cannot be attained. For tasks $k \in [0,1)$, when manager 1 makes the planning decision, he will underprovide $q$ since he bears the full cost of the decision but gets only half of the revenue benefit. By contrast, if the plan is made by manager 2, that manager will overprovide $q$ since by increasing $q$, expected output increases and 2 bears no cost.

Since the profit shares are fixed, without liquidity, the only remaining way to allocate surplus is to modify the degree of integration $\omega$. Given a contract $(\omega, t)$ the two managers subsequently choose $q$ noncooperatively to maximize their corresponding objectives:

\[
\begin{align*}
  u_1(\omega, t) &= \max_{q(k) \in [0,1], k \in [0,1-\omega]} \frac{A}{2} \int_0^2 q(k) dk - \frac{1}{2} \int_0^1 q(k)^2 dk - t \\
  u_2(\omega, t) &= \max_{q(k) \in [0,1], k \in [1-\omega, 2]} \frac{A}{2} \int_0^2 q(k) dk - \frac{1}{2} \int_1^2 q(k)^2 dk + t.
\end{align*}
\]

It is straightforward to see that manager 1 will choose the same level of $q(k)$, namely $q(k) = \frac{A}{2}$ on the assets he controls, and that manager 2 will set $q(k) = 1$ for $k \in [1-\omega, 1)$ and $q(k) = \frac{A}{2}$ for $k \in [1, 2)$. Then, the payoffs associated to a contract $(\omega, t)$ are,

\[
\begin{align*}
  u_1(\omega, t) &= \frac{3}{8} A^2 - \omega \frac{(2 - A)^2}{8} - t \\
  u_2(\omega, t) &= \frac{3}{8} A^2 + \omega \frac{A(2 - A)}{4} + t
\end{align*}
\]

Because reallocating control rights does not affect the feasible set of planning
decisions, a manager gaining control of additional assets cannot be worse off.8

**Proposition 1** A manager’s payoff is nondecreasing in the fraction of assets he owns.

Observe that the total surplus generated by a contract \( \omega \), \( u_1(\omega, t) + u_2(\omega, t) \), is maximal at \( \omega = 0 \) (nonintegration) provided

\[
A < \frac{2}{3}. \quad (3)
\]

We shall focus on this case.9

When agents of types 1 and 2 have liquidity \( l_1 \) and \( l_2 \), the set of feasible payoffs they can attain via contracting is defined by (1) and (2), along with uncontingent transfers that do not exceed the initial liquidities. Specifically, the feasible payoff set is

\[
U(l_1, l_2) = \{(u_1, u_2) : \exists (\omega, t) \in [0, 1] \times [-l_2, l_1], u_i = u_i(\omega, t)\}.
\]

Given the risk neutrality of the managers, ex-ante transfers do not affect total surplus; in particular we have \( u_1(\omega, t) = u_1(\omega, 0) - t \) and \( u_2(\omega, t) = u_2(\omega, 0) + t \). Figure 1 illustrates a typical feasible set when agents have liquidity \( l_1 \) and \( l_2 \).

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8This invariance of the feasible set to transfers of control stems from the absence of investments made before \( q \) is chosen; in particular it extends to cases in which there are noncontractible investments ex post and/or in which sharing rules are flexible. See Legros-Newman (2005).

9If \( A > 2/3 \), efficiency requires giving as much control to type 2 as possible. When type 1 managers have a zero outside option the liquidity distribution of either type does not matter and there is a unique ownership structure in the industry. This is no longer the case if type 1 have a positive outside option; we come back to this case in section 3.4.
Figure 1: Feasible Set

Note that the Pareto frontier when there is no liquidity and when $v_2 \geq 3A^2/8$ is

$$v_2 = -\alpha v_1 + (\alpha + 1) \frac{3}{8} A^2,$$

where $\alpha = 2A / (2 - A) < 1$.

When managers have no liquidity, $t = 0$ and as 1’s payoff decreases, the number of assets 2 owns (weakly) increases. At the same time total surplus is decreasing; thus it is fair to say that here reallocations of ownership are used to transfer surplus, not merely to generate it. Notice as well that this mode of surplus transfer is less efficient than transferring cash; thus any liquidity that the managers have to spare will be used first to meet the surplus division demanded by the market before they transfer ownership.
2.2 Market Equilibrium

Market equilibrium is a partition of the set of agents into coalitions that share surplus on the Pareto frontier; the partition is stable in the sense that no new firm could form and strictly improve the payoffs to its members. The only coalitions that matter are singletons and pairs (which we call “firms”) consisting of one type 1 production unit \( i \in I \) and one type 2 production unit \( j \in J \). Since there is excess supply of type 1 production units, there is at least a measure \( 1 - n \) of type 1 managers who do not find a match and who therefore obtain a surplus of zero. Stability requires that no unmatched type 1 manager can bid up the surplus of a type-2 manager while getting a positive surplus. Necessary conditions for this are that all type 2 managers are matched and that they have a surplus not smaller than \( u_2 (0, 0) = \frac{3}{8} A^2 \).

As is apparent from the construction of the feasible set, when \( v_2 > u_2 (0, 0) \), payoffs on the Pareto frontier are achieved by transferring the liquidity of type 1 only, that is, the 2’s liquidity does not matter. Thus all 2’s are equally good as far as a 1 is concerned and they must therefore receive the same surplus.\(^{10}\)

This “equal treatment” property for the 2’s is an important simplification relative to most assignment models in which there is heterogeneity on both sides of the market.\(^{11}\) Identify the set of firms \( F \) with the index of the type 1 manager in the firm “firm \( i \)” indicates that the firm consists of the \( i \)-th type 1 production unit and a type 2 manager.

\(^{10}\)If in firm \((i,j)\) type 2 \( j \) has a strictly larger surplus than type 2 \( j' \) in the firm \((i',j')\), the firm \((i,j')\) could form and both \( i \) and \( j' \) could be better off since the Pareto frontier is strictly decreasing.

\(^{11}\)This is where the assumption of no third party budget-breaking comes in. Without it, treating all 2's as perfect substitutes regardless of their liquidity would not be possible.
Definition 1  An equilibrium consists of a set of firms $F \subset I$ with Lebesgue measure $n$, a surplus $v^*_2$ received by the type 2 managers, and a surplus function $v^*_1(i)$ for type 1 managers such that:

(i) (feasibility) For all $i \in F$, $(v^*_1(i), v^*_2) \in U(l_1(i), 0)$. For all $i \notin F$, $v^*_1(i) = 0$.

(ii) (stability) For all $i \in I$, for all $j \in J$, for all $(v_1, v_2) \in U(l_1(i), l_2(j))$, either $v_1 \leq v^*_1(i)$ or $v_2 \leq v^*_2$.

2.2.1 Characterizing Market Equilibrium

Since the type-2 managers have the same equilibrium payoff, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2’s. We construct the demand as follows. The amount of surplus a 1 is willing and able to transfer to a 2 depends on how much liquidity he has. The willingness to pay of type 1 is the value of the problem

$$
\max_{(\omega,t)} u_2(\omega, t)
$$

$$
u_1(\omega, 0) \geq t
$$

$$
t \in [0, l_1].
$$

In the contract $(\omega, t)$, the type 1 manager gets $u_1(\omega, t) + l_1$; the opportunity cost of the contract is to be unmatched and get $l_1$; hence the manager is willing to contract when $u_1(\omega, t) \geq 0$ which is equivalent to the condition stated since $u_1(\omega, t) = u_1(\omega, 0) - t$. Simple computations show that the
solution to this program is

\[
\text{If } l_1 \geq \frac{3}{8} A^2, (\omega, t) = \left(0, \frac{3}{8} A^2\right), \quad (5)
\]

\[
\text{If } l_1 < \frac{3}{8} A^2, (\omega, t) = \left(\frac{3A^2 - 8l_1}{(2 - A)^2}, l_1\right).
\]

The willingness of a type 1 manager to pay for matching with a type 2 manager is then

\[
W(i) = \begin{cases} 
\frac{3}{8} A^2 & \text{if } l_1(i) \geq \frac{3}{8} A^2 \\
\frac{3}{8} A^2 + \left(\frac{3}{4} A^2 - 2l_1(i)\right) \frac{A}{2-A} + l_1(i) & \text{if } l_1(i) < \frac{3}{8} A^2
\end{cases} \quad (6)
\]

Since the frontier has slope magnitude less than unity above the 45°-line, and since \(l_1(i)\) is increasing in \(i\), the willingness to pay of \(i\) is nondecreasing in \(i\). If type 2 agents must get a payoff of \(v_2\), the type 1 agents who are willing and able to pay this price is

\[
D(v_2) = 1 - \min\{i \in [0,1] : W(i) \geq v_2\}.
\]

The supply is vertical at \(n\), the measure of 2's. Equilibrium is at the intersection of the two curves: this indicates that \(n\) of the 1's are matched, as claimed above, and that the marginal 1 is receiving zero surplus.

**Proposition 2** The equilibrium set of firms is \(F = [1-n, 1]\) and the equilibrium surplus of type 2 managers is

\[
v_2^* = \min\left\{\frac{3}{4} A^2, W(\bar{l}_1)\right\},
\]
where $\bar{l}_1 = l_1 (1 - n)$.

If $\bar{l}_1 \geq 3A^2/8$, efficiency is obtained since each matched type 1 is able to pay $3A^2/8$ to the type 2 manager; note that in this case the equilibrium surplus of all type 1 managers is zero. We will consider below situations in which $\bar{l}_1 < 3A^2/8$.

In this case, the equilibrium surplus of type 2 managers is $v^*_2 = W(\bar{l}_1) < \frac{3}{4} A^2$. The marginal type 1 manager $1 - n$ has a surplus of 0, but the inframarginal type 1 managers with liquidity $l_1 > \bar{l}_1$ will be able to generate a positive surplus for themselves since they can transfer more liquidity than the marginal type 1. The surplus of an inframarginal type 1 with liquidity $l_1 > \bar{l}_1$ is $v^*_1 = W(\bar{l}_1)$.

Figure 2: The Market for Ownership
$l_1 \geq \bar{l}_1$ when the price is $v^*_2$, it the value of the problem

$$\max_{\omega} u_1 (\omega, t) + l_1$$
$$u_2 (\omega, 0) + t = v^*_2$$
$$t \leq l_1$$

The solution to this problem is $\omega(v^*_2, l_1), t(v^*_2, l_1)$ where

$$\omega(v^*_2, l_1) = 0, t(v^*_2, l_1) = v^*_2 - \frac{3}{8} A^2, \text{ if } l_1 \geq v^*_2 - \frac{3}{8} A^2 \quad (7)$$
$$\omega(v^*_2, l_1) = \frac{4(v^*_2 - \frac{3}{8} A^2 - l_1)}{A(2 - A)} , t(v^*_2, l_1) = l_1 \text{ if } l_1 \leq v^*_2 - \frac{3}{8} A^2.$$ 

In this model, there is a piece-wise linear relationship between the liquidity, the degree of integration $\omega$, the level of output and the managerial welfare. The fact that the degree of integration is a globally convex and decreasing function of liquidity and is an increasing function of the price $v^*_2$ illustrates the internal and external effects we alluded to. The effect of a higher level of liquidity may be overcome by an increase in the price $v^*_2$. Of course, the price itself reflects the liquidity and the technology available in the economy. To study the effects of shocks systematically, we must take account of the fact that $v^*_2$ itself is endogenous, which we do in the next section.

**Lemma 3** The degree of integration $\omega(v^*_2, l_1)$ is piece-wise linear: it is linear nondecreasing in $v^*_2$, nonincreasing in $l_1$ when $l_1 < v^*_2 - \frac{3}{8} A^2$, it is equal to zero when $l_1 \geq v^*_2 - \frac{3}{8} A^2$. 

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3 Comparative Statics of Market Equilibrium

In equilibrium, there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, “richer” firms are less integrated and generate greater surplus for the managers.\textsuperscript{12}

But more liquidity overall can also lead to more integration: if the marginal firm’s liquidity $v^*_2$ rises, possibly by more than an inframarginal firm’s gain in liquidity. As a result, the inframarginal firm may become more integrated, and indeed it is possible that the economy’s average level of integration may increase via this external effect.

We shall consider four types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity, changes in the technological parameter $A$, and a change in the outside option of type 1 managers.

3.1 Relative Scarcity

In order to isolate the “external effect” our first comparative statics exercise involves changes in the tightness of the supplier market, i.e., in the relative scarcities of 1’s and 2’s.

Suppose that the measure of 2’s increases, for instance from entry of downstream producers into the domestic market from overseas. Then just

\textsuperscript{12}Holmström and Milgrom (1994) emphasize a similar cross-sectional variation in organizational variables. In their model, the variation reflects differences in technology but not differences in efficiency, since all firms are surplus maximizing. Here by contrast, the variation stems from differences in liquidity and reflects differences in organizational efficiency.
as in the standard textbook analysis, we represent this by a rightward shift of the supply schedule: the price of 2’s decreases. Indeed, as \( n \) increases the liquidity of the marginal type 1 decreases since \( l_1 (1 - n) \) is decreasing with \( n \). What of course is different from the standard textbook analysis is that this change in price entails (widespread) corporate restructuring.

Let \( F(n) \) be the set of firms when there is a measure \( n \) of type 2 firms. As \( n \) increases to \( \hat{n} \), there is an equilibrium set \( F(\hat{n}) \) where \( F(n) \subset F(\hat{n}) \); that is after the increase in supply, new firms are created but we can consider that previously matched managers stay together. The surplus of all type 1 managers in firms in \( F(n) \) increases. Managers in a firm in \( F(n) \) will restructure (decrease \( \omega \)) in response to the reduction in the equilibrium value of \( v^*_2 \). The analysis is similar in the opposite direction: a decrease in the measure of 2’s leads to an increase in \( v^*_2 \). Thus, we have

**Proposition 4** In response to an increase in the measure of 2’s, the firms remaining in the market become less integrated.

It is worth remarking that if the relative scarcity changes so drastically that the 2’s become more numerous, then 1’s get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have seen, with the role of 1’s and 2’s reversed. The point is that the owners of the integrated firm gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

However, this story is heuristic: increases in demand for the type 1 most likely emanate from entry of new firms (which in turn entails a change in the
liquidity distribution among the active firms) and from increases in productivity (e.g., “skill-biased technical change”). Thus, a general analysis of the effects of changes in relative scarcity requires separate consideration of the effects of changes in liquidity and productivity; we provide this in the next two subsections.

3.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the interplay of the internal and external effects described above. The dependence of the ownership structure $\omega$ on the type-1 liquidity $l_1$ and the equilibrium surplus $v_2^*$ was summarized in Proposition 2 and Lemma 3. Equipped with this result, we can derive simple comparative statics. We focus on the aggregate degree of integration in the market,

Suppose the initial liquidity endowment is $l_1(i)$ and that the economy receives a “shock” that transforms $l_1(i)$ into $\eta(l_1(i))$; the shock function $\eta(\cdot)$ is assumed continuous and increasing. We wish to compare the degree of integration before and after the shock. Let $\omega(v_2^*, l_1)$ be the degree of integration in a firm with a type 1 manager having liquidity $l_1$ when the equilibrium surplus to 2 is $v_2^*$.

The change in the average degree of integration is

$$\int_{1-n}^{1} \omega(v_2^*(\eta(\bar{l}_1)), \eta(l_1(i))) \, di - \int_{1-n}^{1} \omega(v_2^*(\bar{l}_1), l_1(i)) \, di$$

where $\eta(\bar{l}_1)$ and $\bar{l}_1$ are the respective marginal liquidity levels and the notation $v_2^*(\cdot)$ reflects the dependence of the 2’s equilibrium surplus on the marginal
liquidity as articulated in Proposition 2.

We study some special cases that place more structure on the problem.

3.2.1 Positive Shocks to Liquidity

Suppose that the shocks \( \eta(l_1) - l_1 \) are both positive and nondecreasing in \( l_1 \).

Note that a uniform shock in which every type 1 receives the same increase to his endowment is a special case. So is a multiplicative shock in which the percentage increase to the endowment is the same for all 1’s. The shock will increase both the willingness to pay of the type 1’s, which, via the internal effect, reduces the degree of integration, but also will increase the equilibrium surplus to 2, which, via the external effect, has the opposite impact. However, it is a simple matter to demonstrate that in this case, the internal effect dominates: more liquidity implies less integration.

The change in \( v_2^* \) is the change in the willingness to pay \( W(\bar{I}_1) \) times the change in \( 1 - n \)’s liquidity. From (6), \( W'(l_1) = 1 - \alpha < 1 \): this is smaller than the liquidity increase and thus \( 1 - n \) can cover the new price and still buy back some assets; all \( i > 1 - n \) have at least as large an increase in their endowments and can therefore do the same. Of course, negative, nonincreasing shocks yield the opposite changes in surplus and organization.

**Proposition 5** Under positive, nondecreasing, shocks to the liquidity distribution of type 1 the aggregate degree of integration decreases.

To maintain this conclusion, the proviso that the shocks are monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is
higher overall degree of integration. Intuitively, if the positive shock hits only an
small neighborhood of the marginal type 1, the price $v^*_2$ will increase and
the inframarginal unshocked firms will choose to integrate more in response
to the increase in $v^*_2$. This is formally stated below. (Missing proofs in the
text are available in Appendix II.)

**Proposition 6** There exist first order stochastic dominant shifts in the dis-
tribution of type-1 liquidity that lead to more integration.

We turn now to consider other types of distributional changes.

### 3.2.2 Liquidity Heterogeneity and Degree of Integration

Suppose first that $l_1$ and $\eta \circ l_1$ have a single crossing property at $1 - n$:
$\eta(l_1(i)) < l_1(i)$ for $i < 1 - n$ and $\eta(l_1(i)) > l_1(i)$ for $i > 1 - n$. Since
all *matched* 1’s have greater liquidity and the equilibrium surplus $v^*_2$ is by
construction fixed, the surplus accruing to 2 falls in every firm, and the
economy becomes less integrated. If one supposes further that $\int_0^1 l_1(i)di =
\int_0^1 \eta(l_1(i))di$, then in fact the new liquidity distribution (which is essentially
the inverse of the liquidity endowment function) is riskier then the old one
in the sense of second order stochastic dominance (equivalently, it is more
unequal in the sense of Lorenz dominance). This is an instance in which
*increasing inequality may lower integration and raise efficiency.*

Now maintain the common marginal liquidity assumption, and denote
the inverses of the restrictions of $l_1(\cdot)$ and $\eta(l_1(\cdot))$ to $[1 - n, 1]$ as $l^*_1$ and
$(l_1 \circ \eta)^*$: these are just the conditional distributions of liquidity above $\bar{l}_1$.
Suppose that $l^*_1$ is more unequal than $(l_1 \circ \eta)^*$. Then because $\omega$ is linear in
$l_1$ and $v_2^*$ is the same for both distributions, there is less integration under the new distribution.

This suggests the opposite of the previous conclusion: *increasing inequality may raise integration and lower efficiency.* These two results are easily reconciled: while the single-crossing result refers to the distribution for the economy as a whole, the second result refers to the distribution *only among the existing firms.* From the empirical point of view the important distinction is between overall inequality and inequality among the selected sample of matched firms, which in this model at least, can work in opposite directions.

If one is interested in minimizing the degree of integration in the economy (this maximizes the surplus), it is clear that one wants the marginal liquidity as low as possible, so as to minimize the equilibrium price, and one wants to maximize the liquidity of the inframarginal firms. If $\bar{l}_1 = 0$, $v_2^* = (1 + \alpha) 3A^2/8$, and the degree of integration $\omega (v_2^*, l_1)$ for an inframarginal firm with liquidity $l_1$ is as given in (7). If $G (l_1)$ is the distribution of liquidity among the inframarginal type 1’s, the average degree of integration is

$$E [\omega] = \int \omega (v_2^*, l_1) dG (l_1).$$

Because the function $\omega (v_2^*, l_1)$ is globally convex in $l_1$, $E [\omega]$ is minimal when all firms have the same level of liquidity; more generally, there is a simple characterization of the set of distributions that minimize average integration in the economy.

**Proposition 7** Let $L$ be the average liquidity in the economy. The degree of integration is minimized when the marginal type 1 has no liquidity and when
the distribution of liquidity among the inframarginal type 1 agents is such that \( G \) has a support in \([0, \alpha A^2/8]\) when \( L < \alpha A^2/8 \) and \( G \) has support in \([\alpha A^2/8, \infty)\) when \( L > \alpha A^2/8 \).

This leaves considerable freedom to vary the distribution without changing the average level of integration. However, the degree of heterogeneity of ownership structures will be more sensitive to the liquidity distribution.

To simplify, we restrict attention to distributions of liquidity \( K \) when all type 1 are liquidity constrained and belong to firms with a positive \( \omega \). From (7) this is equivalent to having \( K(v_2^* - \frac{3}{8} A^2) = 1 \). We consider in the Appendix the general case where the measure of liquidity constrained type 1 is strictly less than \( n \), that is when there is a positive measure of type 1 in decentralized firms.

Let \( \bar{l}_K^1 \) be the marginal liquidity and \( \mu^K = \frac{1}{n} \int_{\{l_1 \geq \bar{l}_K^1 \}} l_1 dK(l_1) \) and \( \text{var}^K = \frac{1}{n} \int_{\{l_1 \geq \bar{l}_K^1 \}} (l_1 - \mu^K)^2 dK(l_1) \) be the mean and variance of liquidity of the inframarginal type 1. The linearity of the degree of integration in \( l_1 \) implies a monotonic relationship between the first two moments of the distribution of liquidity and those of the distribution of ownership when all firms choose integration.

**Lemma 8** Consider a distribution of liquidity \( K \) and suppose that all firms choose a positive degree of ownership. The mean and the variance of the degree of ownership are

\[
E[\omega] = \omega_0 + a\bar{l}_K^1 - b\mu^K \\
\text{Var}[\omega] = b^2\text{var}^K
\]
where \( \omega_0 = \frac{3A^2}{(2-A)^2} \), \( a = 4 \frac{2-3A}{A(2-A)^2} \), \( b = \frac{4}{A(2-A)} \).

Note that the mean degree of ownership depends on the liquidity of the marginal type 1: this illustrates the role of the external effect since a higher liquidity at the marginal relationship implies a higher degree of ownership of type 2 in other firms. When all firms choose a positive \( \omega \), the variance of \( \omega \) depends only on the variance of liquidity. As we show in the Appendix, when there is a positive measure of type 1 who are not liquidity constrained, the variance of ownership also depends on the marginal liquidity and the first two moments of the distribution of liquidity.

Since \( b \) is greater than 4, a 1% increase in the mean liquidity will decrease the average ownership by more than 4% and a 1% increase in the variance of liquidity will increase the variance of ownership by more than 16%. We are not aware that such a multiplier effect of liquidity has been previously noted in the literature; the magnitude of this multiplier effect is clearly an empirical question.

Equipped with this Lemma it is now easy to compare outcomes for two distributions of liquidity \( G \) and \( H \). Suppose that the marginal level of liquidity is larger at \( H \) than at \( G \) : \( G(\bar{l}_1^G) = H(\bar{l}_1^H) = 1 - n \) implies \( \bar{l}_1^G < \bar{l}_1^H \). It follows that the price of type 2 is greater with \( H \) than with \( G \); in fact from (6), \( v_2^*H = v_2^*G + (1 - \alpha)(\bar{l}_1^H - \bar{l}_1^G) \). Hence each type 1 who is inframarginal with \( H \) uses a greater degree of integration than with \( G \). However this is not incompatible with a decrease in the average degree of integration if the average liquidity increases enough: the internal effect must compensate for the external effect. This is formally stated below. The where we also compare the variances
Proposition 9 Consider two distributions of liquidity $G$ and $H$ for which all firms choose $\omega > 0$.

(i) The mean degree of ownership is lower with $H$ than with $G$ if and only if

\[
(1 - \alpha)(\bar{l}_H^1 - \bar{l}_G^1) \frac{\text{change in price}}{\text{change in average liquidity}} < \frac{\mu_H - \mu_G}{\text{change in average liquidity}}.
\]

(ii) The variance of the degree of ownership is lower in $H$ than in $G$ if and only if the variance of liquidity is lower with $H$ than with $G$.

This proposition confirms that the effect on ownership of a change in the distribution of liquidity depends on the changes in the moments of this distribution. It also implies that there is not a direct relationship between first or second order stochastic changes in the distribution of liquidity and changes in the distribution of ownership.

3.3 Productivity Shocks

The external effect outlined in the previous section offers a propagation mechanism whereby local shocks that affect only a few firms initially may nevertheless entail widespread reorganization. Empirically this implies that to explain why a particular reorganization happens, there is no need to find a smoking gun in the form of a change within that organization: instead the impetus for such change may originate elsewhere in the economy. The same logic applies to other types of shocks, most prominently among them innovating productivity shocks. These are often thought to be the basis of large-scale reorganizations such as merger waves (Jovanovic and Rousseau, 2002).
We model a (positive) productivity shock or technological innovation as an increase in $A$. We suppose the shock inheres in the type 1’s. Suppose that in the initial economy, all firms have the same technology; after a shock, a subset of them, an interval $[i_0, i_1]$, have access to a better technology (for them, $\hat{A} > A$). We restrict ourselves to considering “small” shocks in the sense that $\hat{A} < 2/3$.

Raising $A$ modifies the game that managers play given a contract $\omega$: it is clear from (1) and (2) that both managers obtain a larger surplus from a given contract. Hence the feasible set expands and the type-1’s willingness to pay also increases. What is perhaps less immediate is that there is also more transferability within the firm.

**Lemma 10** Let $A$ be the initial productivity. After a positive productivity shock,

(i) the feasible set expands.

(ii) For any $t < 3A^2/8$, the degree of integration solving $u_1(\omega, t) = 0$ increases.

(iii) there is more transferability in the sense that the slope of the frontier is steeper in the region $v_2 \geq v_1$ when $A$ increases.

**Proof.** (i) From (3), differentiating (1) and (2) with respect to $A$, shows that for any contract $(\omega, t)$, both $u_1(\omega, t)$ and $u_2(\omega, t)$ are increasing in $A$.

(ii) Use (5). (iii) The absolute value of the slope of the frontier in the region $v_2 \geq v_1$ is $\alpha = 2A/(2 - A)$ which is also increasing in $A$. [End of proof]

The willingness to pay (6) depends on the technology available to the
firm; since we assume that some firms have a different technology, we can make explicit the relationship between technology and willingness to pay:

\[ W(i; A_i) = \min \left\{ \frac{3}{4} A_i^2 + \frac{3}{8} A_i^2 + \left( \frac{3}{4} A_i^2 - 2 l_1(i) \right) \frac{A_i}{2 - A_i} + l_1(i) \right\}, \quad (8) \]

with

\[
\begin{aligned}
A_i &= A \quad \text{if } i \notin [i_0, i_1] \\
A_i &= \hat{A} \quad \text{if } i \in [i_0, i_1].
\end{aligned}
\]

Lemma 10(iii) implies that – for a fixed equilibrium surplus for 2 – a shocked firm integrates less since it is able to transfer surplus via \( \omega \) in a more efficient way. Hence when the 2s’ equilibrium surplus is fixed, positive technological shocks lead to less integration in the economy.

However, Lemma 10(ii) implies that when the marginal firm is shocked, the price will increase. Since by (iii) there is more transferability with \( \omega \), liquidity has less value: the inefficiency linked to the use of integration is lower and integration is a better substitute to liquidity transfers. This implies that type 1 agents find it more expensive, in terms of liquidity, to “buy” decision rights or reduce the degree of integration. Therefore, if the 2s’ equilibrium surplus increases, there is a force toward more integration. Unshocked firms certainly integrate more; for shocked firms, we show below that while they benefit internally from the technological shock, the countervailing effect of an increase in the 2s’ equilibrium surplus dominates. The net effect is towards more integration for all firms in the economy if the marginal firm is a shocked firm. Other results are contained in the following proposition:

**Proposition 11** (i) (Inframarginal shocks) If \( i_0 > 1 - n \) the shocked firms become less integrated and the unshocked firms remain unaffected
(ii) (Marginal shocks) If $1 - n \in (i_0, i_1)$ and $1 - n$ is still the marginal type 1 agent, the equilibrium price increases and all firms, shocked and unshocked, integrate more.

(iii) If there is a uniform shock to the technology ($i_0 = 0, i_1 = 1$) each firm integrates more.

Thus the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in “rich” firms (case (i)), only the innovating firms are affected, and they become less integrated. But innovations that occur in “poor” firms (case (ii)) may affect the whole economy: even firms that don’t possess the new technology become more integrated.

Proposition 11 (iii) emphasizes that, in contrast to reduced integration after a positive uniform liquidity shock, a uniform positive productivity shock will have the opposite effect. In this sense the external effect of productivity shocks is more powerful than that for liquidity shocks.

If $A$ inheres on type 1 with the same level of liquidity, and are differentiated, entry of type 2 production units will lead to a marginal relationship that is characterized by a smaller value of $A$ than before entry. By Lemma 10, the price going to type 2 decreases and all previous type 2 will choose a lower level of $\omega$. This argument can be generalized if type 1 are differentiated both by their liquidity endowment and their productivity $A$.

From (6), the willingness to pay of type 1 depends both on $A$ and $l_1$. The market analysis can be extended by ordering the type 1 by their willingness to pay, the marginal type 1 being the agent having a measure $n$ of type 1 with larger willingness to pay. A higher willingness to pay indicates a higher liquidity or a higher value of $A$ but not necessarily of both. When type 1 have
different productivity levels, it is actually possible that units that are more productive than the marginal unit gain control over type 2 assets. Since for a given $A$ and $l_1$, the frontier is decreasing in the payoff going to type 1 and in $\omega$, entry on the downstream market has the unambiguous effect to increase control flowing to previously active type 1. This generalizes Proposition 4 obtained when type 1 are differentiated by liquidity only.

**Proposition 12** Suppose that type 1 are differentiated by their liquidity and technology $A$. Then, entry of type 2 production units will lead to more control by original active type 1.

### 3.4 Outside Options

Until now we have assume that type 1 managers have a zero outside option. Consider now a situation where type 1 managers have a positive outside option $u_1$, that we assume to be the same for all managers.

The willingness to pay is derived as before, except that now the participation constraint of type 1 is $u_1(\omega, 0) \geq u_1 + t$. We obtain,

$$
W(l_1, u_1) = \begin{cases} 
\frac{3}{8}A^2 & \text{if } u_1 + l_1 \leq \frac{3}{8}A^2 \\
\frac{3}{8}A^2 + \left(\frac{3}{8}A^2 - 2(u_1 + l_1)\right) \frac{A}{2-A} + l_1 & \text{if } u_1 + l_1 \geq \frac{3}{8}A^2
\end{cases}
$$

As $A < 2/3$, an increase in $u_1$ will lead to a uniform decrease in integration since the external effect leads to less integration at the marginal relationship. As before, the distribution of the type 2 managers has no effect on integration.

If $A > 2/3$, the absolute value of the slope of the frontier is greater than 1.
In this case, the total surplus maximizing organization entails giving as much control as possible to one of the two managers. It follows that if the type 1 managers have a zero outside option, the market equilibrium is independent of the distribution of liquidity: indeed, the maximum surplus of type 2 is attained when $u_1(\omega, 0) = 0$, that is at $\omega^* = \frac{3A^2}{(2-A)^2}$.

Because the overall frontier is convex, there could be welfare benefits of considering random control structures (lotteries as suggested in Prescott and Townsend 1984): a contract is then a probability $p$ that manager 2 gets control over $\omega^*$ of type 1 tasks, and a probability $1 - p$ that manager 1 getting control over $\omega^*$ of type 1 tasks with the remaining probability. The resulting frontier has slope $-1$. Therefore, liquidity has no intrinsic advantage over lotteries and surplus division can be made (efficiently) by varying the probability that manager 2 gets control.

In practice however, contracts based on lotteries are difficult to implement (for instance there are significant stakes involved in trying to manipulate the lottery). When lotteries cannot be used, all types 1 managers must get the same payoff $u_1$ since transferring their liquidity to type 2 is actually less efficient that transferring them control over decisions. However, it is now the liquidity of type 2 (the short side of the market) that will be used in order to improve the efficiency of contracting. Without liquidity transfer from type 2 to type 1, type 2 must have decision rights $\omega$ less than the efficient level $\omega^*$ in order for type 1 to get a payoff of $u_1 > 0$. By transferring their liquidity ex-ante to type 1, type 2 are able to increase $\omega$ and therefore increase efficiency. The larger the liquidity of the type 2 manager, the larger the control he will obtain in the equilibrium contract. This is illustrated in figure 3, where we
have indicated two type 1 managers with different liquidity levels.\textsuperscript{13} When $A > 2/3$ it is the short side that “buys control” from the long side. Note that as $u_1$ increases, type 2 will have to settle for a lower degree of integration.

\begin{center}
\includegraphics[width=0.5\textwidth]{figure3.png}
\end{center}

Figure 3: Frontier when $A > 2/3$ and role liquidity of type 2

4 Illustrations

4.1 Entry: Supplier Relation in the U.S. Auto Industry

U.S. giants like Chrystler had until the 80s an arm length relationship with their suppliers, often designing their products without the input from suppliers. By contrast Japanese automotive firms embraced a partnership model with suppliers.

\textsuperscript{13}If $A$ becomes larger than $\sqrt{3} - 1$, at $\omega = 1$, we have $u_1(1, 0) > 0$. In this case, the type 1 liquidity will matter again when $u_1 < u_1(1, 0)$. 

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Following a wave of FDI by Japanese firms in the US, Chrystler started reorganizing its relationship with suppliers, first by improving quality control, just-in-time procedures and then by involving suppliers in product and process development, cutting along the way links with half of its previous suppliers. This change in supplier relation at Chrystler has been linked (see for instance Dyer 1996) to the threat posed by the entry of Japanese firms, their dominance on the market for small cars (which was the fast growing segment given the successive oil crises), and the comparatively greater quality of Japanese cars seemingly due to the close cooperation with suppliers for design and development.

In terms of our model, interpret type 2 as the car manufacturers, type 1 as the suppliers, $\omega$ as the degree of control that car manufacturers imposed on suppliers. A “partnership” like arrangement in the spirit of keiretsus is characterized by $\omega$ close to zero, e.g., the supplier have a voice in the design while an arm length relationship is characterized by larger values of $\omega$, e.g., Chrystler imposes its designs on suppliers. The entry of Japanese producers in the US created competition for supplier relationships. The evidence suggests that Chryster retained relationships with some suppliers. Consistent with our model, (Proposition 12) entry should lead to more control flowing to those suppliers, that is to lower values of $\omega$.

### 4.2 Technological Change: Trucking

In recent work, Baker and Hubbard (2004) document how the introduction of on-board computers (OBC) lead to a decrease of driver ownership of trucks. There has been two successive technologies for OBCs: the first gave (very
much like the “black boxes” in airplanes) “ex-post” information about route, time, stops taken by the driver, the second wave of OBCs provided this information in real time. The second generation of OBC yields the same monitoring benefits as the first generation but also allows a better contracting and flexibility in dispatching: it becomes possible for instance to have at a given time the localizations of all the trucks in a fleet and reroute some of them more efficiently if need be. The second generation is therefore akin to an increase in productivity of the relationship and assuming that there is a complementarity between benefits for dispatching and for monitoring, we can model the benefit of the second generation by a multiplicative shock in our model.14

Proposition 11(ii) then implies that all carriers, whether or not they use OBCs, should integrate more, that is carriers should get more control over drivers, which is compatible with carriers using less driver owned trucks. Note that this effect is entirely due to the external effect; indeed, if one were looking only at a single carrier, taking as given the rest of the industry, that carrier should integrate less after adopting OBCs, that is should rely more on driver owned trucks. This result does not depend on our assumption that $A < 2/3$ since from section 3.4, if OBCs lead to $A > 2/3$, the equilibrium will also involve control by the type 2.

14 An alternative scenario is that the OBC has made decisions on a subset of tasks contractible, hence, while the transfer of control for some tasks to type 2 created a large externality for type 1 before, contractibility now makes decisions on these tasks more efficient. We should observe indeed that the decisions on these tasks be contracted upon, leading necessarily to decisions going in the direction preferred by type 2.
4.3 Outside Options: Trucking

In 2006, the trucking industry experienced a significant labor shortage. The probable causes of this shortage are the compensation differential between trucking and construction as well as a change in regulation (Urbina 2006):

- In terms of wages, in 2004, the average annual pay for a truck driver was $34,920 compared with $37,890 for a construction worker. In terms of working conditions, long-haul drivers are often away from their family. Both of these make construction jobs an attractive outside option for truck drivers.

- A 2004 federal regulation put a limit on the driving time between shifts, and has increased the resting time between shifts, reducing the effective productivity.

There was therefore both an increase in the demand for truckers and an increase in outside options. Both effects lead in our model trucking companies to get less control over the truck drivers. This is in fact the only possibility when \( A < \frac{2}{3} \) since the liquidity of type 2 plays no role. When \( A > \frac{2}{3} \), as we have seen in section 3.4, an increase in outside option of type 1 will first be met by a transfer of liquidity of type 2 (akin to a wage increase) and if liquidity binds by a transfer of control to type 1.\(^{15}\) It seems that this pattern finds some echo in the trucking industry after 2004: the wage gap with construction workers has been reduced and more control has been given

\(^{15}\)If we consider also sharing rules of output ex-post as in Legros and Newman (2007), even when \( A < \frac{2}{3} \), we may observe an increase both in the share of output and the control flowing to the truck drivers.
to the workers (who could for instance take their family with them on long hauls).

5 Discussion

If one asks the question “who gets organizational power in a market economy?,” one is tempted to answer “to the scarce goes the power.” There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) which ascribes power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore, 1990; Stole and Zweibel, 1996). Our results suggest that organizational power may emanate from scarcity outside the organization, i.e., from market power. But this result has to be qualified somewhat: Proposition 6 suggests that having more liquidity may actually cause one to lose power, via what we have called the external effect of shocks to fundamentals. Similarly, the possessors of a new technology, if they are inframarginal, will gain ownership (Proposition 11 (i)), but if they are marginal may lose it. This is evidence of the importance of market effects for the allocation of power inside firms and more generally of their importance for the study of organizations.

We now discuss some other implications of the model.

5.1 Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or the
technology available to firms. One can easily extend the model to allow for liquidity that yields a positive return through the period of production. Because liquidity in this model is used only as a means of surplus transfer, and not as a means to purchase new assets, the effects of this can be somewhat surprising. Raising this interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before: formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and by Proposition 5, firms will integrate less if the interest rate increases, and will integrate more if the interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity, by lowering interest rates, may constitute a force for integration above and beyond that suggested by the example in Proposition 6. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward; whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

5.2 Product Market

If we imagine all the firms sell to a competitive product market, then the selling price inheres in \( R \), which we have thus far viewed as exogenous (for instance the supplier market is contained in a small open economy, with prices determined in the world market). But if instead price is determined endogenously in the product market, then shocks to product demand will change the price, which has the effect of changing \( A \) for all firms. Suppose the
price increases. Then from Proposition 11(iii) in the analysis of productivity shocks, all firms become more integrated.

Next, notice that expected output is proportional to $A$ for nonintegrated firms and proportional to $A + \omega (1 - A/2)$ for integrated ones. Integrated firms produce more than nonintegrated ones, and since from Proposition 11 the aggregate $\omega$ increases with $A$, aggregate output rises in response to an increase in $A$. Thus, if product price rises, so does output, and we conclude that the product supply curve is upward sloping. An increase in consumer demand therefore raises equilibrium price: increasing demand results in greater integration.

What is more, the product market price effect now means that more local shocks will result in widespread reorganization: more than just the very poorest firms in the economy may be “marginal.” To see this, suppose a number of perfectly nonintegrated firms innovate. With fixed prices, these firms produce more output, but nothing further happens. With endogenous prices, the increased output in the first instance lowers product price; all other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of “amplification” effect on organizational restructuring.\footnote{Of course the effect is self-limiting because as they become less integrated, they lower their output, causing the price to go up again. As shown in Legros-Newman (2004), these product market effects can be more pronounced in models that rely on somewhat different trade-offs in their basic organizational model than the one considered here.}

Previous work has analyzed how the intensity of product market competition may act as an incentive tool for managers.\footnote{See Hart (1983), Scharfstein (1988) and Schmidt (1997).} In this literature the set of firms and their internal organization are exogenous. Here we wish to empha-
size a causal relation in the opposite direction that becomes apparent once organization is allowed to be endogenous: organizations may affect product market prices, even when there is perfect competition. As discussed in Legros and Newman (2004), the fact that the product market – even a competitive one – can be affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

6 Appendix I: Contracting

We have defined contracts by $\langle \omega, t \rangle$ and equal sharing of the output ex-post. This definition could be restrictive because it ignores the following four potential extensions.

- **Contingent shares.** A contract could specify state contingent revenues $x_i(R), x_i(0)$ to $i = 1, 2$.

- **Debt contract.** Type 1 borrows $B$ from a financial institution in exchange for a repayment of $D$ after output is realized.

- **Ex-post transfers of liquidity.** The total liquidity available in the firm is $L = l_1 + l_2$. This liquidity can be transferred either ex-ante or added to the revenue of the firm ex-post.

- **Asset swapping.** This is a means of effectively committing the managers to high levels of $q$. This commitment is only worthwhile if productivity is sufficiently high relative to costs, which will not be the case given
our parametric case. If assets are to be swapped, we can characterize
the situation via two ownership parameters ψ and ω: manager 1 owns
\( k \in [0, 1 - \omega) \) and \( k \in [2 - \psi, 2) \), and 2 owns the other assets.

We show that our definition of contracting is without loss of generality
by introducing into the contracting model described in the text a moral
hazard element. The incentive compatibility condition associated to this
moral hazard problem will restrict the marginal revenue \( x_i (R) - x_i (0) \) to be
equal to \( R/2 \) for each agent. The result will then follows.

A manager has the opportunity to divert revenue \( R \) in the high state
by choosing an effort \( e \in [0, 1] \) : if the state is high, with probability \( e \)
the perceived output in the firm will be \( R \) while with probability \( 1 - e \) the
perceived output in the firm is 0, in which case the manager diverts a share
\( cR \) and \( (1 - c) R \) is lost; if the state is low, the perceived output in the firm
will be 0 independently of \( e \). Only one manager has the opportunity to divert
(the identity of that manager being chosen by nature).

The ex-post revenue of the firm consists of two components: the risky
component with realizations 0 and \( R \) and a non-risky component denoted by
\( T \), typically the amount of ex-ante liquidity than is pledged (in an escrow
account) to the firm. By choosing \( e \), the manager can “hide” \( R \) but not \( T \).

Let \( x_i (R) \) and \( x_i (0) \) be the revenues to the manager if the perceived
realization of the risky component is \( R \) and 0 respectively. Then, with \( e = 1 \),
the expected revenue to the manager is \( px_i (R) + (1 - p) x_i (0) \). With \( e = 0 \),
the expected revenue is \( p(cR + x_i (0)) + (1 - p) x_i (0) \). Hence \( e = 1 \) is optimal
when \( x_i (R) \geq cR + x_i (0) \), or \( x_i (R) - x_i (0) \geq cR \). Clearly if \( c > 1/2 \), both
incentive compatibility constraints cannot hold. By choosing \( c = 1/2 \), we
have
\[ x_i(R) - x_i(0) = R/2 \]  \hspace{1cm} (9)

as claimed. If \( c < 1/2 \), there is scope for unequal marginal revenues for the two agents, but it still remains true that there is no loss in assuming that \( T = 0 \) and that debt contracts are weakly dominated by non-debt contracts.

Suppose that (9) holds. A contract is \( ((\omega, \psi), (B, D), (x_1, x_2) (t_1, t_2)) \), where we assume without loss of generality that only agent 1 engages in a debt contract. Let \( x_i^* \) be the state contingent revenue equal to \( R/2 \) in state \( R \) and 0 in state 0. We want to show that there exists a contract \( ((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\hat{t}_1, L - \hat{t}_1)) \) that leads to payoffs that are weakly greater for both managers. We establish this result sequentially: first by showing that \( ((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2)) \) is weakly dominated by the contract \( ((\omega, \psi), (0, 0), (x_1^*, x_2^*), (t_1 + x_1(0), t_2 + x_2(0))) \), where neither debt nor ex-post transfers of liquidity are used, second by showing that this contract is dominated by a contract in which only part of the assets of type 1 are reassigned to type 2 \( ((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\hat{t}, L - \hat{t})) \).

Step 1. In a contract \( ((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2)) \), feasibility requires that \( t_1 + t_2 \leq L + B \) and \( t_i \geq 0, i = 1, 2 \). We write \( t = t_1 + t_2 \) the total liquidity ex-ante and \( T = L + B - t \) the liquidity that is pledged to the firm. Ex-post total revenues are then \( T \) and \( T + R \). Managers get state contingent revenues \( x_i(0), x_i(R) \) satisfying budget balancing and limited liability: \( x_1(0) + x_2(0) = T, x_1(R) + x_2(R) = T + R, x_i(0) \geq 0, x_i(R) \geq 0 \).

If there is a debt contract, manager 1 has to repay \( \min \{D, x_1(0)\} \) in state 0 and \( \min \{D, x_1(R)\} \) in state \( R \). Since by (9), we need \( x_2(R) - x_2(0) = R/2 \),
we have \( x_1(R) - x_1(0) = R/2 \), however since manager 1 has to repay the debt, his effective marginal compensation is

\[
x_1(R) - x_1(0) - \min \{ D, x_1(R) \} - \min \{ D, x_1(0) \}.
\]

This is consistent with (9) only if \( \min \{ D, x_1(R) \} = \min \{ D, x_1(0) \} \), or if \( D \leq x_1(0) \). In this case, debt is not risky; the creditor makes a non-negative profit only if \( D \geq B \), but then we need \( x_1(0) \geq B \) and therefore \( x_2(0) \leq L + B - t = B - t \). It follows that the initial contract \( ((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2)) \) is weakly dominated by the contract \( ((\omega, \psi), (0, 0), (x_1^*, x_2^*) (t_1 + x_1(0), t_2 + x_2(0))) \). Since \( \sum_{i=1,2} (t_i + x_i(0)) = L \), there is no liquidity transferred ex-post.

Step 2. Finally we show that swapping of assets is dominated by no swapping of assets

Consider a contract \( ((\omega, \psi), (0, 0), (x_1^*, x_2^*) , (t, L - t)) \) consisting of a swap of assets and ex-ante transfers; we denote such contracts by \( ((\omega, \psi), t) \). We have the following Nash equilibrium payoffs:

\[
u_1(\omega, \psi, t) = \frac{A}{2} \left( (2 - \omega - \psi) \frac{A}{2} + \omega + \psi \right) - \frac{1}{2} \left( \omega + (1 - \omega) \frac{A^2}{4} \right) - t
\]

\[
u_2(\omega, \psi, t) = \frac{A}{2} \left( (2 - \omega - \psi) \frac{A}{2} + \omega + \psi \right) - \frac{1}{2} \left( \psi + (1 - \psi) \frac{A^2}{4} \right) + t
\]

Suppose without loss of generality that \( t > 0 \) and that \( u_2(\omega, \psi, t) - t > u_1(\omega, \psi, t) + t \); then we must have \( \omega > \psi \).
Let $\omega^0 = \omega - \psi A/(1-A/2)$; since $A/(1-A/2) < 1$ and $\omega > \psi$, $\omega^0 > 0$. Then, $u_1(\omega^0, 0, t) = u_1(\omega, \psi, t)$ while $u_2(\omega^0, 0, t) - u_2(\omega, \psi, t) = \psi(2 - A - A^2)/4 > 0$ since $A < 1$. By continuity there exists $\omega < \omega^0$ such that the contract $((\omega, 0), t)$ strictly Pareto dominates the contract $((\omega, \psi), t)$. If $u_2(\omega, \psi, t) - t < u_1(\omega, \psi, t) + t$, a similar argument applies by decreasing the value of $\psi$ appropriately.

7 Appendix II: Proofs

7.1 Proof of Proposition 6

It is enough to provide an example. Consider the liquidity distribution $l_1(i) = \varepsilon i$ where $i \in [0, 1]$ and $\varepsilon < \frac{3}{8} A^2$. Suppose that $n = 1$, that is that the marginal liquidity is 0 and the maximal liquidity is $\varepsilon$. The frontier when there is no liquidity is linear and can be written $v_2 = -\alpha v_1 + v^0$, where $\alpha = \frac{2A}{2-A} \in (0, 1)$ and $v^0 = (\alpha + 1)\frac{3}{8} A^2$. The equilibrium surplus is $v_2^*(0) = v^0$ and if type 1 has liquidity $l_1$, the degree of integration is given by (7).

Define $\eta(l_1)$ by

$$\eta(l_1) = \begin{cases} 
\delta, & \text{if } l_1 \leq \delta \\
 l_1 & \text{if } l_1 \geq \delta.
\end{cases}$$

where we choose $\delta < \varepsilon$. $\eta(l_1)$ is increasing and continuous. The marginal liquidity is now $\eta(0) = \delta$ and the new equilibrium surplus is $v_2^*(\delta) = v^0 + (1-\alpha)\delta$.

Firms with $i \geq \delta/\varepsilon$ have the same liquidity as before but a higher equi-
librium surplus accrues to type 2, and therefore a lower equilibrium surplus accrues to type 1, that is they choose a higher level of integration. From (7), if $\varepsilon_i > \delta$,

$$\omega(v^*_2(\delta), \varepsilon_i) - \omega(v^0, \varepsilon i) = \frac{(1 - \alpha) \delta}{A(2 - A)}$$

By contrast, all firms with $\varepsilon_i < \delta$ are marginal after the shock and have the same degree of integration

$$\omega(v^*_2(\delta), \delta) = \frac{3.4^2 - 8\delta}{(2 - A)^2}$$

while before the shock

$$\omega(v^0, \varepsilon i) = \frac{4v^0 - 3\delta^2 - \varepsilon i}{A(2 - A)}.$$  

There exists $i(\delta)$ such that $\omega(v^*_2(\delta), \delta) - \omega(v^0, \varepsilon i) > 0$ when $i < i(\delta)$ and $\omega(v^*_2(\delta), \delta) - \omega(v^0, \varepsilon i) < 0$ when $i > i(\delta)$. It is immediate that $i(\delta)$ is a strictly increasing function of $\delta$.

Let

$$\Delta(i) = \begin{cases} 
\omega(v^*_2(\delta), \delta) - \omega(v^0, \varepsilon i) & \text{if } i \leq \delta/\varepsilon \\
\frac{(1 - \alpha) \delta}{A(2 - A)} & \text{if } i > \delta/\varepsilon.
\end{cases}$$

In the aggregate, the change in the average degree of integration is

$$\Delta(\delta) = \int_{0}^{i(\delta)} \frac{\Delta(i)}{\langle i(\delta) \rangle} + \int_{i(\delta)}^{\delta/\varepsilon} \frac{\Delta(i)}{\langle i(\delta) \rangle} + \left(1 - \frac{\delta}{\varepsilon}\right) \frac{(1 - \alpha) \delta}{A(2 - A)}.$$  

As $\delta \to 0$, $i(\delta) \to 0$; therefore $\Delta(\delta) > 0$ for $\delta$ small enough since $\Delta'(0) > 0$, proving that integration increases on average.
7.2 Proof of Proposition 7

If \( \bar{l}_1 = 0 \), note that \( v^*_2 = W(0) = (1 + \alpha) \frac{3}{8} A^2 \) and from (7), \( \omega(v^*_2, l_1) \) has a kink at \( l_1 = \alpha \frac{3}{8} A^2 \): for lower values the degree of integration is linear and for larger values it is zero; hence \( \omega(v^*_2, l_1) \) is indeed globally convex in \( l_1 \). Suppose that \( L < \alpha 3A^2/8 \). Let \( L_0 = \int_{l_1 < \alpha 3A^2/8} l_1 dG(l_1) \) and \( L_1 = \int_{l_1 > \alpha 3A^2/8} l_1 dG(l_1) \). Note that by (7), \( \int_{l_1 < \alpha 3A^2/8} \omega(v^*_2, l_1) dG(l_1) = \omega(v^*_2, L_0) \) and that \( \int_{l_1 > \alpha 3A^2/8} \omega(v^*_2, l_1) dG(l_1) = \omega(v^*_2, L_1) \). Hence, \( E\omega = G(\alpha 3A^2/8) \omega(v^*_2, L_0) + (1 - G(\alpha 3A^2/8)) \omega(v^*_2, L_1) \). However since \( \omega(v^*_2, L_1) = 0 \), and since \( \omega \) is globally convex, \( L = G(\alpha 3A^2/8) L_0 + (1 - G(\alpha 3A^2/8)) L_1 \) implies that \( E\omega > \omega(v^*_2, L) \). This shows that \( L_1 = 0 \) and that the support of \( G \) is contained in \([0, \alpha 3A^2/8]\). The same argument applies when \( L > \alpha 3A^2/8 \).

7.3 Proof of Lemma 8

We know from (7) and Proposition 2 that for a given distribution \( K \) the degree of integration is positive when \( l \) belongs to \( [\bar{l}^K_1, v^*_K - \frac{3}{8} A^2] \). In this case we can write \( \omega(v^*_K, l_1) = \omega_0 + a \bar{l}^K_1 - bl_1 \) where \( \omega_0 = \frac{3}{8} A^2 \), \( a = 4 \frac{2-3A}{A(2-A)^2} \), \( b = \frac{4}{A(2-A)} \), note that \( a/b = 1 - \alpha \). Let \( \pi^K = K(v^*_K - \frac{3}{8} A^2) - K(\bar{l}^K_1) \) be the measure of firms choosing a positive \( \omega \).

(i) Let \( \mu^K = \frac{1}{\pi^K} \int_{l_1}^{v^*_K - \frac{3}{8} A^2} l_1 dK(l_1) \) be the conditional mean among firms
choosing a positive $\omega$. We have,

$$E[\omega] = \int \omega(v^*_2, l_1) \frac{dK(l_1)}{n}$$
$$= \frac{1}{n} \int_{l_1}^{v^*_2 - \frac{3}{8} A^2} (\omega_0 + al^*_1 - bl_1) dK(l_1)$$
$$= \frac{\pi^K}{n} (\omega_0 + al^*_1) - b \int_{l_1}^{v^*_2 - \frac{3}{8} A^2} l_1 \frac{dK(l_1)}{n}$$
$$= \frac{\pi^K}{n} (\omega_0 + al^*_1 - b\mu^K).$$

when all firms choose $\omega > 0$, $\pi^K = n$, leading to the expression in the Lemma.

(ii) Let $\text{var}^K = \int_{l_1}^{v^*_2 - \frac{3}{8} A^2} (l_1 - \mu^K)^2 \frac{dK(l_1)}{n}$ be the variance of liquidity among the liquidity constrained type 1, that is those that will be in firms with $\omega > 0$. Direct computations show that the variance of ownership is

$$\text{Var}[\omega] = \int \left[ \omega(v^*_2, l_1) - E[\omega] \right]^2 \frac{dK(l_1)}{n}$$
$$= \int_{l_1}^{v^*_2 - \frac{3}{8} A^2} \left[ \omega(v^*_2, l_1) \right]^2 \frac{dK(l_1)}{n} - E[\omega]^2$$
$$= \frac{\pi}{n} \left(1 - \frac{\pi}{n}\right) (\omega_0 + al^*_1) (\omega_0 + al^*_1 - 2b\mu^K)$$
$$+ \frac{\pi b^2}{n} \left( \int_{l_1}^{v^*_2 - \frac{3}{8} A^2} l^2 \frac{dK(l_1)}{\pi^K} - \frac{\pi}{n} (\mu^K)^2 \right)$$
$$= \frac{\pi}{n} \left(1 - \frac{\pi}{n}\right) (\omega_0 + al^*_1) (E[\omega] - b\mu^K)$$
$$+ \frac{\pi b^2}{n} \left( \text{var}^K - (1 - \frac{\pi}{n})(\mu^K)^2 \right).$$

Since the degree of ownership $\omega$ is positive only if the type 1 is liquidity constrained ($l_1 < v^*_2 - \frac{3}{8} A^2$), the degree of heterogeneity of ownership will
depend on the distribution among these constrained type 1 agents. When all type 1 are constrained, \( \pi^K = n \) and we have as in the Lemma, \( \text{Var}[\omega] = b^2 \left( \int_{l_1}^{v_2^{K}} \frac{2}{b} A^2 l^2 dK(l_1) - (\mu^K)^2 \right) = b^2 \nu \text{Var} K. \)

7.4 Proof of Proposition 9

(i) It is immediate from Lemma 8 that if \( \pi^G = \pi^H, \int \omega (v^G_2, l_1) dH(l_1) < \int \omega (v^G_2, l_1) dG(l_1) \) if and only if \( a \bar{l}_1 - b \mu^H < a \bar{l}_1 - b \mu^G \) or if \( (1 - \alpha) \left( \bar{l}_1^H - \bar{l}_1^G \right) < \mu^H - \mu \) since \( \frac{a}{b} = 1 - \alpha. \)

(ii) If \( \pi^G = \pi^H = n, \) the result is immediate from Lemma 8(ii).

7.5 Proof of Proposition 11

Let

\[ \rho : [0, 1] \rightarrow [0, 1] \]

\[ \rho (i) \geq \rho ( \bar{i} ) \Leftrightarrow W (i) \geq W ( \bar{i} ). \]

be a reordering of the indexes of type 1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type 1 agent is \( i_\pi \) such that the Lebesgue measure of the set \( \{ i : W (i) \geq W (i_\pi) \} \) is \( n \) and the set of equilibrium firms is \( F = \{ i : \pi (i) \geq \pi (i_\pi) \}. \)

Let \( v^*_2 (A) \) be the equilibrium price in the initial situation and \( v^*_2 \left( \bar{A} \right) \) the equilibrium price after the shock to the technology available to agents in \([i_0, i_1].\)

Remark 1 Proposition 11 is concerned with situations where \( i_\mu = 1 - n. \)
However, note that the marginal type may not be $1 - n$. This can happen in two cases.

Case 1: A first possibility is $i_1 < 1 - n$, that is, shocked firms were not matched in the initial economy but because $W(i_1) > v_2^*(A)$, some of these firms will be matched. In this case, the set of “new entrants” are firms with $i \in [i_\pi, i_1]$ while the set of “old firms” are those with index $i \geq k$, where $k \geq 1 - n$ satisfies $i_1 - i_\pi = k - (1 - n)$ (hence firms $i \in [i_\pi, i_1]$ “replace” firms $i \in [1 - n, k]$). Since $W(i_\pi) > v_2^*(A)$, the degree of integration in old firms increases. For new firms, the question is whether the increase in price $W(i_\pi) - W(1 - n)$ is large enough to overcome the internal effect of technology shock pushing towards less integration.

Case 2: Another possibility is $1 - n \in (i_0, i_1)$ and $W(1 - n) > \lim_{\varepsilon \to 0} W(i_1 + \varepsilon)$. Then there exists $k > i_1$ such that $W(k) = W(1 - n)$, and either $i_\pi \in (i_1, k]$ or $i_\pi \in [i_0, 1 - n)$. In either case, if $l_1(i_\pi)$ is low enough, the increase in equilibrium surplus to the 2 may be small enough that the internal effect dominates and shocked firms integrate less.

(i) (Infra marginal shocks) If $i_0 > 1 - n$, then $i_\pi = 1 - n$ and $W(i_\pi) = v_2^*(A)$, then the shocked firms become less integrated while the unshocked firms remain unaffected.

This is a direct consequence of Lemma 10

(ii) (Marginal shocks) If $1 - n \in (i_0, i_1)$ is still the marginal type 1, the equilibrium price increases and all firms, shocked and unshocked, integrate more.

Note that $1 - n$ is still the marginal type if and only if $W(1 - n) \leq$
\[ \lim_{\varepsilon \downarrow 0} W (i_1 + \varepsilon) \], for in this case, all agents \( i > 1 - n \) have higher willingness to pay than \( 1 - n \).

From (8), \( v^*_2 (A) = W (1 - n) \) is increasing in \( A \), hence \( v^*_2 (\hat{A}) > v^*_2 (A) \) and it follows that all unshocked firms \([i_1, 1] \) integrate more.

If the firm \( 1 - n \) did not integrate before the shock (that is chose \( \omega = 0 \)), then all \( i > 1 - n \) firms also chose not to integrate since \( \omega \) is decreasing in the liquidity of type 1. Hence, it is immediate that an increase in \( A \) can only lead to more integration.

Consider now the case where firm \( 1 - n \) integrated before, that is chose a contract with \( \omega > 0 \). If \( i_1 \) chose initially a contract \( \omega = 0 \), there exists \( k \in (1 - n, i_1) \) such that all firms with \( i < k \) integrate \( (\omega > 0) \) and all firms with \( i \geq k \) do not integrate; firms with \( i \geq k \) will necessarily integrate more after the shock. We have \( v^*_2 (A) = W (1 - n; A), v^*_2 (\hat{A}) = W (1 - n; \hat{A}) \), and from (7), (8), for all shocked firms \( i \in [1 - n, k) \), the difference in the degree of integration after and before the shock is

\[
\frac{3\hat{A}^2 - 4\bar{l}_1}{(2 - \hat{A})^2} - \frac{3A^2 - 4\bar{l}_1}{(2 - A)^2} > 0.
\]

(here \( \bar{l}_1 = l_1 (1 - n) \)) and all firms integrate more as claimed.

(iii) If \( i_0 = 0 \) and \( i_1 = 1 \), the arguments for (ii) apply since \( 1 - n \) is still the marginal type 1 manager.
8 References

References


